

EDEXCEL IGCSE MATHS (4MA1) — HIGHER EXTENSION BOOKLET

Edexcel IGCSE (4MA1) Higher Extension

(Higher Tier · Grades 6–9 Extension)

Complete IGCSE Extension Pack: Cross-Reference Guide, Tutorials, Worked Examples & Practice

- 1-Page Syllabus Cross-Reference Guide (how to pair with Grades 4–9 Booklets)
- All 7 Extra IGCSE Topics with Concept Notes & Worked Examples
- 42 Exam-Style Practice Questions with full working-out spaces
- Complete Answer Key Appendix at the back

Student Name: _____

School / Class: _____

Target Grade: _____



SCAN FOR INTERACTIVE IGCSE PRACTICE & WORKED SOLUTIONS

BreakingDownMaths.com

Select "Edexcel IGCSE (4MA1)" on the Home Page

Table of Contents

Complete PEARSON EDEXCEL IGCSE (4MA1) extension syllabus overview and cross-reference guide.

Syllabus Cross-Reference Guide (What to Skip in Main GCSE Booklets)	Page 3
---	--------

PROBABILITY

Formal Set-Builder & Subset Notation	Page 4
--------------------------------------	--------

ALGEBRA

Arithmetic Sequences & Series	Page 8	Functions: Domain & Range	Page 12
Differentiation & Turning Points	Page 27		

GEOMETRY AND MEASURES

Intersecting Chords, Secants & Tangents	Page 16	Magnitude (Modulus) of a Vector	Page 23
---	---------	---------------------------------	---------

RATIO, PROPORTION AND RATES OF CHANGE

Negative Powers & Calculus Kinematics	Page 32
---------------------------------------	---------

Appendix: Answers & Mark Schemes	Page 36
----------------------------------	---------

CROSS-REF CHECKLIST — SYLLABUS BRIDGE

Edexcel IGCSE (4MA1) Cross-Reference Guide

Roughly 90% of Pearson Edexcel International GCSE Mathematics A (4MA1) is identical to UK GCSE (9–1) and is covered in our main Grades 1–9 Booklets. Use this slim Higher Extension Booklet alongside Grades 4–9 to master all 7 extra 4MA1 Higher topics — and skip the 8 UK-GCSE-only topics below.

1. UK-GCSE-ONLY TOPICS TO SKIP IN THE MAIN GRADES 3–9 BOOKLETS

Topic in Main Booklet	Booklet	IGCSE Guidance
3D Solids, Plans & Elevations	Grade 3	Keep 3D faces/edges/vertices & nets; skip front/side/plan elevations
Frequency Trees	Grade 3	UK GCSE only — not examined on Edexcel 4MA1
Product Rule for Counting	Grade 6	UK GCSE only — not examined on Edexcel 4MA1
Box Plots	Grade 6	Skip box plots (keep Cumulative Frequency & IQR!)
Stratified Sampling	Grade 6	UK GCSE only — not examined on Edexcel 4MA1
Capture-Recapture	Grade 6	UK GCSE only — not examined on Edexcel 4MA1
Iteration	Grade 8	Fixed-point iteration $x_{n+1} = g(x_n)$ is not on 4MA1
Equation of a Circle	Grade 9	$x^2 + y^2 = r^2$ tangent equations are not on 4MA1

2. YOUR 7 EXTRA IGCSE TOPICS MASTERED IN THIS BOOKLET

IGCSE Extension Topic	Syllabus Area	Notes	Practice
1. Formal Set-Builder & Subset Notation	Probability	☐	☐
2. Arithmetic Sequences & Series	Algebra	☐	☐
3. Functions: Domain & Range	Algebra	☐	☐
4. Intersecting Chords, Secants & Tangents	Geometry and Measures	☐	☐
5. Magnitude (Modulus) of a Vector	Geometry and Measures	☐	☐
6. Differentiation & Turning Points	Algebra	☐	☐
7. Negative Powers & Calculus Kinematics	Ratio, Proportion and Rates of Change	☐	☐

3. BONUS 4MA1 EXAM-FORMAT & TIME-SAVING TIPS

- **100% Calculator Exam (Papers 1H & 2H):** Both 4MA1 papers allow a scientific calculator — no need to memorise exact trig values ($\sin 30^\circ$, $\cos 45^\circ$, $\tan 60^\circ$) or drill long non-calculator arithmetic.
- **Sequences & Speed–Time Graphs:** Skip quadratic n th term ($an^2 + bn + c$), geometric progressions, and curved velocity–time trapezia — 4MA1 tests **Arithmetic Series** (u_n, S_n) (Topic 2) and **Calculus Kinematics** (Topic 7) instead.
- **Graphical Equations** ($f(x) = mx + c$): To solve e.g. $x^2 - 5x - 1 = 0$ from a plotted curve $y = x^2 - 4x + 1$, rearrange to $x^2 - 4x + 1 = x + 2$ and draw the straight line $y = x + 2$.

Formal Set-Builder & Subset Notation

1 Elements, Cardinality $n(A)$ & Empty Sets

CORE CONCEPT & METHOD

IGCSE formal set language uses compact symbols:

- $x \in A / x \notin A$: x is an element of A / is not an element of A .
- $n(A)$: The **number of elements** in set A (its cardinality).
- $\emptyset$ (or $\{\}$): The **empty set** containing zero elements, so $n(\emptyset) = 0$.
- **Inclusion–Exclusion**: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

MODEL WORKED EXAMPLE

EXAMPLE 1.1

Given $n(\mathcal{E}) = 30$, $n(A) = 14$, $n(B) = 11$ and $n(A \cap B) = 4$, find $n(A \cup B)$ and $n((A \cup B)')$.

