

GCSE MATHEMATICS (9-1) — TUTORIALS & WORKED EXAMPLES

Grade 1 & 2 Mathematics — Tutorials & Examples

(Foundation Tier)

Plain-English Notes, Clear Diagrams, Common Mistake Tips & Worked Examples

- Short, Plain-English Concept Notes & Clear Diagrams
- "Common Mistake" Warnings in every single topic box
- Step-by-Step Worked Examples showing every line of working
- Easy-to-Read Revision Guide (Tutorials & Examples Only)

Student Name: _____

Class / Teacher: _____

Target Grade: _____



SCAN FOR ONLINE PRACTICE & WORKED SOLUTIONS

BreakingDownMaths.com

Interactive Step-by-Step GCSE Mathematics Mastery

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Negative Numbers

1 Adding & Subtracting

□ CORE CONCEPT & METHOD

The Number Line

When adding or subtracting negative numbers, use a number line.

- **Adding** means you move **RIGHT** along the number line.
- **Subtracting** means you move **LEFT** along the number line.

Example 1: $-4 + 6$

Start at -4 and move right 6 steps. You cross zero and land on 2.

Example 2: $-3 - 5$

Start at -3 and move left 5 steps. You land on -8 .

The Double Sign Rules

When you have two signs right next to each other, they combine:

Double Sign	Rule	Result
$++$	Same signs	Positive $+$
$--$	Same signs	Positive $+$
$+-$	Different signs	Negative $-$
$-+$	Different signs	Negative $-$

Example 1: $5 - -3$ becomes $5 + 3$.

Example 2: $6 + -2$ becomes $6 - 2$.



COMMON MISTAKE: Do not use "two minuses make a plus" for $-5 - 3$: start at -5 and move 3 steps LEFT to get -8 .

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Work out the value of $4 - 11$



STEP-BY-STEP SOLUTION:

Start at 4 and move 11 steps left.

You will go past zero and land on -7 .

2 Multiplying & Dividing

CORE CONCEPT & METHOD

The Sign Rules

When multiplying or dividing, look at the signs of both numbers:

Calculation	Signs	Answer
$+$ $\times$ $+$	Same sign	Positive $+$
$-$ $\times$ $-$	Same sign	Positive $+$
$+$ $\times$ $-$	Different sign	Negative $-$
$-$ $\times$ $+$	Different sign	Negative $-$

The same rules apply to division ($\div$).

Example 1:

$$-4 \times -5$$

Same signs (both negative) $\rightarrow$ **positive**. Answer: 20

Example 2:

$$12 \div -3$$

Different signs $\rightarrow$ **negative**. Answer: -4

COMMON MISTAKE: Only use "two negatives make a positive" when multiplying, dividing, or when two signs touch like $4 - (-2)$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Example 1 (Multiplication): Work out the value of -7×4

STEP-BY-STEP SOLUTION:

A negative times a positive gives a **negative** result (different signs $\rightarrow$ negative).

$$7 \times 4 = 28$$

$$\text{So } -7 \times 4 = -28$$

EXAMPLE 2.2**Example 2 (Division):** Work out the value of $-36 \div 9$ **STEP-BY-STEP SOLUTION:****Different signs → negative result:**Work out the division without signs: $36 \div 9 = 4$.

A negative divided by a positive is negative.

So $-36 \div 9 = -4$ **EXAMPLE 2.3****Example 3 (Division):** Work out the value of $-32 \div -8$ **STEP-BY-STEP SOLUTION:****Same signs → positive result:**Work out the division without signs: $32 \div 8 = 4$.

A negative divided by a negative is positive.

So $-32 \div -8 = 4$

Decimals

1 Adding & Subtracting

📖 CORE CONCEPT & METHOD

Align the Decimal Points

When adding or subtracting decimals, the most important rule is to make sure the decimal points line up exactly on top of each other.

Add placeholder zeros to the end of the shorter number if you need to.

COMMON MISTAKE: Never line up the last digits — line up the decimal points and fill any empty gaps with 0 (write 13.6 as 13.60).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Example 1 (Addition): Work out the value of $4.7 + 2.58$

STEP-BY-STEP SOLUTION:

The decimals have different numbers of decimal places, so add a placeholder zero to 4.7 to make it 4.70.

Now line up the decimal points and add:

$$\begin{array}{r} 4.70 \\ +2.58 \\ \hline 7.28 \end{array}$$

Answer: 7.28

EXAMPLE 1.2

Example 2 (Subtraction): Work out the value of $6.4 - 2.75$

STEP-BY-STEP SOLUTION:

Add a placeholder zero to 6.4 to make it 6.40.

Now line up the decimal points and subtract:

$$\begin{array}{r} 6.40 \\ -2.75 \\ \hline 3.65 \end{array}$$

Answer: 3.65

2 Multiplying & Dividing

📖 CORE CONCEPT & METHOD

Multiplying Decimals

Ignore the decimals, multiply the whole numbers using the column method, then count the total number of decimal places in the question and put them back in the answer.

E.g. $0.3 \times 0.2 \rightarrow 3 \times 2 = 6 \rightarrow 0.06$ (2 dp)

Dividing Decimals

Never divide by a decimal! Multiply both numbers by 10 (or 100) until the number you are dividing by is a whole number.

E.g. $1.5 \div 0.3 \rightarrow 15 \div 3 = 5$

COMMON MISTAKE: 0.4×0.3 is 0.12 (two decimal places altogether), NOT 1.2.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Work out the value of 0.6×1.4

STEP-BY-STEP SOLUTION:

Multiply the whole numbers (6 and 14) using the column method:

$$\begin{array}{r} 14 \\ \times 6 \\ \hline 84 \\ \hline \end{array}$$

There are 2 decimal places total in the question.
Put 2 decimal places back into the answer: 0.84

EXAMPLE 2.2

Work out the value of $1.68 \div 0.14$

STEP-BY-STEP SOLUTION:

Multiply both numbers by 100 to make the divisor a whole number:

$$1.68 \times 100 = 168$$

$$0.14 \times 100 = 14$$

Now do the division using the bus stop method:

$$\begin{array}{r} 0 \ 1 \ 2 \\ 14 \overline{) 1 \ 6 \ 8} \end{array}$$

Answer: 12

Ordering Decimals

CORE CONCEPT & METHOD

How to Compare Decimals

To easily compare decimals, add **zero placeholders** to the end of the shorter numbers so that every number has the exact same number of decimal places.

For example, to order **0.5**, **0.46**, and **0.409**, add zeros so they all have 3 decimal places:

0.500

0.460

0.409

Now you can clearly see that 0.409 is the smallest, followed by 0.460, and 0.500 is the largest!

COMMON MISTAKE: 0.421 is NOT bigger than 0.5 — write 0.5 as 0.500 first so every number has the same length.

MODEL WORKED EXAMPLES

EXAMPLE 1

Write these five numbers in order of size. Start with the smallest.

0.42, 0.4, 0.405, 0.45, 0.049

STEP-BY-STEP SOLUTION:

Step 1: Line up the decimal points vertically. The longest decimals have **3 decimal places**.

Step 2: Fill empty spaces on the right with **placeholder zeros** (shown in red) so every number has 3 decimal places, then compare from smallest to largest:

Original	Lined Up Vertically (3 d.p.)	Size Order
0.42	0 . 4 2 0	4th (420)
0.4	0 . 4 0 0	2nd (400)
0.405	0 . 4 0 5	3rd (405)
0.45	0 . 4 5 0	5th — Largest (450)
0.049	0 . 0 4 9	1st — Smallest (049)

Step 3: Write the final answer using the **original numbers**:

0.049, 0.4, 0.405, 0.42, 0.45

NUMBER — APPROXIMATION & BOUNDS

Rounding

1 Rounding to 10, 100, 1000

📖 CORE CONCEPT & METHOD

Look at the digit immediately to the **right** of the place value you are rounding to.

If it is **5 or more**, round UP. If it is **4 or less**, keep it the same.

Replace the remaining digits with zeros!

COMMON MISTAKE: Do not just delete the end digits: ~~4643~~ to the nearest 1000 is 5000, NOT 5.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Round 5,672 to the nearest 100.

STEP-BY-STEP SOLUTION:

Draw a vertical cut-off line straight after the **hundreds** digit (6), then check the digit to the right (7):



Answer: 5,700

2 Decimal Places

📖 CORE CONCEPT & METHOD

Count the digits **after the decimal point**.

To round to 2 decimal places, look at the 3rd decimal digit to decide whether to round the 2nd digit up or leave it alone.

COMMON MISTAKE: If a rounded decimal ends in zero (like $25.98 \rightarrow 26.0$ to 1 d.p.), you **MUST** keep the .0.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Round 27.4681 to 2 decimal places.

STEP-BY-STEP SOLUTION:

$$\begin{array}{r} 27.46 \quad 81 \\ \downarrow \\ 27.47 \end{array}$$

The second decimal place is 6. The digit to the right is 8. Because it is 5 or more, we round UP.

The 6 becomes 7.

Answer: 27.47.

3 Significant Figures**CORE CONCEPT & METHOD**

Significant figures start from the FIRST non-zero digit.

For example, in 0.00345, the first sig fig is 3.

In 3450, the first sig fig is 3.

Remember to fill in zeros at the end if you need to keep the place value of a large number!

COMMON MISTAKE: Leading zeros at the start (0.00 . . .) do NOT count as significant figures — start counting at the first non-zero digit.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Round 0.07284 to 2 significant figures.

STEP-BY-STEP SOLUTION:

The first significant figure is 7. The second is 2. The digit to the right is 8. Because it is 5 or more, we round UP.

The 2 becomes 3.

Answer: 0.073.

EXAMPLE 3.2

Round 6431 to 2 significant figures.

STEP-BY-STEP SOLUTION:

The first significant figure is 6. The second is 4. The digit to the right is 3.

Because it is less than 5, the 4 stays the same.

Crucial step: We must fill in the remaining spaces with zeros to keep the number in the thousands.

Answer: 6400.

Money Problems

□ CORE CONCEPT & METHOD

Solving Money Problems

Break the problem down into two steps:

1. Find the **total cost** of everything you bought. Be careful if you bought more than one of the same item!
2. Find the **change** by subtracting the total cost from the money you handed to the cashier.

COMMON MISTAKE: Never mix pounds and pence in one sum: change 49p into £0.49 before adding to pounds.

MODEL WORKED EXAMPLES

EXAMPLE 1

Maya buys 4 notebooks for 65p each and pays with a £10 note. How much change does she get?

STEP-BY-STEP SOLUTION:

$$\text{Cost} = 4 \times 65\text{p} = 260\text{p} = £2.60.$$

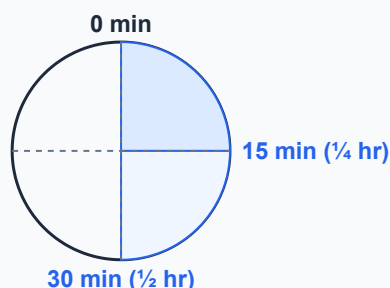
$$\text{Change} = £10.00 - £2.60 = £7.40$$

Time & Timetables

1 Converting Hours, Minutes & Fractional Hours

□ CORE CONCEPT & METHOD

Because there are **60 minutes in 1 hour** (not 100), time does not use base 10. For example, 1.5 hours means 1 hour and 30 minutes ($1\frac{1}{2}$ hours), *not* 1 hour 50 minutes.



Hours → Minutes: $\times 60$

Minutes → Hours: $\div 60$

$\frac{3}{4}$ hr = 45 min

Fraction of an Hour	Calculation	Minutes
$\frac{1}{4}$ hour (0.25 hr)	$60 \div 4$	15 minutes
$\frac{1}{3}$ hour	$60 \div 3$	20 minutes
$\frac{1}{2}$ hour (0.5 hr)	$60 \div 2$	30 minutes
$\frac{3}{4}$ hour (0.75 hr)	3×15	45 minutes
$\frac{1}{5}$ hour (0.2 hr)	$60 \div 5$	12 minutes

- **12-Hour to 24-Hour Clock:** For **am** times, keep the hour the same (use a leading zero like 08:25, except 12:XX am becomes 00:xx). For **pm** times after 12:59 pm, **add 12 to the hours** (e.g. 3:40 pm → 15:40).

COMMON MISTAKE: Time is out of 60, not 100: 1.5 hours means 1 hour 30 mins, NOT 1 hour 50 mins.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Convert $3\frac{3}{4}$ hours into minutes.

STEP-BY-STEP SOLUTION:

Step 1: Convert the whole hours into minutes

$$3 \times 60 = 180 \text{ minutes}$$

Step 2: Convert $\frac{3}{4}$ of an hour into minutes

$$\frac{3}{4} \times 60 = 45 \text{ minutes}$$

Step 3: Add them together

$$180 + 45 = 225 \text{ minutes}$$

225

EXAMPLE 1.2

Work out the difference, in minutes, between $2\frac{1}{4}$ hours and 98 minutes.

STEP-BY-STEP SOLUTION:

Step 1: Convert $2\frac{1}{4}$ hours into minutes

$$2 \times 60 + 15 = 120 + 15 = 135 \text{ minutes}$$

Step 2: Subtract 98 minutes from 135 minutes

$$135 - 98 = 37 \text{ minutes}$$

37

EXAMPLE 1.3

Write 4:35 pm as a 24-hour clock time.

STEP-BY-STEP SOLUTION:

Step 1: Check whether the time is am or pm

Since the time is after 1:00 pm, add 12 to the hour:

$$4 + 12 = 16$$

Step 2: Keep the minutes unchanged

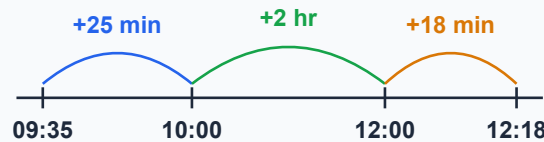
$$16:35$$

16 : 35

2 Elapsed Time & Multi-Stage Journeys

□ CORE CONCEPT & METHOD

Never use normal column subtraction for times (because 1 hour = 60 minutes, not 100). Instead, use a **time number line** and jump to the **next whole hour** first.