STEP-BY-STEP SOLUTION:

$$n(A \cup B) = 14 + 11 - 4 = 21.$$

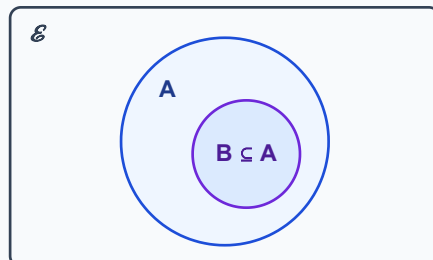
$$n((A \cup B)') = 30 - 21 = 9.$$

21; 9

2 Subsets ($\subseteq$) & Combined Operations

CORE CONCEPT & METHOD

$B \subseteq A$ means B is a **subset** of A : every element of B also belongs to A .



- When $B \subseteq A$, we automatically have $A \cap B = B$ and $A \cup B = A$.
- A finite set with k elements has exactly 2^k subsets (including $\emptyset$ and the set itself).

MODEL WORKED EXAMPLE

EXAMPLE 2.1

Given $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4\}$, list $A' \cap B$.

STEP-BY-STEP SOLUTION:

$A' = \{1, 3, 5, 7\}$. Intersecting with $B = \{1, 2, 3, 4\}$ gives $\{1, 3\}$.

$\{1, 3\}$

3 Set-Builder Notation & Algebraic Venn Diagrams

CORE CONCEPT & METHOD

Set-builder notation $\{x : \text{rule}\}$ reads as "*the set of all x such that...*". Always check the universal set $\mathcal{E}$ (e.g. integers $\mathbb{Z}$ vs positive integers $\mathbb{Z}^+$) before listing elements!

In algebraic Venn diagrams, write an expression for **each separate region** (especially subtracting the overlap x from the outer parts!) and equate the sum of all regions to $n(\mathcal{E})$.

MODEL WORKED EXAMPLE

EXAMPLE 3.1

Let $\mathcal{E} = \{\text{integers}\}$, $A = \{x : 2x + 1 \geq 5\}$ and $B = \{x : x < 6\}$. List $A \cap B$.

STEP-BY-STEP SOLUTION:

$2x + 1 \geq 5 \implies x \geq 2$. Combining $x \geq 2$ and $x < 6$ for integers gives $\{2, 3, 4, 5\}$.

$$\{2, 3, 4, 5\}$$

EDEXCEL IGCSE (4MA1) EXAM PRACTICE**Total: 11 marks****Formal Set-Builder & Subset Notation — Practice Questions**

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1**(1 mark)**

Given the set $B = \{\text{positive factors of } 36\}$, write down the value of $n(B)$.

WORKING OUT SPACE

Answer:**Question 2****(1 mark)**

Let $M = \{x : x \text{ is a factor of } 24\}$. How many integers k satisfy BOTH $k \in M$ and $k > 7$?

WORKING OUT SPACE

Answer:**Question 3****(2 marks)**

Sets P and Q satisfy $P \subseteq Q$, with $n(\mathcal{E}) = 30$, $n(P) = 8$ and $n(Q) = 21$.
Work out $n(P' \cap Q)$.

WORKING OUT SPACE

Answer:

Question 4**(2 marks)**

$A = \{1, 4, 9, 16\}$, $B = \{2, 4, 8, 16\}$ and $C = \{4, 6, 8, 10\}$ (within universal set $\mathcal{E} = \{1, 2, \dots, 20\}$).
List the elements of $A \cap B \cap C$.

WORKING OUT SPACE

Answer:

Question 5**(2 marks)**

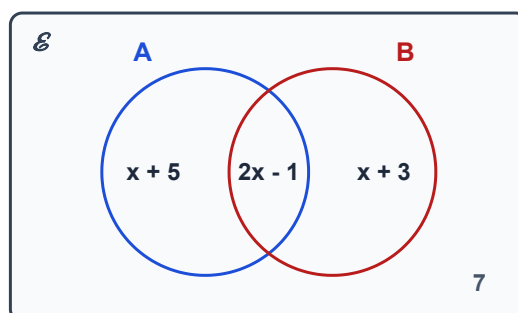
$\mathcal{E} = \{\text{integers}\}$, $A = \{x : 3x + 2 > -4\}$ and $B = \{x : 2x + 1 \leq 9\}$.
List all the elements of $A \cap B$.

WORKING OUT SPACE

Answer:

Question 6**(3 marks)**

The Venn diagram shows the number of elements in each region of $\mathcal{E}$.



Given that $n(\mathcal{E}) = 30$, find $n(A \cup B)$.

WORKING OUT SPACE

Answer:

Arithmetic Sequences & Series

1 Arithmetic Progressions (u_n)

CORE CONCEPT & METHOD

An **arithmetic sequence** (or progression) has a constant **common difference** d from one term to the next. If the first term is a , the n th term u_n is:

$$u_n = a + (n - 1)d$$

For example, the 10th term is $a + 9d$ and the 25th term is $a + 24d$.

MODEL WORKED EXAMPLE

EXAMPLE 1.1

In an arithmetic sequence, the 4th term is 17 and the 10th term is 41. Find the 20th term.

STEP-BY-STEP SOLUTION:

1. Write two equations: $a + 3d = 17$ and $a + 9d = 41$.
2. Subtract: $6d = 24 \implies d = 4$, so $a = 17 - 3(4) = 5$.
3. Find $u_{20} = 5 + 19(4) = 81$.

81

2 Sum of an Arithmetic Series (S_n)

CORE CONCEPT & METHOD

When the terms of an arithmetic sequence are added together, we get an **arithmetic series**. The sum S_n of the first n terms is:

$$S_n = \frac{n}{2} [2a + (n - 1)d] \quad \text{or} \quad S_n = \frac{n}{2} (a + l)$$

Use the second version whenever you know the first term a and the last term l !

MODEL WORKED EXAMPLE

EXAMPLE 2.1

Find the sum of the first 30 terms of the arithmetic series $5 + 9 + 13 + 17 + \dots$

STEP-BY-STEP SOLUTION:

Here $a = 5$, $d = 4$ and $n = 30$.

$$S_{30} = \frac{30}{2} [2(5) + (30 - 1)(4)] = 15 \times (10 + 116) = 15 \times 126 = 1890.$$

1890

3 Simultaneous & Quadratic Series Problems

CORE CONCEPT & METHOD

Higher Grade 8–9 exam problems test three powerful techniques:

- **Finding n given S_n :** Substitute a , d and S_n to form a quadratic equation in n , then choose the positive integer root!
- **Using S_n formulas:** Because $S_n = u_1 + u_2 + \cdots + u_{n-1} + u_n$, the n th term is always $u_n = S_n - S_{n-1}$ (and $a = S_1$).
- **Sum of multiples:** Identify the first multiple a and last multiple l in the interval, find $n = \frac{l - a}{d} + 1$, then use $\frac{n}{2}(a + l)$.

MODEL WORKED EXAMPLE

EXAMPLE 3.1

The sum of the first n terms of a series is $S_n = 3n^2 + 4n$. Work out the 6th term.

STEP-BY-STEP SOLUTION:

Use $u_6 = S_6 - S_5$:

$$S_6 = 3(6)^2 + 4(6) = 108 + 24 = 132$$

$$S_5 = 3(5)^2 + 4(5) = 75 + 20 = 95$$

$$u_6 = 132 - 95 = 37.$$

37

EDEXCEL IGCSE (4MA1) EXAM PRACTICE

Total: 14 marks

Arithmetic Sequences & Series — Practice Questions

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1**(2 marks)**

Here are the first four terms of an arithmetic sequence:

 $85, 82, 79, 76, \dots$

Work out the 25th term of this sequence.

WORKING OUT SPACE

Answer:

Question 2**(3 marks)**

In an arithmetic sequence, the 3rd term is 5 and the 9th term is 41.

Work out the 15th term of the sequence.

WORKING OUT SPACE

Answer:

Question 3**(2 marks)**

Find the sum of the first 25 terms of the arithmetic series:

 $8 + 12 + 16 + 20 + \dots$

WORKING OUT SPACE

Answer:

Question 4**(2 marks)**

Evaluate:

$$\sum_{r=1}^{18} (5r + 3)$$

WORKING OUT SPACE

Answer:**Question 5****(3 marks)**

The sum of the first n terms of the arithmetic series $5 + 9 + 13 + 17 + \dots$ is 275.
Form and solve a quadratic equation to find the value of n .

WORKING OUT SPACE

Answer:**Question 6****(2 marks)**

The sum of the first n terms of an arithmetic series is given by $S_n = 4n^2 - n$.
Work out the 6th term of the series.

WORKING OUT SPACE

Answer:

Functions: Domain & Range

1 Excluded Values & Natural Domains

CORE CONCEPT & METHOD

The **domain** of a function is the set of allowed **input** (x) values. Two rules restrict real domains:

- **Never divide by zero:** In $f(x) = \frac{a}{g(x)}$, solve $g(x) = 0$ to find the **excluded** value(s) of x .
- **No negative square roots:** In $f(x) = \sqrt{g(x)}$, require $g(x) \geq 0$.

MODEL WORKED EXAMPLE

EXAMPLE 1.1

State the value of x that must be excluded from any domain of $f(x) = \frac{5}{2x-8}$, and find the largest possible domain of $g(x) = \sqrt{x-3}$.

STEP-BY-STEP SOLUTION:

For $f(x)$: set $2x - 8 = 0 \implies x = 4$ must be excluded.

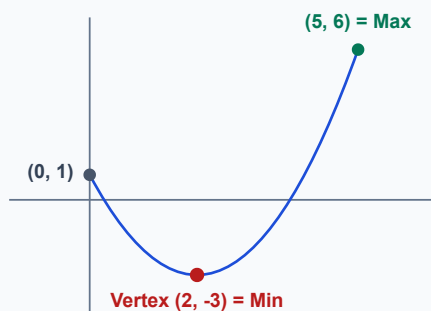
For $g(x)$: require $x - 3 \geq 0 \implies x \geq 3$.

$$x = 4; \quad x \geq 3$$

2 Finding the Range of a Function

CORE CONCEPT & METHOD

The **range** of f is the set of all possible **output values** $f(x)$ produced by the domain.



Quadratic Turning-Point Trap: On $f(x) = (x-2)^2 - 3$ for $0 \leq x \leq 5$, do *not* only test $x = 0$ and $x = 5$! Because the vertex $x = 2$ lies inside $[0, 5]$, the minimum output is $f(2) = -3$, giving $-3 \leq f(x) \leq 6$.

MODEL WORKED EXAMPLE

EXAMPLE 2.1

Find the range of $f(x) = (x-3)^2 + 4$ for all real x .

STEP-BY-STEP SOLUTION:

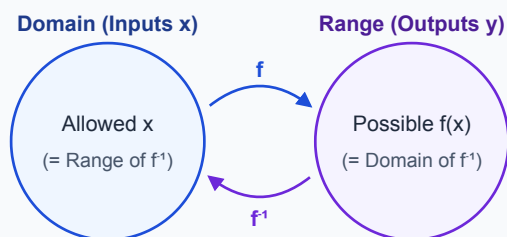
Because $(x-3)^2 \geq 0$ for all real x , adding 4 gives $(x-3)^2 + 4 \geq 4$.

$$f(x) \geq 4$$

3 Domain & Range of Inverse Functions

CORE CONCEPT & METHOD

Because an inverse function f^{-1} swaps inputs and outputs:



Domain of $f^{-1} = \text{Range of } f$ and Range of $f^{-1} = \text{Domain of } f$

MODEL WORKED EXAMPLE

EXAMPLE 3.1

The function $f(x) = (x - 2)^2 + 5$ is defined for $x \geq 2$. State the domain and range of $f^{-1}(x)$.