- **Finding Duration:** Count the minutes from the start time up to the next whole hour, add the whole hours, and then add the final minutes.
- **Adding a Duration in Minutes:** First convert the duration into *hours and minutes* by dividing by 60 (e.g. 145 minutes = 2 hours 25 minutes), then add the hours followed by the minutes.
- **Multi-Stage Journeys:** Add each leg of the journey (including any waiting or rest stops) step by step on the timeline.

COMMON MISTAKE: Never subtract times in columns like normal decimals — jump along a timeline to the next whole hour first.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Tariq leaves home at 09:40.

He walks for 18 minutes to the library, spends 1 hour 25 minutes studying, and then walks 18 minutes back home. At what time does Tariq arrive back home? Give your answer in 24-hour clock format.

STEP-BY-STEP SOLUTION:

Step 1: Add the walk to the library

$$09:40 + 18 \text{ minutes} = 09:58$$

Step 2: Add 1 hour 25 minutes at the library

$$09:58 + 1 \text{ hour} = 10:58$$

$$10:58 + 25 \text{ minutes} = 11:23$$

Step 3: Add the 18 minute walk home

$$11:23 + 18 \text{ minutes} = 11:41$$

11 : 41

EXAMPLE 2.2

A film lasts for 138 minutes. It starts at 14:35.

At what time does the film finish? Give your answer in 24-hour clock format.

STEP-BY-STEP SOLUTION:

Step 1: Convert 138 minutes into hours and minutes

Since $2 \times 60 = 120$, we have:

$$138 \text{ minutes} = 2 \text{ hours } 18 \text{ minutes}$$

Step 2: Add 2 hours 18 minutes to 14:35

$$14:35 + 2 \text{ hours} = 16:35$$

$$16:35 + 18 \text{ minutes} = 16:53$$

$$16 : 53$$

EXAMPLE 2.3

Elena drives from Bristol to York. She sets off at 08:15 and drives for $1\frac{3}{4}$ hours before stopping for a 25-minute coffee break. She then drives for a further 85 minutes to reach York.

At what time does Elena arrive in York? Give your answer in 24-hour clock format.

STEP-BY-STEP SOLUTION:

Step 1: Convert all durations into hours and minutes

First leg = $1\frac{3}{4}$ hours = 1 hour 45 minutes.

Coffee break = 25 minutes.

Second leg = 85 minutes = 1 hour 25 minutes.

Step 2: Add each stage to the start time 08:15

$$08:15 + 1 \text{ hr } 45 \text{ min} = 10:00$$

$$10:00 + 25 \text{ min} = 10:25$$

$$10:25 + 1 \text{ hr } 25 \text{ min} = 11:50$$

$$11 : 50$$

3 Reading Bus & Train Timetables

□ CORE CONCEPT & METHOD

In a bus or train timetable, each **column** represents one single bus or train journey from top to bottom, and each **row** represents a station or bus stop.

Station	Train A	Train B	Train C
Millbrook	07:42	08:15	08:40
Oakham	07:58	-	08:56
Redbridge	08:19	08:43	09:17

- **Dashes (–):** A dash means the bus or train *does not stop* at that station (for example, Train B is an express train that skips Oakham).
- **Journey Duration:** Stay in the **same column** and count the time from the departure station row to the arrival station row.
- **Planning with Deadlines:** When someone must arrive at a destination by a deadline and has a walk from the station, first **subtract the walking time** from the deadline to find the latest arrival time at the destination station, then pick the latest train column that arrives *on or before* that time.

COMMON MISTAKE: A dash (–) means the bus or train does NOT stop at that station, so pick a column that stops at both places.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Here is part of a train timetable from Silverton to Harboro.

Station	Train 1	Train 2	Train 3
Silverton	08:36	09:05	09:38
Cranford	08:52	-	09:54
Harboro	09:21	09:42	10:23

Work out how many minutes the 08:36 train takes to travel from **Silverton to Harboro**.

STEP-BY-STEP SOLUTION:

Step 1: Find the departure and arrival times in the first column

Departs Silverton at 08:36; arrives Harboro at 09:21.

Step 2: Count up to 09:00 and then to 09:21

From 08:36 to 09:00 is 24 minutes.

From 09:00 to 09:21 is 21 minutes.

$$24 + 21 = 45 \text{ minutes}$$

45

EXAMPLE 3.2

Using the timetable below, work out how many minutes **faster** the 10:15 express bus is than the 09:45 bus from **Newbridge to Portsmouth Gate**.

Stop	Bus 1	Bus 2	Bus 3
Newbridge	09:45	10:15	10:45
Elmswell	10:08	-	11:08
Portsmouth Gate	10:37	10:53	11:37

STEP-BY-STEP SOLUTION:

Step 1: Find the duration of the 09:45 bus

From 09:45 to 10:37 is $15 + 37 = 52$ minutes.

Step 2: Find the duration of the 10:15 express bus

From 10:15 to 10:53 is $53 - 15 = 38$ minutes.

Step 3: Subtract the durations

$$52 - 38 = 14 \text{ minutes}$$

14

EXAMPLE 3.3

Here is a bus timetable from Highfield to Market Square.

Bus Stop	Bus A	Bus B	Bus C	Bus D
Highfield	07:50	08:10	08:25	08:45
Station Road	08:06	08:26	08:41	09:01
Market Square	08:22	08:42	08:57	09:17

Nadia lives a 9-minute walk from the **Highfield** bus stop. Her office is an 8-minute walk from **Market Square**. She must arrive at her office by 09:00.

What is the **latest time** Nadia can leave her house? Give your answer in 24-hour clock format.

STEP-BY-STEP SOLUTION:

Step 1: Find the latest arrival time at Market Square

Subtract the 8-minute walk to the office from 09:00:

$$09:00 - 8 \text{ minutes} = 08:52$$

Step 2: Choose the latest bus arriving at Market Square by 08:52

Bus C arrives at 08:57 (too late), so Nadia must catch **Bus B**, which leaves Highfield at 08:10 and arrives at 08:42.

Step 3: Subtract the 9-minute walk to Highfield

$$08:10 - 9 \text{ minutes} = 08:01$$

08 : 01

BIDMAS

1 Order of Operations

📖 CORE CONCEPT & METHOD

The Order of Operations

When a calculation has more than one symbol, you must follow a strict order known as **BIDMAS**:

- Brackets
- Indices (Powers/Roots)
- Division & Multiplication (Left to right)
- Addition & Subtraction (Left to right)

Example:

$$3 + 4 \times 5$$

Multiplication comes before addition. Do $4 \times 5 = 20$ first.

Then $3 + 20 = 23$.

COMMON MISTAKE: Do NOT just work from left to right: in $2 + 3 \times 4$, always multiply first ($3 \times 4 = 12$, then $+2 = 14$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Example 1 (Multiply/Divide before Add/Subtract): Work out $15 - 3 \times 4$

STEP-BY-STEP SOLUTION:

Step 1: Multiplication comes before subtraction (BIDMAS).

$$3 \times 4 = 12$$

Step 2: Now do the subtraction.

$$15 - 12 = 3$$

Answer: 3

EXAMPLE 1.2

Example 2 (Brackets): Work out $(4 + 7) \times 3$

STEP-BY-STEP SOLUTION:

Step 1: Brackets are always first — calculate inside the brackets.

$$4 + 7 = 11$$

Step 2: Now multiply.

$$11 \times 3 = 33$$

Answer: 33

Without brackets it would be $4 + 7 \times 3 = 4 + 21 = 25$ — a completely different answer!

EXAMPLE 1.3**Example 3 (Indices):** Work out $5 + 4^2 \times 2$ **STEP-BY-STEP SOLUTION:****Step 1:** Indices (powers) come before multiplication. Calculate 4^2 first.

$$4^2 = 16$$

Step 2: Now do the multiplication.

$$16 \times 2 = 32$$

Step 3: Finally, do the addition.

$$5 + 32 = 37$$

Answer: 37**2 Brackets and Indices****BOOK CORE CONCEPT & METHOD****Using Brackets**

Brackets are the very first letter in **BIDMAS**. This means you must calculate whatever is inside the brackets before doing anything else.

Example 1:

$$(3 + 4) \times 5$$

Do the brackets first: $3 + 4 = 7$ Then multiply: $7 \times 5 = 35$.**Using Indices**

Indices (powers) come second.

Example 2:

$$3^2 + 4$$

Index first: $3^2 = 9$ Then add: $9 + 4 = 13$.

COMMON MISTAKE: Work out indices BEFORE multiplying or adding: in $16 + 5^2$, work out $5^2 = 25$ first, NOT $(16 + 5)^2$.

MODEL WORKED EXAMPLES**EXAMPLE 2.1****Work out the value of** $(3 + 4)^2$ **STEP-BY-STEP SOLUTION:**Brackets first: $(3 + 4) = 7$ Then index (power): $7^2 = 49$

Powers & Roots

1 Square Numbers, Cube Numbers & Higher Powers

□ CORE CONCEPT & METHOD

A **power** (or *index*) tells you how many times to multiply a base number by itself.

$$4^3 = 4 \times 4 \times 4 = 64$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Warning: $4^2 = 4 \times 4 = 16$ (NOT $4 \times 2 = 8$)

n	1	2	3	4	5	6	7	8	9	10
Square (n^2)	1	4	9	16	25	36	49	64	81	100
Cube (n^3)	1	8	27	64	125	216	343	512	729	1000

- **Larger Squares to Memorise:** $11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$.
- **Powers of 10:** The power tells you how many zeros follow the 1: $10^2 = 100$, $10^3 = 1000$, $10^4 = 10\,000$.
- **Index Form:** Repeated multiplication of the same number can be written compactly as a power, e.g. $7 \times 7 \times 7 \times 7 = 7^4$.

COMMON MISTAKE: 3^2 means $3 \times 3 = 9$ (multiply by itself), NOT $3 \times 2 = 6$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Work out the value of 13^2 and the value of 5^3 .

STEP-BY-STEP SOLUTION:

Step 1: Multiply 13 by itself

$$13^2 = 13 \times 13 = 169$$

Step 2: Multiply 5 by itself three times

$$5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$$

$$13^2 = 169, \quad 5^3 = 125$$

EXAMPLE 1.2

Work out the value of $6 \times 10^3 + 2^5$.

STEP-BY-STEP SOLUTION:

Step 1: Evaluate each power first

$$10^3 = 1000 \implies 6 \times 10^3 = 6000$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Step 2: Add the two values

$$6000 + 32 = 6032$$

$$\underline{6032}$$

EXAMPLE 1.3

Write $9 \times 9 \times 9$ as a power of 9.

STEP-BY-STEP SOLUTION:

Step 1: Count how many 9s are multiplied together

The base number 9 is multiplied by itself 3 times:

$$9 \times 9 \times 9 = 9^3$$

$$\underline{9^3}$$

2 Square Roots & Cube Roots

□ CORE CONCEPT & METHOD

Finding a **root** is the inverse (opposite) operation of raising a number to a power.

- **Square Root ($\sqrt{x}$):** Asks "What positive number multiplied by itself gives x ?"
For example, since $8^2 = 64$, we know that $\sqrt{64} = 8$.
- **Cube Root ($\sqrt[3]{x}$):** Asks "What number multiplied by itself three times gives x ?"
For example, since $4^3 = 4 \times 4 \times 4 = 64$, we know that $\sqrt[3]{64} = 4$.
- **Order of Operations (BIDMAS):** In calculations combining powers, roots, and addition/subtraction, always work out the **powers and roots first** before adding or subtracting.

$$\sqrt{49} : \boxed{} \times \boxed{} = 49$$

same number

$$7 \times 7 = 49 \rightarrow \sqrt{49} = 7$$

$$\sqrt[3]{27} : \boxed{} \times \boxed{} \times \boxed{} = 27$$

same number

$$3 \times 3 \times 3 = 27 \rightarrow \sqrt[3]{27} = 3$$

COMMON MISTAKE: Do NOT divide by 2 to find a square root: $\sqrt{49} = 7$ (since $7 \times 7 = 49$), NOT 24.5.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Work out the value of $\sqrt{144}$.

STEP-BY-STEP SOLUTION:

Step 1: Find the positive number that squares to give 144

Since $12 \times 12 = 144$:

$$\sqrt{144} = 12$$

12

EXAMPLE 2.2

Work out the value of $\sqrt[3]{216}$.

STEP-BY-STEP SOLUTION:

Step 1: Find the number that cubes to give 216

Test small integers: $5^3 = 125$ and $6^3 = 6 \times 6 \times 6 = 216$.

$$\sqrt[3]{216} = 6$$

6

EXAMPLE 2.3

Work out the value of $4^3 - \sqrt{81}$.

STEP-BY-STEP SOLUTION:

Step 1: Evaluate the cube and the square root separately

$$4^3 = 4 \times 4 \times 4 = 64$$

$$\sqrt{81} = 9$$

Step 2: Subtract the results

$$64 - 9 = 55$$

55

3 Identifying Squares, Cubes & Powers from a List**□ CORE CONCEPT & METHOD**

In GCSE Foundation exams, you are often given a list of numbers and asked to pick out a number with a specific property.

- **Square Numbers up to 150:** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.
- **Cube Numbers up to 150:** 1, 8, 27, 64, 125.
- **Powers of 2:** $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$.
- **Both a Square and a Cube:** Notice that 1 ($1^2 = 1^3$) and 64 ($8^2 = 4^3$) are **both** square numbers and cube numbers!

COMMON MISTAKE: Check every word in the question carefully — students often pick a square number (25) when asked for a cube number (27).

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Here is a list of numbers:

12 18 27 36 45 54

From the list, write down the **square number**.

STEP-BY-STEP SOLUTION:

Step 1: Compare the list against the square numbers (1, 4, 9, 16, 25, 36, 49, ...)

Since $6^2 = 6 \times 6 = 36$, the square number in the list is 36.

36

EXAMPLE 3.2

Here is a list of numbers:

15 24 32 40 50 60

From the list, write down the number that is a **power of 2**.

STEP-BY-STEP SOLUTION:

Step 1: List the powers of 2

$$2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16, \quad 2^5 = 32$$

Step 2: Match with the given list

The number 32 ($= 2^5$) is in the list.

32

EXAMPLE 3.3

Find the number between 20 and 90 that is **both** a square number and a cube number.

STEP-BY-STEP SOLUTION:

Step 1: List the cube numbers between 20 and 90

$$3^3 = 27, \quad 4^3 = 64$$

Step 2: Check which of these is also a square number

Since $8^2 = 8 \times 8 = 64$, the number 64 is both a square number and a cube number.

64

Factors & Multiples

□ CORE CONCEPT & METHOD

Factors are numbers that divide exactly into another number without leaving a remainder. They always come in pairs!

Multiples are just the numbers in the times table of that number.

COMMON MISTAKE: Do not confuse factors and multiples: factors of 12 fit **INSIDE** 12 (1, 2, 3, 4, 6, 12), and 1 is **NOT** a prime number.

MODEL WORKED EXAMPLES

EXAMPLE 1

List all the factors of 28

STEP-BY-STEP SOLUTION:

1. Start from 1: $1 \times 28 = 28$
2. Try 2: $2 \times 14 = 28$
3. Try 3: $3 \times \square$ — **does not work** ($28 \div 3$ leaves a remainder)
4. Try 4: $4 \times 7 = 28$
5. Try 5: $5 \times \square$ — **does not work**
6. Try 6: $6 \times \square$ — **does not work**
7. Try 7: **Already have 7** (from 4×7), **so stop!**
8. List all factors in order: 1, 2, 4, 7, 14, 28

1, 2, 4, 7, 14, 28

EXAMPLE 2

List the first 5 multiples of 6

STEP-BY-STEP SOLUTION:

1. $1 \times 6 = 6$
2. $2 \times 6 = 12$
3. $3 \times 6 = 18$
4. $4 \times 6 = 24$
5. $5 \times 6 = 30$

6, 12, 18, 24, 30

NUMBER — FRACTIONS & DECIMALS

Fractions (Simplify, <, =, >)

1 Writing & Simplifying Fractions

CORE CONCEPT & METHOD

Before we order fractions, we need to know how to simplify them! To **simplify** a fraction, we want to write it using the smallest numbers possible.

$$\frac{4}{8} = \frac{1}{2}$$

Divide both top and bottom by the same number (4):

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

To simplify, find a number that divides perfectly into **both** the top (numerator) and the bottom (denominator). Keep dividing until you can't go any further!

COMMON MISTAKE: You must divide the top AND bottom by the exact same number — stop only when nothing else goes into both.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Simplify: $\frac{10}{15}$

STEP-BY-STEP SOLUTION:

Both numbers are multiples of 5, so we can divide them both by 5:

$$\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

2 Equivalent Fractions

□ CORE CONCEPT & METHOD

Equivalent fractions look different but are actually the same size. To find a missing number, figure out what the bottom was multiplied by, and do the **exact same thing** to the top!

$$\frac{2}{3} = \frac{\boxed{}}{15}$$

COMMON MISTAKE: Never add the same number to the top and bottom — always MULTIPLY or DIVIDE both by the same number.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Find the missing number: $\frac{3}{4} = \frac{?}{24}$

STEP-BY-STEP SOLUTION:

1. The bottom goes from 4 to 24. That means we multiplied by 6! ($4 \times 6 = 24$)
2. Do the same to the top: $3 \times 6 = 18$

The missing number is **18**.

$$\frac{3}{4} = \frac{18}{24}$$

3 Comparing (Same Denom)

□ CORE CONCEPT & METHOD

Comparing fractions is super easy if the bottom numbers (denominators) are the same! Just look at the top numbers.

- < means Less Than
- > means Greater Than
- = means Equal To

Remember, the crocodile always eats the bigger number!

COMMON MISTAKE: Remember the crocodile mouth opens towards the BIGGER fraction: $\frac{2}{5} < \frac{4}{5}$.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Compare: $\frac{3}{7}$ — $\frac{5}{7}$

STEP-BY-STEP SOLUTION:

Since the bottoms are both 7, we just compare the tops:

$$\begin{array}{ccc} 3 & \xrightarrow{\text{less than}} & 5 \\ \hline 7 & \xrightarrow{\text{Same}} & 7 \end{array}$$

The crocodile eats the bigger fraction, so: <

$$\frac{3}{7} < \frac{5}{7}$$

4 Comparing (Diff Denom)

CORE CONCEPT & METHOD

If the bottom numbers are different, we can't compare them straight away! We have to find a **Common Denominator** first.

Use Cross Multiply

Just like when adding fractions, multiply each fraction by the **other fraction's denominator** to create a perfect match!

$$\begin{array}{ccc} ? & & ? \\ \hline A & \text{---} & B \end{array}$$

$\swarrow \quad \searrow$
 $\times B \quad \quad \times A$

COMMON MISTAKE: A bigger bottom number makes SMALLER slices: $\frac{1}{5}$ is smaller than $\frac{1}{3}$, not bigger.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

Compare: $\frac{3}{5}$ — $\frac{2}{3}$

STEP-BY-STEP SOLUTION:

1. Find a common denominator by multiplying the bottoms: $5 \times 3 = 15$

2. **Cross Multiply** to convert the fractions! Multiply the first fraction by 3 and the second fraction by 5:

$$\begin{array}{ccc} \frac{3}{5} & \text{---} & \frac{2}{3} \\ & \swarrow \quad \searrow & \\ & \text{x 3} \quad \text{x 5} & \end{array}$$

3. Compare the new fractions:

$$\begin{array}{ccc} 9 & \xrightarrow{\text{less than}} & 10 \\ \hline 15 & \xrightarrow{\text{Same}} & 15 \end{array}$$

The crocodile eats the bigger fraction, so: <

$$\frac{3}{5} < \frac{2}{3}$$

5 Ordering Lists

CORE CONCEPT & METHOD

When ordering a whole list of fractions, find a **Common Denominator** that they all fit into.

Convert them all, order the numerators, and write down the **original** fractions in that order.

Watch out! Always check if the question asks for *smallest to largest* or *largest to smallest*!

COMMON MISTAKE: Write your final answer using the ORIGINAL fractions from the question, and check "smallest" vs "largest".

MODEL WORKED EXAMPLES

EXAMPLE 5.1Order from **smallest to largest**:

$$\frac{1}{2}, \frac{3}{4}, \frac{3}{5}, \frac{5}{5}$$

STEP-BY-STEP SOLUTION:

1. Find a common denominator for 2, 4, 5, 5. The lowest common multiple is 20!

2. Convert all fractions to 20ths:

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

$$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

$$\frac{5 \times 4}{5 \times 4} = \frac{20}{20}$$

3. Order from smallest to largest: $\frac{10}{20}, \frac{12}{20}, \frac{15}{20}, \frac{20}{20}$ 4. Write down the **original** fractions:

$$\frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{5}{5}$$

EXAMPLE 5.2Order from **largest to smallest**:

$$\frac{2}{3}, \frac{7}{12}, \frac{3}{4}$$

STEP-BY-STEP SOLUTION:

1. Find a common denominator for 3, 12, 4. The lowest common multiple is 12!

2. Convert all fractions to 12ths:

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{7 \times 1}{12 \times 1} = \frac{7}{12}$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

3. Order from **largest to smallest**: $\frac{9}{12}, \frac{8}{12}, \frac{7}{12}$ 4. Write down the **original** fractions:

$$\frac{3}{4}, \frac{2}{3}, \frac{7}{12}$$

Pictograms

1 Interpreting Pictograms

📖 CORE CONCEPT & METHOD

A pictogram uses pictures or symbols to represent data.

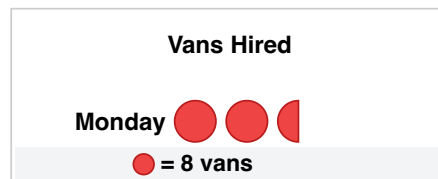
You **MUST** always look at the **key** to see how much one full symbol is worth! If a full circle is worth 8, then a half circle is worth 4, and a quarter circle is worth 2.

COMMON MISTAKE: Always check the Key first — one picture rarely stands for 1 item!

MODEL WORKED EXAMPLES

EXAMPLE 1.1

A pictogram shows circles representing vans.



How many vans does 2 and a half circles represent?

STEP-BY-STEP SOLUTION:

2 full circles: $2 \times 8 = 16$ vans.

Half a circle: $8 \div 2 = 4$ vans.

Total = $16 + 4 = 20$ vans.

2 Totals and Missing Keys

📖 CORE CONCEPT & METHOD

To find the total, calculate the value of each row and add them up.

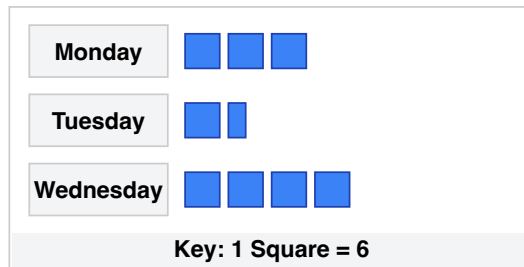
Sometimes the key is missing! If you know the total number for a row, you can divide it by the number of symbols in that row to find out what one symbol is worth.

COMMON MISTAKE: Half a symbol means half the Key value, and a quarter of a symbol means divide the Key by 4.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

If Monday has 3 symbols, Tuesday has 1.5 symbols, and Wednesday has 4 symbols:



What is the total value of all the symbols shown?

STEP-BY-STEP SOLUTION:

Total symbols = $3 + 1.5 + 4 = 8.5$.

Multiply by the key: $8.5 \times 6 = 51$.

3 Pictogram Problem Solving**CORE CONCEPT & METHOD**

Some questions will give you facts like: "*Wednesday was 3 times as large as Thursday*" and give you the combined total.

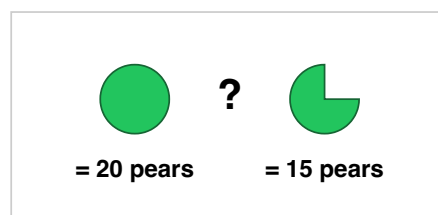
You can use ratios or simple algebra to find the exact amounts before you even look at the symbols!

COMMON MISTAKE: If Thursday is "3 times Wednesday", there are 4 equal shares in total ($1 + 3 = 4$), so divide the total by 4, not 3.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

If 1 symbol represents 20 pears, how do you draw 15 pears?

**STEP-BY-STEP SOLUTION:**

Divide the amount by the key: $15 \div 20 = \frac{3}{4}$.

You need to draw **three-quarters** of a symbol (shown above).

Fractions of an Amount

1 Unit Fractions of an Amount

□ CORE CONCEPT & METHOD

In maths, the word 'of' is just a secret code for **Multiply!**

$$\text{'of'} = \times$$

Because it's just multiplication, we can use the exact same trick we learned for multiplying fractions and whole numbers: just write the whole number as a fraction over 1!

COMMON MISTAKE: To find $\frac{1}{6}$ of 30, DIVIDE by the bottom number ($30 \div 6 = 5$) — do not multiply by 6.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

What is $\frac{1}{4}$ of 28?

STEP-BY-STEP SOLUTION:

1. 'of' means multiply: $\frac{1}{4} \times 28$

2. Write the whole number as a fraction: $\frac{1}{4} \times \frac{28}{1}$

3. Multiply the tops and bottoms: $\frac{1 \times 28}{4 \times 1} = \frac{28}{4}$

4. Simplify ($28 \div 4$): 7

$$\frac{1}{4} \text{ of } 28 = 7$$

2 Fractions of an Amount

□ CORE CONCEPT & METHOD

The exact same rule applies when the top number of the fraction is bigger than 1. Just replace 'of' with a multiply sign, put the whole number over 1, and multiply across!

COMMON MISTAKE: Easier to divide by the bottom number FIRST, then multiply by the top number.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

What is $\frac{3}{5}$ of 35?

STEP-BY-STEP SOLUTION:

Method 1 (Better method — Divide first, then multiply):

1. Divide by the bottom number: $\frac{35}{5} = 7$
2. Multiply by the top number: $7 \times 3 = 21$

Method 2 (Other method — Multiply first, then divide):

1. 'of' means multiply: $\frac{3}{5} \times 35$
2. Write the whole number as a fraction: $\frac{3}{5} \times \frac{35}{1}$
3. Multiply the tops and bottoms: $\frac{3 \times 35}{5 \times 1} = \frac{105}{5}$
4. Simplify ($105 \div 5$): 21

$$\frac{3}{5} \text{ of } 35 = 21$$

3 Applied Fractions of an Amount**□ CORE CONCEPT & METHOD**

Fractions of an amount can be used in real life problems like sales, sharing sweets, or finding differences!

Reverse Fractions: If you know $\frac{3}{4}$ of a number is 30, first find $\frac{1}{4}$ by dividing by 3 ($30 \div 3 = 10$). Then find the whole number by multiplying by 4 ($10 \times 4 = 40$).

COMMON MISTAKE: If you are told " $\frac{3}{4}$ of a book is 27 pages", divide by the top ($27 \div 3 = 9$) then times by the bottom ($9 \times 4 = 36$).

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Tariq is saving up for a skateboard.

$\frac{3}{5}$ of his target amount is £42.

Work out the total price of the skateboard.