STEP-BY-STEP SOLUTION:

1. For $x \geq 2$, the range of f is $f(x) \geq 5$, so the **domain of** f^{-1} is $x \geq 5$.
2. The domain of f is $x \geq 2$, so the **range of** f^{-1} is $f^{-1}(x) \geq 2$.

Domain: $x \geq 5$, Range: $f^{-1}(x) \geq 2$

EDEXCEL IGCSE (4MA1) EXAM PRACTICE

Total: 13 marks

Functions: Domain & Range — Practice Questions

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1

(2 marks)

The function f is defined by $f(x) = \frac{4x + 1}{2x + 6}$.

State the value of x that must be **excluded** from any domain of f .

WORKING OUT SPACE

Answer:

Question 2

(2 marks)

The real-valued function h is defined by $h(x) = \sqrt{15 - 3x}$.

Write down an inequality in x for the largest possible domain of h .

WORKING OUT SPACE

Answer:

Question 3

(2 marks)

Find the range of the function $f(x) = 5x + 1$ for the domain $x > -1$.

Write your answer as an inequality in $f(x)$ or y (e.g. $f(x) \geq 2$).

WORKING OUT SPACE

Answer:

Question 4**(2 marks)**

The function f is defined by $f(x) = (x - 1)^2 + 2$ for the domain $-3 \leq x \leq 3$.

The range of f is $a \leq f(x) \leq b$. Work out the values of a and b .

WORKING OUT SPACE

Answer:**Question 5****(3 marks)**

The one-to-one function f is defined by $f(x) = x^2 + 4x - 3$ for $x \geq -2$.

Complete the square to find the range of f , and hence state the **domain of the inverse function** $f^{-1}(x)$.

WORKING OUT SPACE

Answer:**Question 6****(2 marks)**

The function f is defined by $f(x) = (x - 4)^2 - 1$ for the domain $x \geq 4$.

Work out the value of $f^{-1}(35)$.

WORKING OUT SPACE

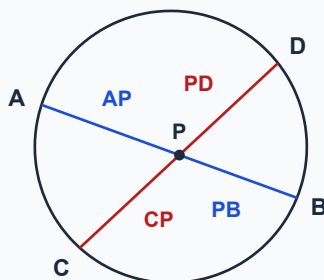
Answer:

Intersecting Chords, Secants & Tangents

1 Internal Intersecting Chords

CORE CONCEPT & METHOD

When two chords AB and CD intersect at a point P **inside** a circle, the products of their segments are equal:



$$AP \times PB = CP \times PD$$

MODEL WORKED EXAMPLE

EXAMPLE 1.1

Chords AB and CD intersect inside a circle at P . Given $AP = 6$ cm, $PB = x$ cm, $CP = 4$ cm and $PD = 12$ cm, find x .

STEP-BY-STEP SOLUTION:

Apply $AP \times PB = CP \times PD$:

$$6 \times x = 4 \times 12$$

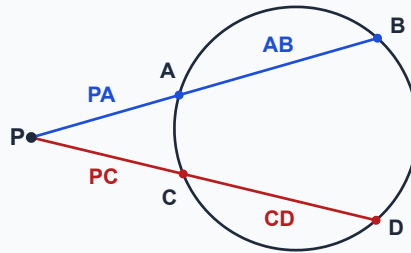
$$6x = 48 \implies x = 8$$

$$x = 8$$

2 External Intersecting Secants

CORE CONCEPT & METHOD

When two secants PAB and PCD meet at an **external point** P , multiply the **outside part** by the **whole secant length**:



Exam Trap Alert: $PA \times PB = PC \times PD$, where $PB = PA + AB$ and $PD = PC + CD$.
Never multiply $PA \times AB$! Always use *Outside* $\times$ *Whole*.

MODEL WORKED EXAMPLE

EXAMPLE 2.1

From an external point P , secants PAB and PCD satisfy $PA = 5$ cm, $AB = 7$ cm, and $PC = 4$ cm. Find CD .

STEP-BY-STEP SOLUTION:

1. Whole length $PB = 5 + 7 = 12$ cm, and whole length $PD = 4 + CD$.

2. Use $PA \times PB = PC \times PD$:

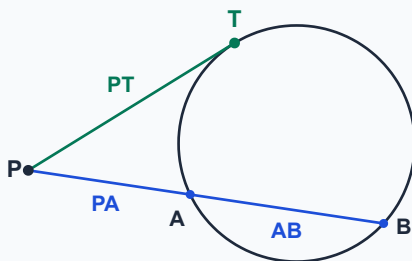
$$5 \times 12 = 4(4 + CD) \implies 60 = 4(4 + CD) \implies 15 = 4 + CD \implies CD = 11 \text{ cm.}$$

11 cm

3 Tangent–Secant Theorem

CORE CONCEPT & METHOD

When a tangent PT and a secant PAB are drawn from the same external point P , the square of the tangent equals **outside** $\times$ **whole**:



$$PT^2 = PA \times PB \quad (\text{where } PB = PA + AB)$$

MODEL WORKED EXAMPLE

EXAMPLE 3.1

PT is a tangent and PAB is a secant. Given $PA = 4$ cm and $AB = 5$ cm, find PT .