STEP-BY-STEP SOLUTION:

1. We know $\frac{3}{5} = 42$
2. Find $\frac{1}{5}$ by dividing by the top number (3): $42 \div 3 = 14$
3. Find the whole amount ($\frac{5}{5}$) by multiplying by the bottom number (5): $14 \times 5 = 70$

$$\text{Total price} = \text{£}70$$

FDP Conversions

1 Fractions to Decimals

📖 CORE CONCEPT & METHOD

Method 1 (The Easy Way): Try to make the bottom number exactly **10, 100, or 1000!** Just multiply the top and bottom by the same number.

Method 2 (The Bus Stop): If the bottom number doesn't multiply nicely (like an 8), remember the fraction bar just means **divide!** Use short division to divide the top number by the bottom number.

COMMON MISTAKE: $\frac{1}{5}$ is 0.2 (NOT 0.5 or 1.5) — divide the top number by the bottom number ($1 \div 5$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Example 1 (Method 1 — The Easy Way): Write $\frac{3}{5}$ as a decimal.

STEP-BY-STEP SOLUTION:

Method 1: Equivalent Fractions

Let's try to make the bottom number a 10. We can do this by multiplying top and bottom by 2!

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

Since it's 6 'tenths', the decimal is 0.6.

$$\frac{3}{5} = 0.6$$

EXAMPLE 1.2

Example 2 (Method 2 — The Bus Stop): Write $\frac{5}{8}$ as a decimal.

STEP-BY-STEP SOLUTION:**Method 2: Short Division (Bus Stop Method)**

8 doesn't easily multiply to make 10 or 100, but remember the fraction bar just means **divide**:

$$\frac{5}{8} = 5 \div 8$$

Set up short division with 5.000 inside the bus stop and 8 outside:

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \\ \underline{50} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Step-by-step:

- 8 into 5 goes 0 times, remainder 5. Write 0 on top and place the decimal point.
- Carry 5 to make 50: 8 into 50 goes 6 times ($8 \times 6 = 48$), remainder 2.
- Carry 2 to make 20: 8 into 20 goes 2 times ($8 \times 2 = 16$), remainder 4.
- Carry 4 to make 40: 8 into 40 goes 5 times ($8 \times 5 = 40$) exactly with no remainder!

$$\frac{5}{8} = 0.625$$

2 Decimals to Percentages**CORE CONCEPT & METHOD**

The word **Percent** literally means 'out of 100'.

To turn a decimal into a percentage, just **multiply by 100!** (This moves the decimal point 2 places to the right).

COMMON MISTAKE: Multiply by 100 carefully: $0.02 = 2\%$, whereas $0.2 = 20\%$.

MODEL WORKED EXAMPLES**EXAMPLE 2.1**

Write 0.65 as a percentage.

STEP-BY-STEP SOLUTION:

Multiply the decimal by 100!

$$0.65 \times 100 = 65$$

$$0.65 = 65\%$$

3 Fractions to Percentages

□ CORE CONCEPT & METHOD

You have two superpowers to solve this:

- **Way 1 (Make it out of 100):** Change the fraction so the bottom number is 100. The top number is your percentage!
- **Way 2 (Multiply by 100):** Multiply the fraction directly by 100!

COMMON MISTAKE: Do NOT just write the bottom number as the percentage: $\frac{1}{4}$ is 25% (out of 100), NOT 4%.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Example 1 (Way 1 — Make it out of 100): Write $\frac{4}{5}$ as a percentage.

STEP-BY-STEP SOLUTION:

Make the bottom number 100!

Step 1: What do we multiply 5 by to get 100?

$$5 \times 20 = 100$$

Step 2: Multiply the top and bottom by 20:

$$\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$$

Step 3: Since it is out of 100, the percentage is 80%.

80%

EXAMPLE 3.2

Example 2 (Way 2 — Multiply by 100): Write $\frac{7}{20}$ as a percentage.

STEP-BY-STEP SOLUTION:

Multiply the fraction directly by 100:

$$\frac{7}{20} \times 100$$

Step 1: Divide 100 by the bottom number (denominator):

$$100 \div 20 = 5$$

Step 2: Multiply by the top number (numerator):

$$5 \times 7 = 35$$

So $\frac{7}{20} = 35\%$.

35%

4 Percentages to Fractions

📖 CORE CONCEPT & METHOD

Going backwards is super easy! Since 'percent' means 'out of 100', just put the percentage number **over 100** and then simplify the fraction.

COMMON MISTAKE: Do not leave 12% as $\frac{12}{100}$ if the question asks for simplest form — cancel down to $\frac{3}{25}$.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

Write 35% as a fraction in its simplest form.

STEP-BY-STEP SOLUTION:

1. Put it over 100: $\frac{35}{100}$

2. Simplify! Both end in 5 or 0, so divide by 5:

$$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

$$35\% = \frac{7}{20}$$

5 Decimals to Fractions

📖 CORE CONCEPT & METHOD

Look at the place value! One decimal place means 'tenths' (over 10). Two decimal places means 'hundredths' (over 100). Put the number over 10 or 100 and simplify!

COMMON MISTAKE: Check the place value column: 0.03 is 3 hundredths ($\frac{3}{100}$), NOT $\frac{3}{10}$.

MODEL WORKED EXAMPLES

EXAMPLE 5.1

Write 0.45 as a fraction in its simplest form.

STEP-BY-STEP SOLUTION:

1. Two decimal places means it goes over 100: $\frac{45}{100}$

2. Simplify! Divide top and bottom by 5:

$$\frac{45 \div 5}{100 \div 5} = \frac{9}{20}$$

$$0.45 = \frac{9}{20}$$

6 Percentages to Decimals

📖 CORE CONCEPT & METHOD

Since percent means 'out of 100', turning a percentage into a decimal means you just need to **divide by 100!** (Move the decimal point 2 places to the left).

COMMON MISTAKE: Don't forget to divide by 100 — so 7% is 0.07, NOT 0.7.

MODEL WORKED EXAMPLES

EXAMPLE 6.1

Write 32% as a decimal.

STEP-BY-STEP SOLUTION:

Divide the percentage by 100!

$$32 \div 100 = 0.32$$

$$32\% = 0.32$$

Intro to Algebra

1 The Basics (Variables)

📖 CORE CONCEPT & METHOD

In algebra, letters (called **variables**) are just placeholders for numbers.

Mathematicians are lazy! We don't write the multiply sign ($\times$) because it looks too much like the letter x . So, $2 \times y$ is just written as $2y$. And $a \times b$ is written as ab . If a number multiplies itself ($y \times y$), we write y^2 . Also, we always put the number **before** the letter (write $5x$, not $x5$).

COMMON MISTAKE: Write the number IN FRONT of the letter ($6a$, never $a6$), and remember $1x$ is written simply as x .

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Write $6 \times q$ without the multiplication sign.

STEP-BY-STEP SOLUTION:

We just push the number and the letter together! The number always goes first.

Answer: $6q$

$$6q$$

EXAMPLE 1.2

Write $c \times d$ without the multiplication sign.

STEP-BY-STEP SOLUTION:

Just push the two letters together!

Answer: cd

$$cd$$

EXAMPLE 1.3

Write $w \times w$ in its simplest form.

STEP-BY-STEP SOLUTION:

A number (or letter) multiplied by itself is squared!

Answer: w^2

$$w^2$$

2 Collecting Like Terms

□ CORE CONCEPT & METHOD

You can only add or subtract terms if they have the **exact same letters!**

Power Trap: A squared letter (x^2) is a completely different object to a normal letter (x). You cannot add them together!

Be careful with negatives, the sign in front belongs to the term.

COMMON MISTAKE: Never squash different letters together: $3x + 2y$ stays as $3x + 2y$, NOT $5xy$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Simplify $7a + 2b - 3a + 6b$

STEP-BY-STEP SOLUTION:

Group the a 's: $7a - 3a = 4a$

Group the b 's: $+2b + 6b = +8b$

Put them together for the final answer!

$$4a + 8b$$

EXAMPLE 2.2

Simplify $6x^2 + 4x - 2x^2 - 9x$

STEP-BY-STEP SOLUTION:

Trap! You cannot combine x^2 and x . Treat them as separate groups!

Group the squares: $6x^2 - 2x^2 = 4x^2$

Group the regular x 's: $+4x - 9x = -5x$

$$4x^2 - 5x$$

EXAMPLE 2.3

Simplify $8x^2y + 5xy^2 - 3x^2y$

STEP-BY-STEP SOLUTION:

Trap! x^2y is NOT the same as xy^2 . Look closely at where the squared symbol is.

Group the x^2y terms: $8x^2y - 3x^2y = 5x^2y$

The xy^2 term has no partner, so it stays exactly as it is.

$$5x^2y + 5xy^2$$

3 Multiplying & Dividing

📖 CORE CONCEPT & METHOD

When multiplying terms, multiply the numbers normally, then **ADD** the powers for any matching letters!

For division, divide the numbers normally, and **SUBTRACT** the bottom power from the top power. If there are more of a letter on the bottom, it stays on the bottom!

COMMON MISTAKE: Watch negative signs carefully: a negative times a negative gives a POSITIVE ($-3x^2 \times -6x^2 = +18x^4$).

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Simplify $5a \times 3b$

STEP-BY-STEP SOLUTION:

1. Multiply the numbers: $5 \times 3 = 15$
2. Multiply the letters: $a \times b = ab$
3. Push them together!

$$15ab$$

EXAMPLE 3.2

Simplify $6y \times 3y$

STEP-BY-STEP SOLUTION:

1. Multiply the numbers: $6 \times 3 = 18$
2. Multiply the letters: $y^1 \times y^1 = y^{1+1} = y^2$
3. Push them together!

$$18y^2$$

EXAMPLE 3.3

Simplify $4x^3y^2 \times -5xy^4$

STEP-BY-STEP SOLUTION:

1. Multiply the numbers: $4 \times -5 = -20$
2. Add the powers for x : $x^3 \times x^1 = x^4$
3. Add the powers for y : $y^2 \times y^4 = y^6$

$$-20x^4y^6$$

EXAMPLE 3.4

Simplify $\frac{24x^4y^2}{6x^3y^5}$

STEP-BY-STEP SOLUTION:

1. Divide the numbers: $24 \div 6 = 4$
2. Subtract x powers (top minus bottom): $4 - 3 = 1$, so x^1 (or just x) stays on top.
3. For y , there are 2 on top and 5 on the bottom. Since there are more on the bottom, we are left with y^3 on the bottom!

$$\frac{4x}{y^3}$$

4 Substitution**□ CORE CONCEPT & METHOD**

Substitution just means **swapping** a letter for a specific number.

Top Tip: Always put the number in brackets when you swap it in so you don't get confused! And remember BIDMAS (do indices and multiplication before addition!).

COMMON MISTAKE: Always multiply — do NOT add the numbers or join the digits together: for $x = -3$, $4x = 4 \times (-3) = -12$, NOT $4 + (-3) = 1$ (and for $x = 3$, $4x = 4 \times 3 = 12$, NOT 43).

MODEL WORKED EXAMPLES**EXAMPLE 4.1**

If $x = 5$, work out the value of $4x + 3$

STEP-BY-STEP SOLUTION:

Swap the x for a 5 in brackets:

$$4(5) + 3$$

Remember $4(5)$ means 4×5 .

$$= 20 + 3$$

$$= 23$$

$$23$$

EXAMPLE 4.2

If $a = -3$ and $b = 8$, work out the value of $2a + b$

STEP-BY-STEP SOLUTION:

Swap the letters for the numbers in brackets:

$$2(-3) + (8)$$

$$2 \times -3 = -6$$

$$= -6 + 8$$

$$= 2$$

$$2$$

EXAMPLE 4.3

If $x = -5$ and $y = 2$, work out the value of $x^2 - y^3$

STEP-BY-STEP SOLUTION:

Swap the letters for the numbers in brackets:

$$(-5)^2 - (2)^3$$

Warning! $(-5)^2$ means -5×-5 , which is $+25$.

$(2)^3$ means $2 \times 2 \times 2$, which is 8.

$$= 25 - 8$$

$$= 17$$

$$17$$

5 Writing Expressions

□ CORE CONCEPT & METHOD

Algebra is just a language! We can translate English sentences into algebraic expressions.

- '3 more than x ' is $x + 3$
- 'Double y ' is $2y$
- 'Half of p ' is $\frac{p}{2}$

COMMON MISTAKE: Order matters for subtraction: "9 younger than n " is $n - 9$, NOT $9 - n$.

MODEL WORKED EXAMPLES

EXAMPLE 5.1

Zara has y stickers in her album. She collects 8 more. Write an expression for how many stickers she has now.

STEP-BY-STEP SOLUTION:

'8 more' means we need to **add** 8 to our starting amount.

Starting amount: y

Add 8: $+8$

$$y + 8$$

EXAMPLE 5.2

A museum pass costs p pounds. Write an expression for the cost of 6 museum passes.

STEP-BY-STEP SOLUTION:

If one pass is p , then six passes is $6 \times p$.

In algebra, we write this without the multiply sign!

$$6p$$

EXAMPLE 5.3

A croissant costs $\pounds c$ and a muffin costs $\pounds m$. Write an expression for the cost of 3 croissants and 5 muffins.

STEP-BY-STEP SOLUTION:

Treat the two items separately, then add them together:

$$\text{Cost of 3 croissants} = 3 \times c = 3c$$

$$\text{Cost of 5 muffins} = 5 \times m = 5m$$

Total cost is them added together!

$$3c + 5m$$

EXAMPLE 5.4

Liam is k years old. Maya is 7 years older than Liam. Write an expression for Maya's age.

STEP-BY-STEP SOLUTION:

'Older' means Maya's age is Liam's age **plus** 7.

Liam's age: k

Maya's age: $k + 7$

$$k + 7$$

6 Formula Substitution

□ CORE CONCEPT & METHOD

Formulas are just rules written with letters. Often they come from Science (like $v = u + at$ for final velocity) or Geometry (like $A = \frac{1}{2}bh$ for area of a triangle).