STEP-BY-STEP SOLUTION:

1. Find the whole secant length: $PB = 4 + 5 = 9$ cm.

2. Apply $PT^2 = PA \times PB$:

$$PT^2 = 4 \times 9 = 36 \implies PT = 6 \text{ cm.}$$

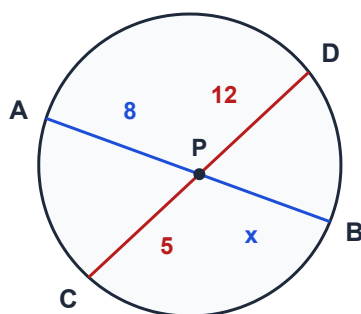
6 cm

EDEXCEL IGCSE (4MA1) EXAM PRACTICE**Total: 16 marks****Intersecting Chords, Secants & Tangents — Practice Questions**

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1**(2 marks)**

Chords AB and CD intersect at P inside the circle.



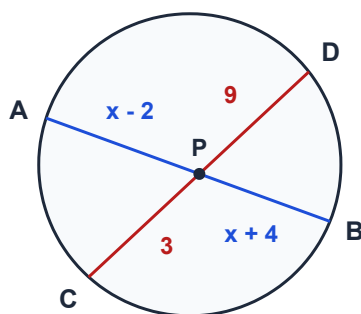
Work out the value of x .

WORKING OUT SPACE

Answer:

Question 2**(3 marks)**

Chords AB and CD intersect at P inside a circle.



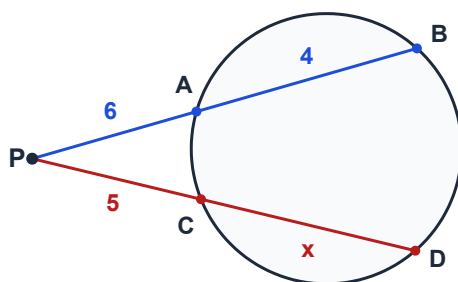
Form an equation and find the value of x .

WORKING OUT SPACE

Answer:

Question 3**(3 marks)**

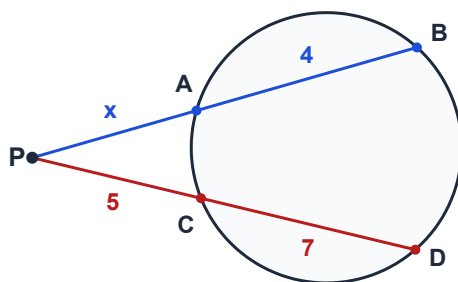
PAB and PCD are two secants to a circle from an external point P .



Work out the value of x (the length of chord CD).

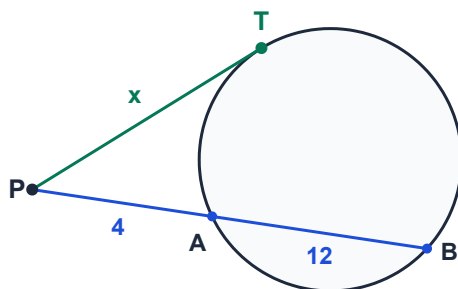
WORKING OUT SPACE

Answer:

Question 4**(3 marks)** PAB and PCD are secants to a circle.Form and solve a quadratic equation to find the value of x .

WORKING OUT SPACE

Answer:

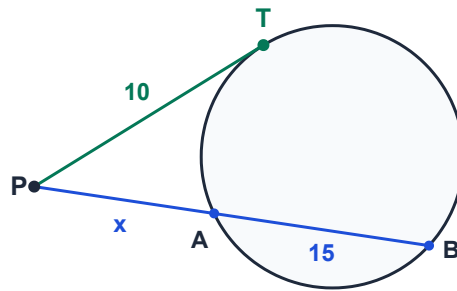
Question 5**(2 marks)** PT is the tangent to the circle at T , and PAB is a secant.Work out the length $PT = x$.

WORKING OUT SPACE

Answer:

Question 6**(3 marks)**

PT is a tangent of length 10 cm and AB is a chord of length 15 cm.



Find the length $PA = x$.

WORKING OUT SPACE

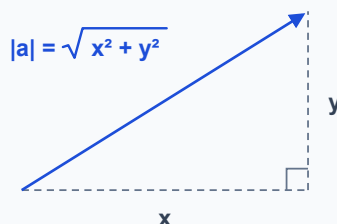
Answer:

Magnitude (Modulus) of a Vector

1 Modulus of a Column Vector

CORE CONCEPT & METHOD

The **magnitude** (or **modulus**) of a vector $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$, written $|\mathbf{a}|$, is its length found by Pythagoras' theorem:



$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

MODEL WORKED EXAMPLE

EXAMPLE 1.1

Given $\mathbf{a} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$, work out $|\mathbf{a}|$.

STEP-BY-STEP SOLUTION:

$$|\mathbf{a}| = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$$

10

2 Combined & Displacement Vectors $|\overrightarrow{AB}|$

CORE CONCEPT & METHOD

Crucial Rule: $|\mathbf{a} + \mathbf{b}|$ is *not* equal to $|\mathbf{a}| + |\mathbf{b}|$!

- Always combine the vectors into a **single column vector first**, then apply $\sqrt{x^2 + y^2}$.
- For two points $A(x_1, y_1)$ and $B(x_2, y_2)$, first find $\overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$, then find $|\overrightarrow{AB}|$.

MODEL WORKED EXAMPLE

EXAMPLE 2.1

A is $(-1, 4)$ and B is $(4, -8)$. Work out $|\overrightarrow{AB}|$.

STEP-BY-STEP SOLUTION:

$$1. \overrightarrow{AB} = \begin{pmatrix} 4 - (-1) \\ -8 - 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}.$$

$$2. |\overrightarrow{AB}| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = 13.$$

13

3 Missing Components & Parallel Vectors

CORE CONCEPT & METHOD

When you are given the magnitude $|\mathbf{v}| = m$ and asked to find an unknown k , square both sides ($x^2 + y^2 = m^2$) so the square root disappears!

Remember: unless k is restricted (e.g. $k > 0$), taking a square root gives **two solutions** ($\pm$). To scale a vector $\mathbf{p}$ to a new length L , multiply $\mathbf{p}$ by $\frac{L}{|\mathbf{p}|}$.

MODEL WORKED EXAMPLE

EXAMPLE 3.1

Vector $\mathbf{v} = \begin{pmatrix} k \\ -8 \end{pmatrix}$ has $|\mathbf{v}| = 10$ with $k < 0$. Find k .

STEP-BY-STEP SOLUTION:

$$k^2 + (-8)^2 = 10^2 \implies k^2 + 64 = 100 \implies k^2 = 36 \implies k = \pm 6.$$

Since $k < 0$, $k = -6$.

$$k = -6$$

EDEXCEL IGCSE (4MA1) EXAM PRACTICE

Total: 13 marks

Magnitude (Modulus) of a Vector — Practice Questions*Answer all 6 questions. Show clear algebraic working in the spaces provided.***Question 1****(2 marks)**

Given that $\mathbf{a} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$, work out $|\mathbf{a}|$.

WORKING OUT SPACE

Answer:

Question 2**(2 marks)**

A vector is given in component form as $\mathbf{p} = -10\mathbf{i} + 24\mathbf{j}$.
Calculate the magnitude $|\mathbf{p}|$.

WORKING OUT SPACE

Answer:

Question 3**(2 marks)**

Given that $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, work out $|3\mathbf{a} - \mathbf{b}|$.

WORKING OUT SPACE

Answer:

Question 4**(2 marks)**

Given that $\overrightarrow{AB} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -3 \\ -12 \end{pmatrix}$, work out $|\overrightarrow{AC}|$.

WORKING OUT SPACE

Answer:**Question 5****(2 marks)**

Given that $\mathbf{a} = \begin{pmatrix} k \\ -12 \end{pmatrix}$ where $k < 0$, and $|\mathbf{a}| = 13$, find the value of k .

WORKING OUT SPACE

Answer:**Question 6****(3 marks)**

Let $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

Given that $k > 0$ and $|\mathbf{a} + k\mathbf{b}| = 13$, find the value of k .

WORKING OUT SPACE

Answer:

Differentiation & Turning Points

1 Differentiating Polynomials

CORE CONCEPT & METHOD

Differentiation finds the **gradient function** $\frac{dy}{dx}$ (or $f'(x)$) of a curve at any point.

The Power Rule: If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$

"Multiply by the power, then subtract 1 from the power."

- **Linear terms (kx):** Differentiate to k (since $x^1 \rightarrow 1x^0 = 1$).
- **Constants (c):** Differentiate to 0 (a horizontal line $y = c$ has gradient 0).
- **Brackets or fractions:** Always **expand brackets** or **divide through** so y is written as separate powers of x before differentiating!

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Given that $y = 4x^3 - 7x^2 + 9x - 12$, find $\frac{dy}{dx}$.

STEP-BY-STEP SOLUTION:

Differentiate each term separately:

$$4x^3 \rightarrow 4 \times 3x^{3-1} = 12x^2$$

$$-7x^2 \rightarrow -7 \times 2x^{2-1} = -14x$$

$$+9x \rightarrow +9 \text{ and } -12 \rightarrow 0.$$

$$12x^2 - 14x + 9$$

EXAMPLE 1.2

Find $\frac{dy}{dx}$ when $y = x(2x - 5)^2$.

STEP-BY-STEP SOLUTION:

1. Expand the brackets first:

$$y = x(4x^2 - 20x + 25) = 4x^3 - 20x^2 + 25x$$

2. Apply the power rule to each term:

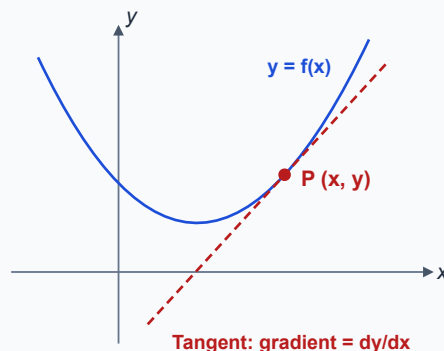
$$\frac{dy}{dx} = 12x^2 - 40x + 25$$

$$12x^2 - 40x + 25$$

2 Gradients & Equations of Tangents

CORE CONCEPT & METHOD

The derivative $\frac{dy}{dx}$ gives the **exact gradient of the tangent** to the curve at any x -coordinate.



- **Gradient at $x = a$:** Find $\frac{dy}{dx}$, then substitute $x = a$.
- **Where is the gradient equal to m ?** Solve the equation $\frac{dy}{dx} = m$ for x .
- **Equation of the tangent at (x_1, y_1) :** Find $m = \frac{dy}{dx}$ at x_1 , then substitute m and (x_1, y_1) into $y = mx + c$.

MODEL WORKED EXAMPLE

EXAMPLE 2.1

Find the equation of the tangent to the curve $y = x^2 - 3x + 5$ at the point $(4, 9)$.

STEP-BY-STEP SOLUTION:

1. Differentiate to find the gradient function:

$$\frac{dy}{dx} = 2x - 3$$

2. Substitute $x = 4$ to get the tangent gradient:

$$m = 2(4) - 3 = 5$$

3. Substitute $m = 5$ and $(4, 9)$ into $y = mx + c$:

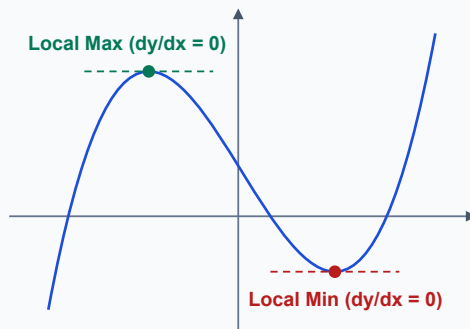
$$9 = 5(4) + c \implies c = -11$$

$$y = 5x - 11$$

3 Stationary Points & Optimisation

CORE CONCEPT & METHOD

At a **stationary point** (turning point), the tangent is completely horizontal, so $\frac{dy}{dx} = 0$.