To use a formula, just replace each letter with the number you are given. Always remember **BIDMAS!**

COMMON MISTAKE: Put negative numbers inside brackets before squaring: $(-2)^2 = +4$, NOT -4 .

MODEL WORKED EXAMPLES**EXAMPLE 6.1**

The formula for the perimeter of a rectangle is $P = 2l + 2w$. Work out the value of P when $l = 7$ and $w = 4$.

STEP-BY-STEP SOLUTION:

Swap the letters for the numbers in brackets:

$$P = 2(7) + 2(4)$$

Multiply first:

$$P = 14 + 8$$

$$P = 22$$

$$22$$

EXAMPLE 6.2

A formula is $v = u + at$. Work out v when $u = 15$, $a = -4$ and $t = 6$.

STEP-BY-STEP SOLUTION:

Swap the letters for the numbers:

$$v = (15) + (-4 \times 6)$$

Multiply first:

$$-4 \times 6 = -24$$

Add:

$$v = 15 + (-24)$$

$$v = -9$$

$$-9$$

ALGEBRA — EXPRESSIONS & FORMULAE

Function Machines

1 Function Machines

□ CORE CONCEPT & METHOD

A **function machine** changes an input number into an output number by following a rule.

To find the **output**, push the input through the machine from left to right.

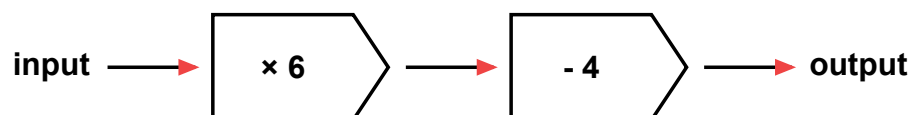
To find the **input**, you must work backwards from the output using the **inverse** (opposite) operations!

COMMON MISTAKE: When going BACKWARDS from Output to Input, reverse BOTH the order AND every sign ($+$ $\leftrightarrow$ $-$, $\times$ $\leftrightarrow$ $\div$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Here is a number machine.



What is the output when the input is 8?

STEP-BY-STEP SOLUTION:

Work left to right:

$$8 \times 6 = 48$$

$$48 - 4 = 44$$

44

2 Missing Operations & Equations

□ CORE CONCEPT & METHOD

Sometimes a machine is broken and missing an operation! Put the input through the first box, then ask yourself: *what do I need to do to get to the output?*

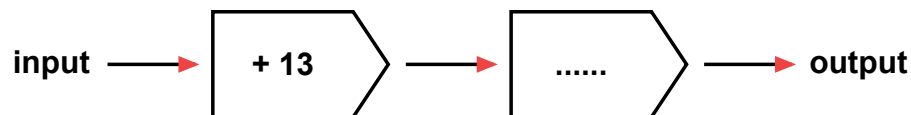
If a question asks when the input and output are the **same value**, you can use algebra! Let the input be x and the output be x , then solve the equation.

COMMON MISTAKE: If Input = Output, call the unknown number x , follow the machine to form an equation, and solve for x .

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Here is a number machine.



When the input is 11 the output is 3.

Find the missing operation.

STEP-BY-STEP SOLUTION:

Input is 11.

After the first box (+13), we have $11 + 13 = 24$.

We need to get from 24 to the output (3).

How do we get from 24 to 3? We could subtract 21, or divide by 8.

Usually these are simple operations. Let's use -21 or $\div 8$ (either works!).

$$-21$$

3 Real-Life Formulae**□ CORE CONCEPT & METHOD**

Formulas in real life are just function machines in disguise! Just follow the instructions step by step.

COMMON MISTAKE: Subtract the fixed starting charge FIRST before dividing by the cost per mile.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

A rule to calculate a courier delivery fee is:

£3.50 booking fee plus £1.80 per mile

Work out how much an 8 mile courier delivery would cost.

STEP-BY-STEP SOLUTION:

8 miles means we do $8 \times 1.80 = 14.40$

Then add the booking fee of £3.50:

$14.40 + 3.50 = 17.90$

$$£17.90$$

Linear Equations

1 One-Step Equations

□ CORE CONCEPT & METHOD

An equation is like a set of weighing scales. Whatever you do to one side, you must do to the other to keep it balanced!

To find x , do the **opposite operation** to get it by itself.

COMMON MISTAKE: Always do the OPPOSITE operation to both sides: for $\frac{p}{6} = 7$, multiply by 6 ($p = 42$), do not divide.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Solve $x + 7 = 16$

STEP-BY-STEP SOLUTION:

The opposite of adding 7 is **subtracting 7**. Do this to both sides to balance the equation!

$$\begin{array}{rcl} x + 7 & = & 16 \\ -7 & & -7 \\ \hline x & = & 9 \end{array}$$

$$x = 9$$

EXAMPLE 1.2

Solve $4y = 28$

STEP-BY-STEP SOLUTION:

Remember that $4y$ means $4 \times y$. The opposite of multiplying by 4 is **dividing by 4**.

$$\begin{array}{rcl} 4y & = & 28 \\ \div 4 & & \div 4 \\ \hline y & = & 7 \end{array}$$

$$y = 7$$

EXAMPLE 1.3

Solve $\frac{x}{6} = 5$

STEP-BY-STEP SOLUTION:

The opposite of dividing by 6 is **multiplying by 6**. Do this to both sides to balance the equation!

$$\begin{array}{rcl} \frac{x}{6} & = & 5 \\ \times 6 & & \times 6 \\ \hline x & = & 30 \end{array}$$

$$x = 30$$

2 Two-Step Equations

□ CORE CONCEPT & METHOD

When there are two steps, always get rid of the plain number (+ or -) first, and deal with the multiplier last!

COMMON MISTAKE: Undo the + or - number FIRST before dividing — in $4x - 6 = 30$, add 6 first to get $4x = 36$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Solve $3x + 2 = 17$

STEP-BY-STEP SOLUTION:

1. Do the opposite of +2:

$$\begin{array}{rcl} 3x + 2 & = & 17 \\ -2 & & -2 \\ \hline 3x & = & 15 \end{array}$$

2. Do the opposite of $\times 3$:

$$\begin{array}{rcl} 3x & = & 15 \\ \div 3 & & \div 3 \\ \hline x & = & 5 \end{array}$$

$$x = 5$$

EXAMPLE 2.2

Tariq has x stickers.

Meera has 8 more stickers than Tariq.

Kieran has 2 fewer stickers than Tariq.

Together they have 66 stickers in total.

Form an equation and solve it to find how many stickers Tariq has.

STEP-BY-STEP SOLUTION:

1. Write an expression for each person:

Tariq = x

Meera = $x + 8$

Kieran = $x - 2$

2. Add them all together to equal 66:

$$x + (x + 8) + (x - 2) = 66$$

Collect the terms ($x + x + x = 3x$ and $8 - 2 = 6$):

$$3x + 6 = 66$$

3. Solve it!

$$\begin{array}{rcl} 3x + 6 & = & 66 \\ -6 & & -6 \\ \hline 3x & = & 60 \end{array}$$

Now divide by 3:

$$\begin{array}{rcl} 3x & = & 60 \\ \div 3 & & \div 3 \\ \hline x & = & 20 \end{array}$$

$$x = 20$$

EXAMPLE 2.3

A student museum pass costs £ x .

An adult museum pass costs £11.

Nadia bought 4 student passes and 3 adult passes.

The total cost was £65.

Form an equation to find the price of a student museum pass.

STEP-BY-STEP SOLUTION:

1. Find the cost of the adult passes first:

$$3 \times £11 = £33$$

2. Set up the total cost equation:

4 student passes plus the adult passes equals £65.

$$4x + 33 = 65$$

3. Solve it!

$$\begin{array}{rcl} 4x + 33 & = & 65 \\ -33 & & -33 \\ \hline 4x & = & 32 \end{array}$$

Now divide by 4:

$$\begin{array}{rcl} 4x & = & 32 \\ \div 4 & & \div 4 \\ \hline x & = & 8 \end{array}$$

$$x = 8$$

EXAMPLE 2.4

Solve $\frac{4x}{3} - 5 = 3$

STEP-BY-STEP SOLUTION:

1. Deal with the plain number first. The opposite of -5 is $+5$:

$$\begin{array}{rcl} \frac{4x}{3} - 5 & = & 3 \\ +5 & & +5 \\ \hline \frac{4x}{3} & = & 8 \end{array}$$

2. Now deal with the fraction. The opposite of dividing by 3 is **multiplying by 3**:

$$\begin{array}{rcl} \frac{4x}{3} & = & 8 \\ \times 3 & & \times 3 \\ \hline 4x & = & 24 \end{array}$$

3. Finally, the opposite of multiplying by 4 is **dividing by 4**:

$$\begin{array}{rcl} 4x & = & 24 \\ \div 4 & & \div 4 \\ \hline x & = & 6 \end{array}$$

$$x = 6$$

Coordinates & Midpoints

1 Coordinates in 4 Quadrants & Horizontal/Vertical Lines

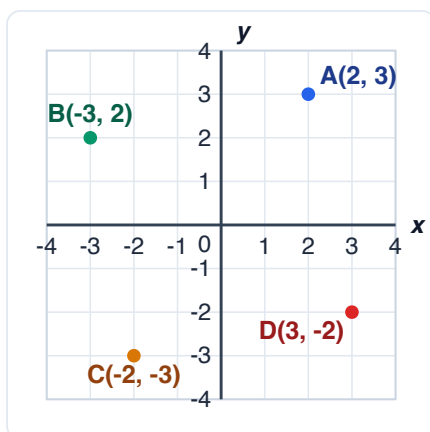
□ CORE CONCEPT & METHOD

Coordinates in 4 Quadrants & Horizontal/Vertical Lines

Every point on a Cartesian grid is written as an ordered pair (x, y) : always move horizontally along the x -axis first, then vertically parallel to the y -axis.

(Along the corridor, up the stairs)

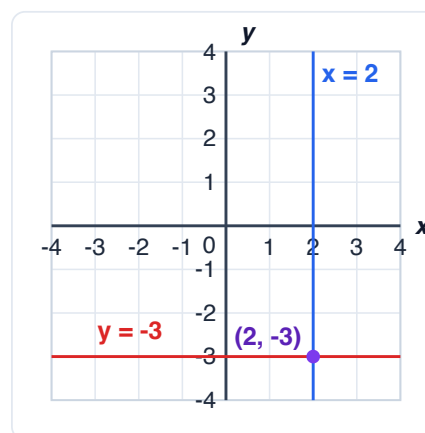
1. Reading (x, y) in All 4 Quadrants



Start at the origin $(0, 0)$:

- **x -coordinate first:** positive = right, negative = left.
- **y -coordinate second:** positive = up, negative = down.

2. Vertical ($x = a$) & Horizontal ($y = b$) Lines



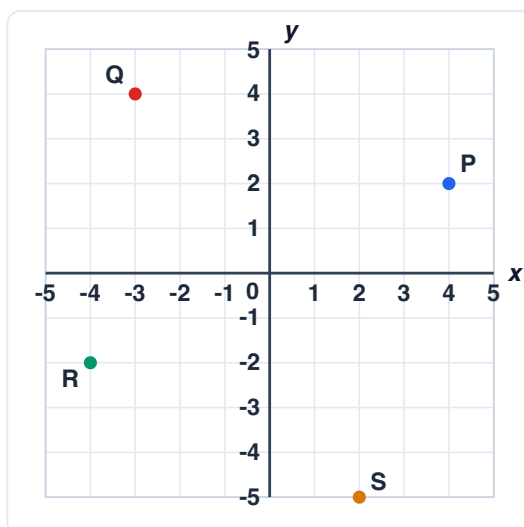
- A **vertical line** crosses the x -axis at a , so every point on it has $x = a$ (e.g. $x = 2$).
- A **horizontal line** crosses the y -axis at b , so every point on it has $y = b$ (e.g. $y = -3$).
- The lines $x = 2$ and $y = -3$ intersect at $(2, -3)$.

COMMON MISTAKE: Go **ALONG** (x) before **UP/DOWN** (y), and remember $y = 3$ is a **HORIZONTAL** line cutting the y -axis at 3.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Four points P , Q , R , and S are plotted on the coordinate grid below.



Write down the coordinates of point Q .

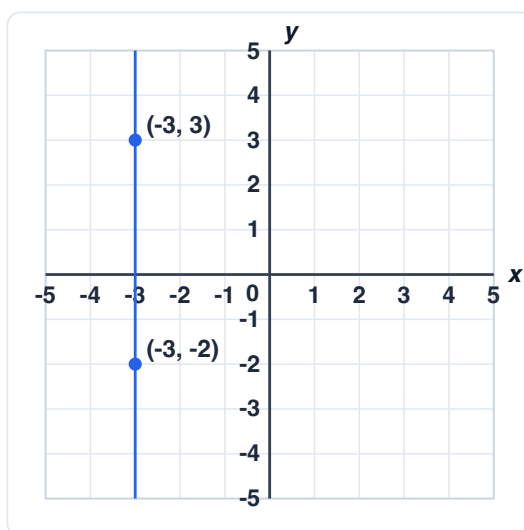
STEP-BY-STEP SOLUTION:

1. Start at the origin $(0, 0)$ and move horizontally along the x -axis to align with Q : this is at $x = -3$ (3 units left).
2. Move vertically up to Q : this is at $y = 4$ (4 units up).
3. Write the coordinates in the form $(x, y) = (-3, 4)$.

$$(-3, 4)$$

EXAMPLE 1.2

Write down the equation of the straight line shown in blue on the coordinate grid below.