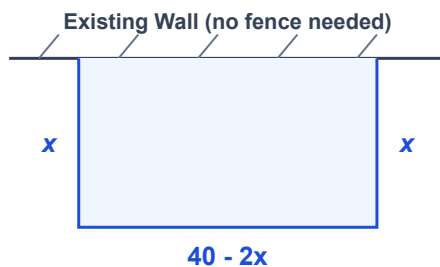


- **Step 1:** Solve $\frac{dy}{dx} = 0$ to find the x -coordinate(s).
- **Step 2:** Substitute x back into the original equation $y = f(x)$ to find y .
- **Max vs Min on a positive cubic ($+ax^3$):** The curve rises first then falls then rises ($\nearrow \searrow \nearrow$), so the **smaller** x is the local maximum and the **larger** x is the local minimum!

MODEL WORKED EXAMPLE

EXAMPLE 3.1

A farmer uses 40 m of fencing for three sides of a rectangular enclosure against a wall, giving area $A = 40x - 2x^2 \text{ m}^2$.



Find the maximum possible area.

STEP-BY-STEP SOLUTION:

1. At maximum area, $\frac{dA}{dx} = 0$:

$$\frac{dA}{dx} = 40 - 4x = 0 \implies x = 10 \text{ m}$$

2. Substitute $x = 10$ into A :

$$A_{\max} = 40(10) - 2(10)^2 = 400 - 200 = 200 \text{ m}^2$$

$$200 \text{ m}^2$$

EDEXCEL IGCSE (4MA1) EXAM PRACTICE

Total: 17 marks

Differentiation & Turning Points — Practice Questions

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1

(2 marks)

Given that $y = 2x^3 + 6x^2 - 3x + 11$, find $\frac{dy}{dx}$.

WORKING OUT SPACE

Answer:

Question 2

(2 marks)

Expand and simplify first to find $\frac{dy}{dx}$ when $y = (2x + 3)(x - 4)$.

WORKING OUT SPACE

Answer:

Question 3

(2 marks)

Find the gradient of the curve $y = x^3 + 4x^2 - 5x + 7$ at the point where $x = -2$.

WORKING OUT SPACE

Answer:

Question 4**(3 marks)**

Find the equation of the tangent to the curve $y = x^3 - 2x^2 + x$ at the point $(2, 2)$.

Give your answer in the form $y = mx + c$.

WORKING OUT SPACE

Answer:**Question 5****(4 marks)**

A curve has equation $y = x^3 - 3x^2 - 9x + 7$.

Use differentiation to find the coordinates of its two stationary points, and state which is the local maximum.

WORKING OUT SPACE

Answer:**Question 6****(4 marks)**

A rectangular paddock against a long hedge is divided into two equal pens by an internal fence parallel to the ends of length x m (so three sides have length x m). Given that 120 m of fencing is used in total, form an expression in terms of x for the total area A , and use differentiation to find the maximum total area of the paddock (in m^2).

WORKING OUT SPACE

Answer:

Negative Powers & Calculus Kinematics

1 Differentiating Negative Powers

CORE CONCEPT & METHOD

In Edexcel IGCSE (4MA1), you also differentiate fractions with x in the denominator by rewriting them as **negative powers** first: $\frac{a}{x^n} = ax^{-n}$.

Watch the negative sign when subtracting 1 from the power!

$$x^{-1} \rightarrow -1x^{-2} = -\frac{1}{x^2} \cdot x^{-2} \rightarrow -2x^{-3} = -\frac{2}{x^3}$$

3D Solid Optimisation: When a cuboid or cylinder of fixed volume has surface area $S = ax^2 + \frac{k}{x}$, rewrite as $ax^2 + kx^{-1}$ and solve $\frac{dS}{dx} = 0$ to find the minimum surface area.

MODEL WORKED EXAMPLE

EXAMPLE 1.1

Given that $y = 2x^2 + \frac{12}{x}$, find the gradient of the curve where $x = 2$.

STEP-BY-STEP SOLUTION:

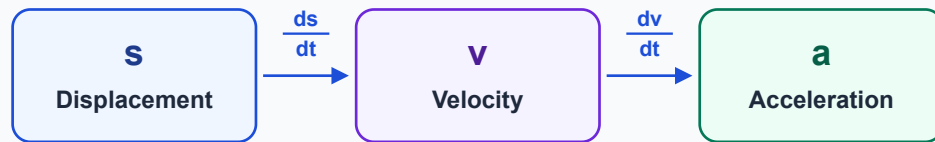
1. Rewrite using negative indices: $y = 2x^2 + 12x^{-1}$
2. Differentiate: $\frac{dy}{dx} = 4x - 12x^{-2} = 4x - \frac{12}{x^2}$
3. Substitute $x = 2$: $\frac{dy}{dx} = 4(2) - \frac{12}{2^2} = 8 - 3 = 5$

5

2 Displacement, Velocity & Acceleration

CORE CONCEPT & METHOD

When a particle moves along a straight line with displacement s at time t , calculus links **displacement (s)**, **velocity (v)**, and **acceleration (a)**:



- **Velocity:** $v = \frac{ds}{dt}$ (differentiate displacement once with respect to t).
- **Acceleration:** $a = \frac{dv}{dt}$ (differentiate velocity with respect to t).