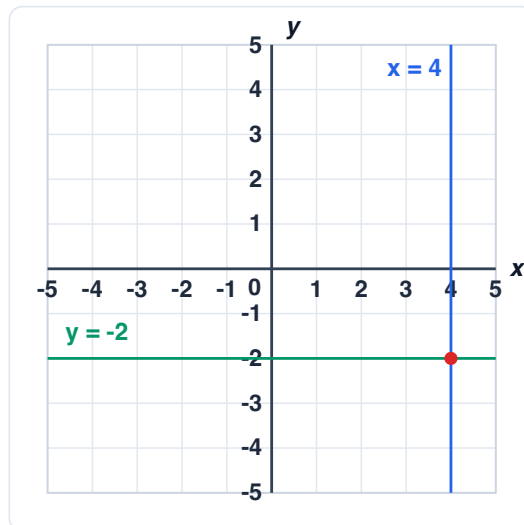
**STEP-BY-STEP SOLUTION:**

1. Every point on the vertical line has the exact same x -coordinate, such as $(-3, 3)$, $(-3, 0)$, and $(-3, -2)$.
2. Since the line crosses the x -axis at -3 , its equation is $x = -3$.

$$x = -3$$

EXAMPLE 1.3

The vertical line $x = 4$ and the horizontal line $y = -2$ are drawn on the coordinate grid below.



Write down the coordinates of their point of intersection.

STEP-BY-STEP SOLUTION:

1. Every point on the vertical line $x = 4$ has an x -coordinate of 4.
2. Every point on the horizontal line $y = -2$ has a y -coordinate of -2 .
3. Therefore, the two lines intersect at $(4, -2)$.

$(4, -2)$

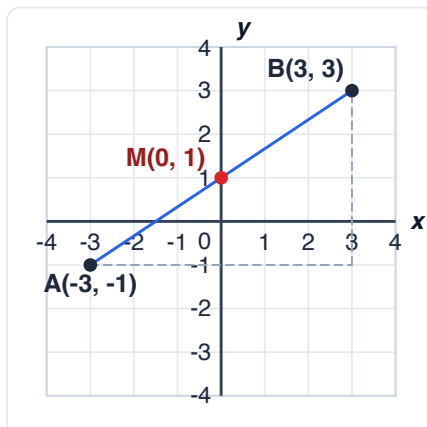
2 Midpoint of a Line Segment

□ CORE CONCEPT & METHOD

Midpoint of a Line Segment

The **midpoint** M of a line segment AB is the exact halfway point between $A(x_1, y_1)$ and $B(x_2, y_2)$.

1. Visual Midpoint on a Grid



From $A(-3, -1)$ to $B(3, 3)$, the horizontal run is 6 units and the vertical rise is 4 units. Halfway is 3 units right and 2 units up, giving $M(0, 1)$.

2. The Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Add the two x -coordinates and divide by 2 (the mean of the x 's).
- Add the two y -coordinates and divide by 2 (the mean of the y 's).
- Take care with negative signs: e.g.

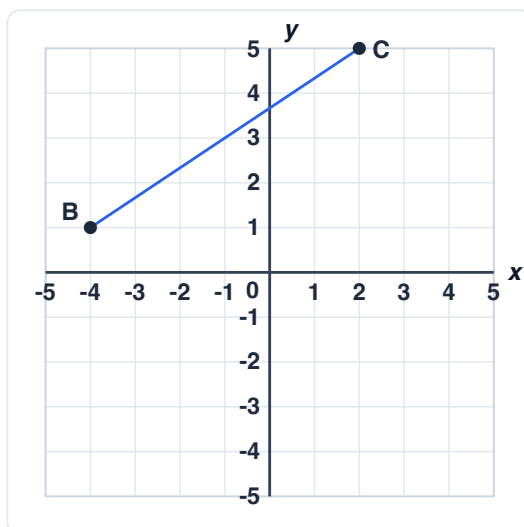
$$\frac{-5 + 2}{2} = \frac{-3}{2} = -1.5.$$

COMMON MISTAKE: To find a midpoint, ADD the coordinates and divide by 2 — never subtract them.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Line segment BC is plotted on the coordinate grid below.



Find the coordinates of the midpoint of BC .

STEP-BY-STEP SOLUTION:

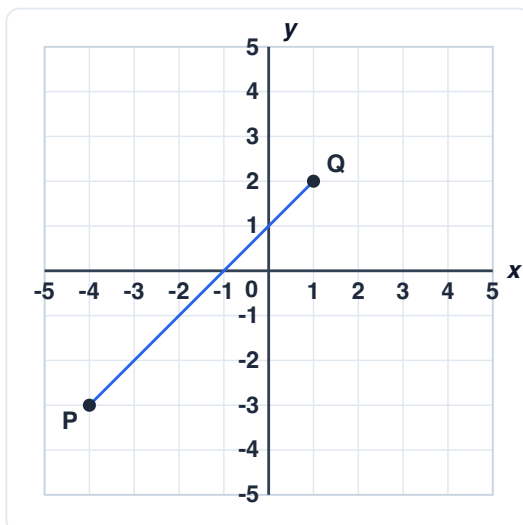
1. Read the coordinates of the endpoints from the grid: $B(-4, 1)$ and $C(2, 5)$.
2. Average the x -coordinates and the y -coordinates:

$$M = \left(\frac{-4 + 2}{2}, \frac{1 + 5}{2} \right) = \left(\frac{-2}{2}, \frac{6}{2} \right) = (-1, 3)$$

$$(-1, 3)$$

EXAMPLE 2.2

Line segment PQ is shown on the coordinate grid below.



Work out the coordinates of the midpoint of PQ .

STEP-BY-STEP SOLUTION:

1. Read the coordinates of the endpoints from the grid: $P(-4, -3)$ and $Q(1, 2)$, then substitute into the midpoint formula:

$$x_M = \frac{-4 + 1}{2} = \frac{-3}{2} = -1.5$$

$$y_M = \frac{-3 + 2}{2} = \frac{-1}{2} = -0.5$$

2. The midpoint is $(-1.5, -0.5)$.

$$(-1.5, -0.5)$$

EXAMPLE 2.3

Point A has coordinates $(-14, 26)$ and point B has coordinates $(8, -10)$.

Find the coordinates of the midpoint of line segment AB .

STEP-BY-STEP SOLUTION:

1. Use the midpoint formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$:

$$x_M = \frac{-14 + 8}{2} = \frac{-6}{2} = -3$$

$$y_M = \frac{26 + (-10)}{2} = \frac{16}{2} = 8$$

2. The coordinates of the midpoint are $(-3, 8)$.

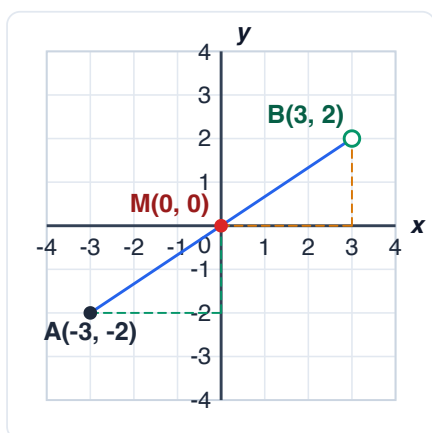
$$(-3, 8)$$

3 Reverse Midpoints & Coordinates of Shapes

CORE CONCEPT & METHOD

Reverse Midpoints & Coordinates of Shapes

Use equal coordinate steps (translation vectors) to find an unknown endpoint from a midpoint or to complete squares, rectangles, parallelograms, and isosceles triangles.

1. Finding a Missing Endpoint (**$A \rightarrow M \rightarrow B$)**

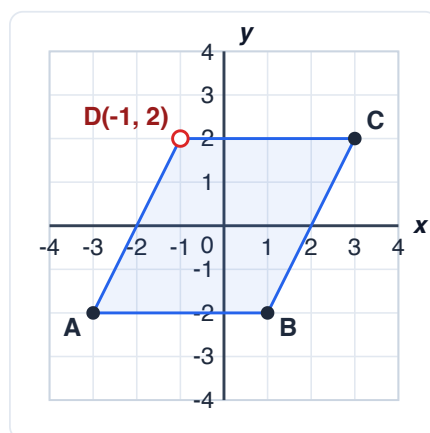
The step from A to M is identical to the step from M to B :

$$\bullet x: -3 \xrightarrow{+3} 0 \xrightarrow{+3} 3$$

$$\bullet y: -2 \xrightarrow{+2} 0 \xrightarrow{+2} 2$$

Or algebraically: $x_B = 2x_M - x_A$ and

$$y_B = 2y_M - y_A.$$

2. Missing Vertex of a Parallelogram /**Square**

In any parallelogram $ABCD$, opposite sides are parallel and equal in length, so the step from B to A equals the step from C to D :

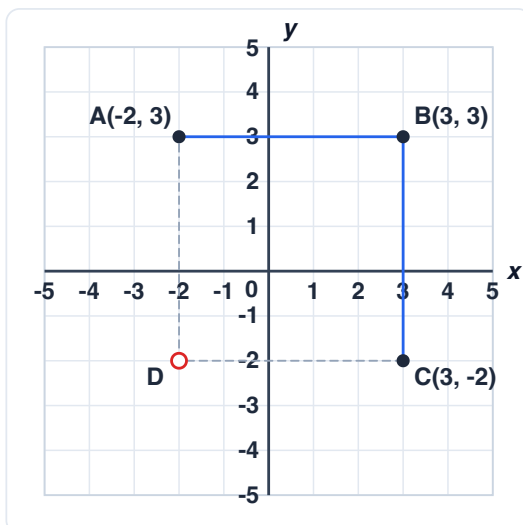
From $B(1, -2)$ to $A(-3, -2)$ is 4 units left, so from $C(3, 2)$ go 4 units left to $D(-1, 2)$.

COMMON MISTAKE: Do not average the given end and the midpoint — count the jump from A to M , then repeat the exact same jump to B .

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Three vertices $A(-2, 3)$, $B(3, 3)$, and $C(3, -2)$ of a square $ABCD$ are plotted on the grid below.



Find the coordinates of vertex D .

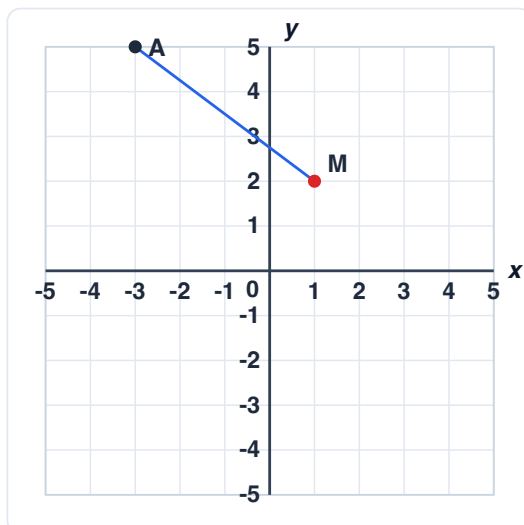
STEP-BY-STEP SOLUTION:

1. Side AB is horizontal of length 5 units (from $x = -2$ to $x = 3$) and side BC is vertical of length 5 units (from $y = 3$ to $y = -2$).
2. Vertex D is directly below A ($x = -2$) and directly to the left of C ($y = -2$).
3. Therefore, the coordinates of D are $(-2, -2)$.

$(-2, -2)$

EXAMPLE 3.2

Point A and the midpoint M of line segment AB are plotted on the coordinate grid below.



Find the coordinates of point B .

STEP-BY-STEP SOLUTION:

1. Read from the grid: $A(-3, 5)$ and midpoint $M(1, 2)$. Find the step from A to M :

- x -step: from -3 to 1 is $+4$
- y -step: from 5 to 2 is -3

2. Apply the exact same step from $M(1, 2)$ to reach B :

$$x_B = 1 + 4 = 5, \quad y_B = 2 - 3 = -1$$

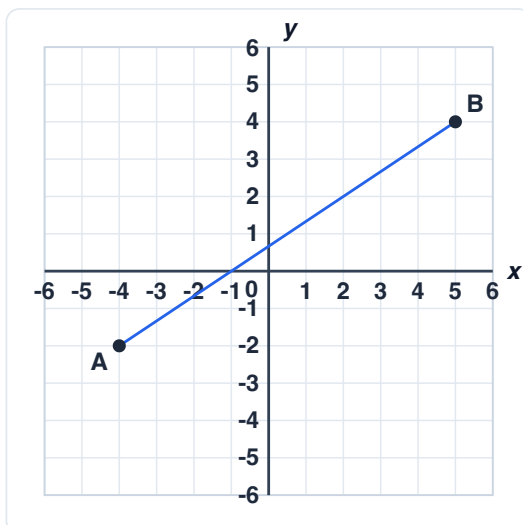
3. Thus B has coordinates $(5, -1)$.

$(5, -1)$

EXAMPLE 3.3

Line segment AB is drawn on the grid below.

Point P lies on AB such that $AP : PB = 1 : 2$.



Work out the coordinates of point P .

STEP-BY-STEP SOLUTION:

1. Read from the grid: $A(-4, -2)$ and $B(5, 4)$. The ratio $AP : PB = 1 : 2$ means P is $\frac{1}{1+2} = \frac{1}{3}$ of the way from A to B .

2. Calculate the total change in x and y from $A(-4, -2)$ to $B(5, 4)$:

$$\Delta x = 5 - (-4) = 9, \quad \Delta y = 4 - (-2) = 6$$

3. Add $\frac{1}{3}$ of each change to the coordinates of A :

$$x_P = -4 + \frac{1}{3}(9) = -4 + 3 = -1$$

$$y_P = -2 + \frac{1}{3}(6) = -2 + 2 = 0$$

4. Point P has coordinates $(-1, 0)$.

$$(-1, 0)$$

Angles in Lines, Triangles & Quadrilaterals

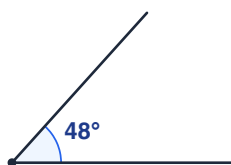
1 Angles at a Point & on a Straight Line

□ CORE CONCEPT & METHOD

Types of Angles & Core Angle Facts

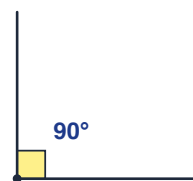
Master the four angle classifications and the fundamental rules for angles on straight lines and around a point.

1. Acute Angle ($< 90^\circ$)



An **acute angle** is strictly less than 90° ($0^\circ < \theta < 90^\circ$).

2. Right Angle ($= 90^\circ$)



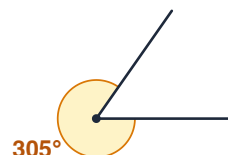
A **right angle** is exactly 90° (a quarter turn), shown by a square corner symbol. Angles inside a right angle sum to 90° .

3. Obtuse Angle (90° to 180°)



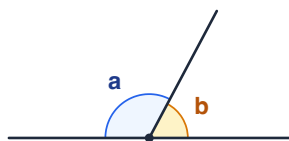
An **obtuse angle** is greater than 90° but less than 180° ($90^\circ < \theta < 180^\circ$).

4. Reflex Angle (180° to 360°)



A **reflex angle** is greater than 180° and less than 360° ($180^\circ < \theta < 360^\circ$).

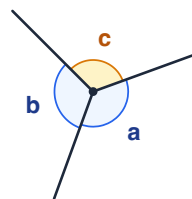
5. Angles on a Straight Line



Adjacent angles on a straight line (a half turn) always add up to 180° :

$$a + b = 180^\circ$$

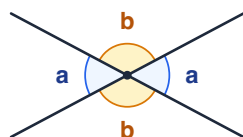
6. Angles Around a Point



Angles meeting around a single point (a full turn) always add up to 360° :

$$a + b + c = 360^\circ$$

7. Vertically Opposite Angles



When two straight lines intersect, the **vertically opposite angles are equal** ($a = a$ and $b = b$).

Exam Rule — Stating Geometric Reasons: Always use the word Angles when stating a reason:

- e.g. Angles on a straight line add up to 180° .
- Angles around a point add up to 360° .

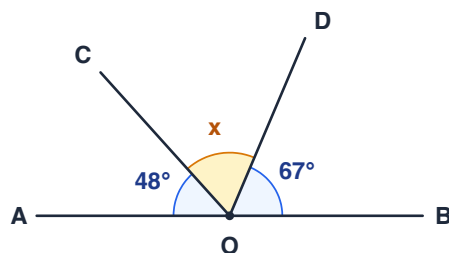
COMMON MISTAKE: Straight lines add to 180° (NOT 100°), and a little square corner always means 90° even with no number written.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

AOB is a straight line.

Angle $AOC = 48^\circ$ and angle $BOD = 67^\circ$.



Work out the size of angle x .

STEP-BY-STEP SOLUTION:

1. Angles on a straight line add up to 180° .

2. Add the known angles:

$$48^\circ + 67^\circ = 115^\circ$$

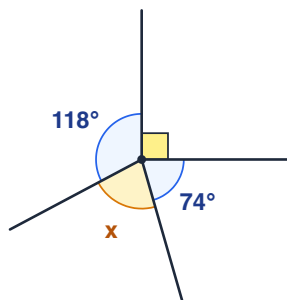
3. Subtract from 180° :

$$x = 180^\circ - 115^\circ = 65^\circ$$

$$65^\circ$$

EXAMPLE 1.2

Four rays meet at a point.



Work out the size of angle x .

STEP-BY-STEP SOLUTION:

1. Angles around a point add up to 360° , and the square symbol represents a right angle (90°).

2. Add the three known angles:

$$90^\circ + 118^\circ + 74^\circ = 282^\circ$$

3. Subtract from 360° :

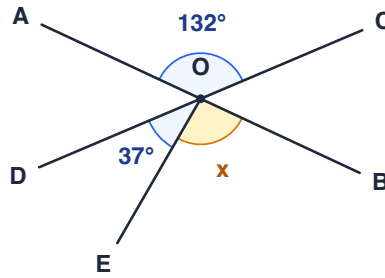
$$x = 360^\circ - 282^\circ = 78^\circ$$

$$78^\circ$$

EXAMPLE 1.3

Straight lines AB and CD intersect at O .

Angle $AOC = 132^\circ$ and angle $DOE = 37^\circ$.



Work out the size of angle x .

STEP-BY-STEP SOLUTION:

1. Vertically opposite angles are equal, so:

$$\angle BOD = \angle AOC = 132^\circ$$

2. Since ray OE splits $\angle BOD$:

$$x = 132^\circ - 37^\circ = 95^\circ$$

$$95^\circ$$

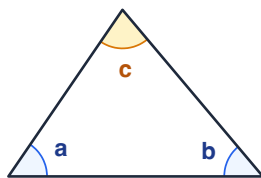
2 Angles in Triangles & Quadrilaterals

□ CORE CONCEPT & METHOD

Angles in Triangles & Quadrilaterals

Every triangle has angles adding to 180° , and every quadrilateral can be split into two triangles so its angles add to 360° .

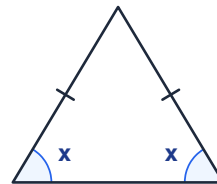
1. Triangle Angle Sum (180°)



The three interior angles in any triangle always add up to 180° :

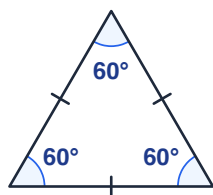
$$a + b + c = 180^\circ$$

2. Isosceles Triangle



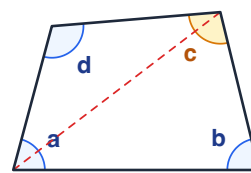
An **isosceles triangle** has two equal sides (marked with ticks) and **two equal base angles** opposite those sides.

3. Equilateral Triangle



An **equilateral triangle** has three equal sides and three equal angles of 60° each ($180^\circ \div 3 = 60^\circ$).

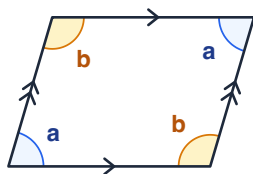
4. General Quadrilateral (360°)



A diagonal splits any quadrilateral into 2 triangles ($2 \times 180^\circ$), so its 4 interior angles sum to 360° :

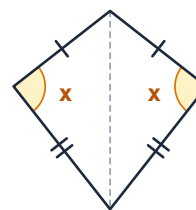
$$a + b + c + d = 360^\circ$$

5. Parallelogram & Rhombus



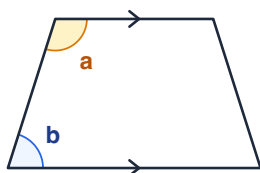
In a **parallelogram** or **rhombus**, **opposite angles are equal** ($a = a, b = b$) and adjacent angles add up to 180° ($a + b = 180^\circ$). (In a **square** or **rectangle**, all four angles are 90° .)

6. Kite



A **kite** has two pairs of equal adjacent sides and **one pair of opposite angles that are equal** (the angles x between the unequal sides).

7. Trapezium



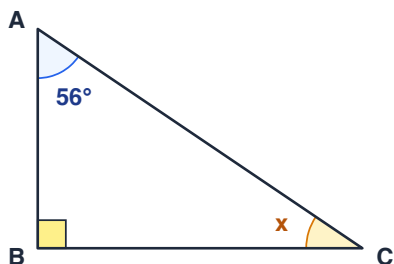
A **trapezium** has one pair of parallel sides. Each pair of co-interior angles between the parallel sides adds up to 180° ($a + b = 180^\circ$).

COMMON MISTAKE: Tick marks on two sides mean an ISOSCELES triangle — the two angles opposite those equal sides are equal.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Triangle ABC is right-angled at B , and angle $BAC = 56^\circ$.



Work out the size of angle x .

STEP-BY-STEP SOLUTION:

- Angles in a triangle add up to 180° , and $\angle ABC = 90^\circ$.
- Add the known angles:

$$90^\circ + 56^\circ = 146^\circ$$

- Subtract from 180° :

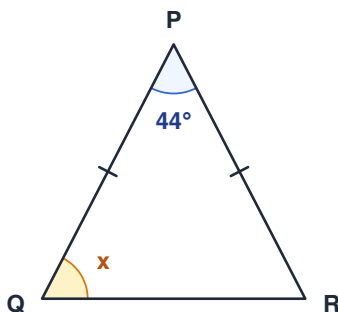
$$x = 180^\circ - 146^\circ = 34^\circ$$

$$34^\circ$$

EXAMPLE 2.2

Triangle PQR is isosceles with $PQ = PR$.

Angle $QPR = 44^\circ$.



Work out the size of base angle x .

STEP-BY-STEP SOLUTION:

- Because $PQ = PR$, the two base angles at Q and R are equal ($\angle PQR = \angle PRQ = x$).
- Subtract the vertex angle from 180° :

$$180^\circ - 44^\circ = 136^\circ$$

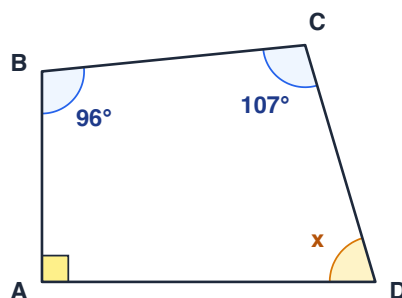
- Divide equally between the two base angles:

$$x = 136^\circ \div 2 = 68^\circ$$

$$68^\circ$$

EXAMPLE 2.3

In quadrilateral $ABCD$, angle $DAB = 90^\circ$, angle $ABC = 96^\circ$, and angle $BCD = 107^\circ$.



Work out the size of angle x .

STEP-BY-STEP SOLUTION:

1. The interior angles in a quadrilateral add up to 360° .
2. Add the three known angles:

$$90^\circ + 96^\circ + 107^\circ = 293^\circ$$

3. Subtract from 360° :

$$x = 360^\circ - 293^\circ = 67^\circ$$

$$67^\circ$$

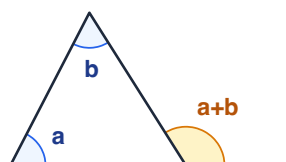
3 Multi-Step & Algebraic Angle Problems

□ CORE CONCEPT & METHOD

Multi-Step, Compound & Algebraic Angle Problems

Higher-tariff GCSE questions combine multiple polygons, hidden isosceles triangles, ratios, and algebraic expressions.

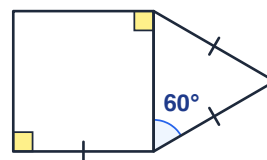
1. Exterior Angle of a Triangle



When one side of a triangle is extended, the **exterior angle equals the sum of the two interior opposite angles**:

$$\text{Exterior Angle} = a + b$$

2. Compound Polygons (Square + Equilateral)



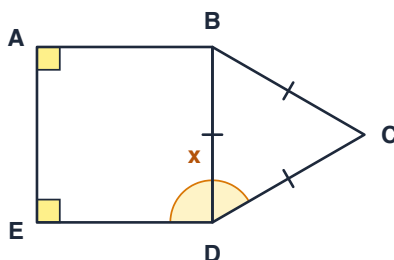
When a square shares a side with an equilateral triangle, all sides have the same length! Mark 90° in the square, 60° in the equilateral triangle, and watch for hidden isosceles triangles.

COMMON MISTAKE: A reflex angle goes the LONG way around the outside ($> 180^\circ$), so subtract the inside angle from 360° .

MODEL WORKED EXAMPLES

EXAMPLE 3.1

$ABDE$ is a square and BCD is an equilateral triangle.



Work out the size of obtuse angle EDC (x).

STEP-BY-STEP SOLUTION:

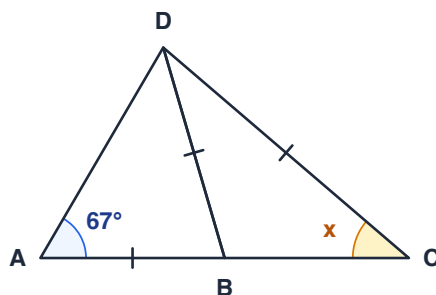
1. Each interior angle of square $ABDE$ is 90° , so $\angle EDB = 90^\circ$.
2. Each interior angle of equilateral triangle BCD is 60° , so $\angle BDC = 60^\circ$.
3. Add the two adjacent angles at vertex D :

$$x = 90^\circ + 60^\circ = 150^\circ$$

$$150^\circ$$

EXAMPLE 3.2

ABC is a straight line. $AB = BD = DC$ and angle $BAD = 67^\circ$.



Work out the size of angle BCD (x).

STEP-BY-STEP SOLUTION:

1. In isosceles triangle ABD , since $AB = BD$, $\angle ADB = \angle BAD = 67^\circ$.
2. Angles in $\triangle ABD$ sum to 180° :

$$\angle ABD = 180^\circ - (67^\circ + 67^\circ) = 46^\circ$$

3. Angles on straight line ABC sum to 180° :

$$\angle DBC = 180^\circ - 46^\circ = 134^\circ$$

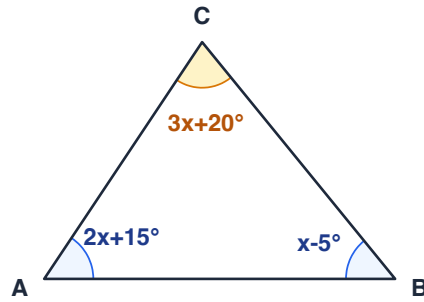
4. Since $BD = DC$, $\triangle BDC$ is isosceles:

$$x = (180^\circ - 134^\circ) \div 2 = 23^\circ$$

$$23^\circ$$

EXAMPLE 3.3

The three interior angles of triangle ABC are $(2x + 15)^\circ$, $(3x + 20)^\circ$, and $(x - 5)^\circ$.



Form an equation and work out the value of x .