MODEL WORKED EXAMPLE

EXAMPLE 2.1

A particle has displacement $s = 2t^3 - 5t^2 + 8t$ metres after t seconds. Find its velocity and acceleration when $t = 3$.

STEP-BY-STEP SOLUTION:

1. Velocity: $v = \frac{ds}{dt} = 6t^2 - 10t + 8$. At $t = 3$: $v = 6(9) - 30 + 8 = 32$ m/s.

2. Acceleration: $a = \frac{dv}{dt} = 12t - 10$. At $t = 3$: $a = 12(3) - 10 = 26$ m/s².

$$v = 32 \text{ m/s}, a = 26 \text{ m/s}^2$$

3 Instantaneous Rest & Max Displacement

CORE CONCEPT & METHOD

Key kinematics phrases in exam questions translate directly into equations:

- "Instantaneously at rest" or "changes direction" or "maximum height/displacement" $\implies$ set $v = 0$!
- "Returns to O " $\implies$ set $s = 0$.
- "Constant/maximum velocity" $\implies$ set $a = 0$.

MODEL WORKED EXAMPLE

EXAMPLE 3.1

A particle moves with displacement $s = t^3 - 6t^2 + 9t$ metres ($t \geq 0$). Find its displacement from O when it **first** comes to rest.

STEP-BY-STEP SOLUTION:

1. Set $v = \frac{ds}{dt} = 0$:

$$3t^2 - 12t + 9 = 0 \implies 3(t-1)(t-3) = 0 \implies t = 1 \text{ s or } t = 3 \text{ s.}$$

2. It **first** comes to rest at $t = 1$ s. Substitute $t = 1$ into s :

$$s = (1)^3 - 6(1)^2 + 9(1) = 4 \text{ m.}$$

$$4 \text{ m}$$

EDEXCEL IGCSE (4MA1) EXAM PRACTICE

Total: 16 marks

Negative Powers & Calculus Kinematics — Practice Questions

Answer all 6 questions. Show clear algebraic working in the spaces provided.

Question 1

(2 marks)

Given that $y = 5x^2 - 2x^{-3}$, find $\frac{dy}{dx}$.

You may leave your answer with negative indices or as a fraction.

WORKING OUT SPACE

Answer:

Question 2

(4 marks)

An open-top rectangular water tank has a square base of side x m and a volume of 32 m^3 , so its total outer surface area is given by $S = x^2 + \frac{128}{x} \text{ m}^2$ ($x > 0$).

Use differentiation to find the **minimum possible outer surface area** of the tank (in m^2).

WORKING OUT SPACE

Answer:

Question 3

(2 marks)

A particle moves along a straight line so that its displacement, s m, from a fixed point O at time t s is given by $s = t^3 - 9t^2 + 24t - 3$.

Find an expression for the velocity, v m/s, of the particle at time t .

WORKING OUT SPACE

Answer:

Question 4**(3 marks)**

The displacement of a particle from O at time t s is $s = 3t^3 - 12t^2 + 7t$ metres.

Work out the **acceleration** (in m/s^2) of the particle when $t = 2$.

WORKING OUT SPACE

Answer:**Question 5****(2 marks)**

For $t > 0$, a particle has displacement $s = t^3 - 27t + 10$ metres from O .

Find the value of t when the particle is instantaneously at rest.

WORKING OUT SPACE

Answer:**Question 6****(3 marks)**

A particle moves along a straight line with displacement $s = t^3 - 9t^2 + 15t + 4$ metres at time t s ($t \geq 0$).

Find the two values of t for which the particle is instantaneously at rest.

WORKING OUT SPACE

Answer:

APPENDIX

Answers & Mark Schemes

Final answers and mark allocations for all 42 PEARSON EDEXCEL IGCSE (4MA1) practice questions. Scan the cover QR code for interactive step-by-step worked solutions online.

Topic 1: Formal Set-Builder & Subset Notation

Q1:	9	[1m]
Q2:	3	[1m]
Q3:	13	[2m]
Q4:	{4}	[2m]
Q5:	{-1, 0, 1, 2, 3, 4}	[2m]
Q6:	23	[3m]

Topic 2: Arithmetic Sequences & Series

Q1:	13	[2m]
Q2:	77	[3m]
Q3:	1400	[2m]
Q4:	909	[2m]
Q5:	11	[3m]
Q6:	43	[2m]

Topic 3: Functions: Domain & Range

Q1:	-3	[2m]
Q2:	$x \leq 5$	[2m]
Q3:	$f(x) > -4$	[2m]
Q4:	$a = 2, b = 18$	[2m]
Q5:	$x \geq -7$	[3m]
Q6:	10	[2m]

Topic 4: Intersecting Chords, Secants & Tangents

Q1:	7.5	[2m]
Q2:	5	[3m]
Q3:	7	[3m]
Q4:	6	[3m]
Q5:	8	[2m]
Q6:	5	[3m]

Topic 5: Magnitude (Modulus) of a Vector

Q1:	13	[2m]
Q2:	26	[2m]
Q3:	13	[2m]
Q4:	17	[2m]
Q5:	-5	[2m]
Q6:	2	[3m]

Topic 6: Differentiation & Turning Points

Q1:	$6x^2 + 12x - 3$	[2m]
Q2:	$4x - 5$	[2m]
Q3:	-9	[2m]
Q4:	$y = 5x - 8$	[3m]
Q5:	Max (-1, 12), Min (3, -20)	[4m]
Q6:	$A = 120x - 3x^2; 1200$	[4m]

Topic 7: Negative Powers & Calculus Kinematics

Q1:	$10x + 6x^{-4}$	[2m]
Q2:	48	[4m]
Q3:	$3t^2 - 18t + 24$	[2m]
Q4:	12	[3m]
Q5:	3	[2m]
Q6:	$t = 1, 5$	[3m]