STEP-BY-STEP SOLUTION:

1. Angles in a triangle add up to 180° :

$$(2x + 15) + (3x + 20) + (x - 5) = 180$$

2. Collect like terms:

$$6x + 30 = 180$$

3. Solve for x :

$$6x = 150 \implies x = 25$$

25

Area and Perimeter

1 Squares and Rectangles

□ CORE CONCEPT & METHOD

Area is the space inside a shape (measured in squares, e.g., cm^2).

Perimeter is the total distance around the outside edge.

COMMON MISTAKE: Do not muddle Area and Perimeter: Perimeter = add ALL outside edges (including unlabelled sides); Area = length $\times$ width.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

A rectangle has a length of 11 cm and a width of 6 cm. Find **(a)** its area and **(b)** its perimeter.



STEP-BY-STEP SOLUTION:

(a) Area:

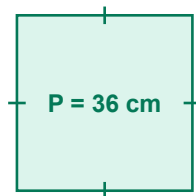
$$\text{Area} = \text{length} \times \text{width} = 11 \times 6 = 66 \text{ cm}^2$$

(b) Perimeter:

$$\text{Perimeter} = 2(l + w) = 2(11 + 6) = 2 \times 17 = 34 \text{ cm}$$

EXAMPLE 1.2

A square has a perimeter of 36 cm. Find the area of the square.



STEP-BY-STEP SOLUTION:

First find the side length from the perimeter:

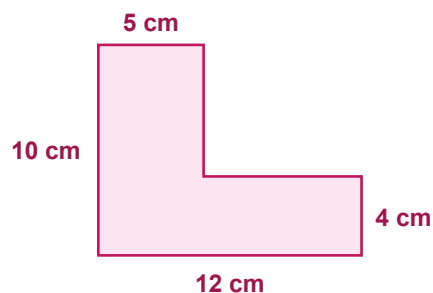
$$\text{side} = 36 \div 4 = 9 \text{ cm}$$

Then find the area:

$$\text{Area} = 9 \times 9 = 81 \text{ cm}^2$$

EXAMPLE 1.3

Find the area and perimeter of this compound shape.

**STEP-BY-STEP SOLUTION:****1. Find the missing sides:**

Inner horizontal = $12 - 5 = 7$ cm

Inner vertical = $10 - 4 = 6$ cm

2. Area (split vertically into two rectangles):

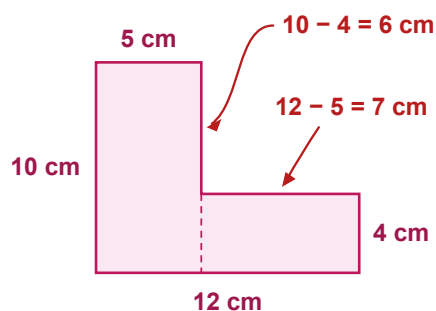
Left rectangle: $5 \times 10 = 50$ cm²

Right rectangle: $7 \times 4 = 28$ cm²

Total Area = $50 + 28 = 78$ cm²

3. Perimeter:

Add all the outside edges: $10 + 12 + 4 + 7 + 6 + 5 = 44$ cm

**2 Triangles & Parallelograms****CORE CONCEPT & METHOD**

Area of a Triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

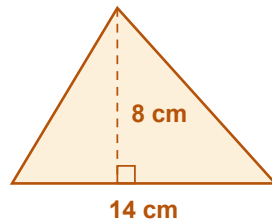
Area of a Parallelogram = base $\times$ perpendicular height

COMMON MISTAKE: Don't forget to HALVE base $\times$ height for a triangle, and always use the straight-up (90°) height, never the slanted edge.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Find the area of a triangle with a base of 14 cm and a perpendicular height of 8 cm.

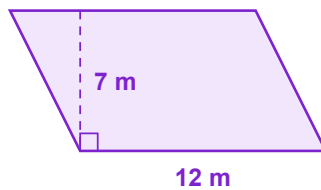
**STEP-BY-STEP SOLUTION:**

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \frac{1}{2} \times 14 \times 8 = 56 \text{ cm}^2$$

EXAMPLE 2.2

A parallelogram has a base of 12 m and a perpendicular height of 7 m. Find its area.

**STEP-BY-STEP SOLUTION:**

$$\text{Area} = \text{base} \times \text{perpendicular height}$$

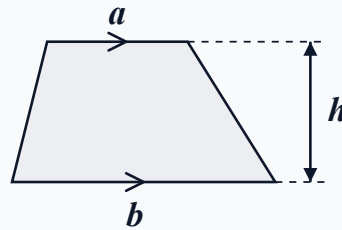
$$\text{Area} = 12 \times 7 = 84 \text{ m}^2$$

Remember to use the **perpendicular** height, not the slant side.

3 Trapeziums

□ CORE CONCEPT & METHOD

$$\text{Area of a Trapezium} = \frac{1}{2}(a + b)h$$



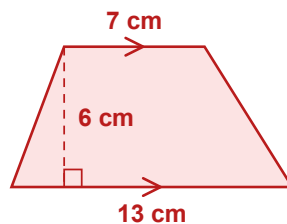
Add the parallel sides together, divide by 2, then multiply by the height!

COMMON MISTAKE: Add the two PARALLEL sides ($a + b$) inside the brackets first BEFORE halving and multiplying by the height.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

A trapezium has parallel sides of 7 cm and 13 cm. Its perpendicular height is 6 cm. Find its area.



STEP-BY-STEP SOLUTION:

Formula: $\frac{1}{2}(a + b)h$

Add parallel sides: $7 + 13 = 20$

Divide by 2: $20 \div 2 = 10$

Multiply by height: $10 \times 6 = 60 \text{ cm}^2$.

4 Grid & Logic Puzzles

□ CORE CONCEPT & METHOD

For shapes drawn on a centimetre grid, simply count the squares to find the base and perpendicular height!

For puzzles like '*the length is twice the width*', use algebra! Let the width be x and the length be $2x$, then solve for x using the area or perimeter given.

COMMON MISTAKE: Count the grid SQUARES (gaps between lines) for lengths, not the grid corner dots.

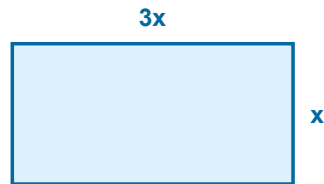
MODEL WORKED EXAMPLES

EXAMPLE 4.1

The length of a rectangle is three times the width of the rectangle.

The perimeter of the rectangle is 40 cm.

Find the area of the rectangle.

**STEP-BY-STEP SOLUTION:**

1. Let width = x . Length = $3x$.
2. Perimeter = $x + 3x + x + 3x = 8x$.
3. So, $8x = 40$, which means $x = 5$.
4. The width is 5 cm and the length is 15 cm.
5. Area = $5 \times 15 = 75 \text{ cm}^2$.

PROBABILITY — BASIC PROBABILITY

Probability Scale & Basics

1 The Probability Scale

□ CORE CONCEPT & METHOD

A probability is between 0 and 1 inclusive ($0 \leq P \leq 1$), written as a fraction, decimal, or percentage.

- **0** means Impossible
- **0.5 (or 1/2)** means an Even Chance
- **1** means Certain

COMMON MISTAKE: Never write a probability in words ("likely") or ratios ($1 : 2$) when asked for a number — write a fraction, decimal, or percentage from 0 to 1 inclusive.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

A box contains 4 green and 4 yellow marbles. Which value on the probability scale best describes the chance of picking a green marble at random?

STEP-BY-STEP SOLUTION:

There are 4 green marbles out of 8 in total, so $P(\text{green}) = \frac{4}{8} = 0.5$.

This sits exactly in the **middle** of the probability scale — an **Even Chance**.

EXAMPLE 1.2

Mark each event on the probability scale with its correct value:

- Picking a 10 from a set of cards numbered 1 to 8
- Landing on tails when flipping a fair coin
- A day of the week having the letter 'y' in its name

STEP-BY-STEP SOLUTION:

- Picking a 10 from cards 1 to 8: **Impossible** — probability = 0
- Landing on tails on a fair coin: **Even Chance** — probability = 0.5
- A day of the week having 'y' in its name: **Certain** — probability = 1

2 Basic Probability

□ CORE CONCEPT & METHOD

To find a probability:

$$P = \frac{\text{Number of successful outcomes}}{\text{Total number of possible outcomes}}$$

COMMON MISTAKE: Count ALL items for the bottom number of the fraction, not just the items that do NOT match.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

A crate contains 6 cartons of apple juice, 5 cartons of orange juice and 4 cartons of mango juice. A carton is picked at random. Find the probability of picking an orange juice carton.

STEP-BY-STEP SOLUTION:

Total cartons = $6 + 5 + 4 = 15$.

Number of orange cartons = 5.

$$P(\text{orange}) = \frac{5}{15} = \frac{1}{3}$$

EXAMPLE 2.2

There are 30 members in a drama club. 18 are juniors and the rest are seniors. One member is chosen at random. What is the probability of choosing a senior?

STEP-BY-STEP SOLUTION:

Number of seniors = $30 - 18 = 12$.

$$P(\text{senior}) = \frac{12}{30} = \frac{2}{5} = 0.4$$

3 The 'Not' Rule**CORE CONCEPT & METHOD**

The probability of something happening PLUS the probability of it NOT happening always equals 1.

$$P(\text{Not happens}) = 1 - P(\text{Happens})$$

COMMON MISTAKE: Line up decimals carefully when subtracting from 1: $1 - 0.74 = 0.26$, NOT 0.36.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

The probability that a train arrives early is 0.15. What is the probability that the train does NOT arrive early?

STEP-BY-STEP SOLUTION:

Step 1: Use the rule $P(\text{Not}) = 1 - P(\text{Happens})$.

$$1 - 0.15 = 0.85$$

The probability the train does not arrive early is 0.85.

EXAMPLE 3.2

The probability of a seed germinating is $\frac{4}{9}$. What is the probability of the seed NOT germinating?

STEP-BY-STEP SOLUTION:

Step 1: Use the rule $P(\text{Not}) = 1 - P(\text{Happens})$.

To subtract $\frac{4}{9}$ from 1, think of 1 as a full fraction: $\frac{9}{9}$.

$$\frac{9}{9} - \frac{4}{9} = \frac{5}{9}$$

The probability of not germinating is $\frac{5}{9}$.

4 Probability Tables

□ CORE CONCEPT & METHOD

If a table shows all the possible outcomes of an event, their probabilities **must add up to 1**.

COMMON MISTAKE: All probabilities in the table must add up to 1 (or 100%) — add the known numbers first, then subtract from 1.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

A biased spinner can land on Purple, Silver, Gold, or Teal. The table shows the probabilities that it lands on Purple, Silver, or Gold.

Colour	Purple	Silver	Gold	Teal
Probability	0.25	0.35	0.15	?

Work out the probability that it lands on Teal.

STEP-BY-STEP SOLUTION:

Step 1: Add up the known probabilities.

$$0.25 + 0.35 + 0.15 = 0.75$$

Step 2: Subtract the total from 1 because all probabilities must add up to 1.

$$1 - 0.75 = 0.25$$

The probability it lands on Teal is 0.25.

5 Spinners & Scales

□ CORE CONCEPT & METHOD

To find the probability of a **spinner**, count the total number of sections and the number of sections with the target letter/number!

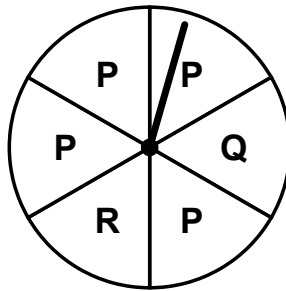
If a cross is placed on a **probability scale**, estimate its value based on how close it is to 0, 0.5, or 1.

COMMON MISTAKE: Check the total number of equal sections on the spinner — do not assume every spinner has 4 or 10 parts.

MODEL WORKED EXAMPLES

EXAMPLE 5.1

Maya spins a fair 6-sided spinner.



Write down the probability that the spinner lands on **P**.

STEP-BY-STEP SOLUTION:

Step 1: Count the total number of sections on the spinner. There are 6 sections.

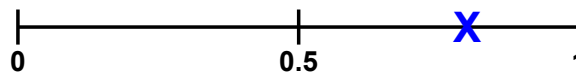
Step 2: Count how many sections have the letter P. There are 4 sections with P.

$$\text{Probability} = \frac{4}{6}$$

The probability is $\frac{4}{6}$ (or $\frac{2}{3}$).

EXAMPLE 5.2

The probability of a team winning a cup match is marked with a cross (X) on the probability scale.



Write down an estimate for this probability.

STEP-BY-STEP SOLUTION:

Step 1: Look at where the cross is positioned between 0 and 1.

It is past the 0.5 mark, and is three-fifths of the way from 0.5 to 1.

Step 2: Estimate a decimal value. Since it is 80% of the way along the scale, the probability is 0.8 (or $\frac{4}{5}$).

Estimated probability: 0.8.

PROBABILITY — COMBINATIONS

Systematic Listing

1 Counting Combinations

📖 CORE CONCEPT & METHOD

To find the total number of combinations, use the **Product Rule for Counting**: just multiply the number of options together!

If you have 4 Sandwiches and 5 Drinks, there are $4 \times 5 = 20$ possible meal deals.

COMMON MISTAKE: Multiply the number of choices ($6 \times 7 = 42$) rather than adding them ($6 + 7 = 13$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

A coin is tossed and a fair 4-sided spinner numbered 1, 2, 3, and 4 is spun. List all possible outcomes.

STEP-BY-STEP SOLUTION:

Use a systematic approach. The coin can be Heads (H) or Tails (T).

(H, 1), (H, 2), (H, 3), (H, 4)

(T, 1), (T, 2), (T, 3), (T, 4)

2 Digit Combinations

📖 CORE CONCEPT & METHOD

If you are making a 3-digit number from 4 cards (without reusing them):

- You have 4 choices for the first digit.
- You have 3 choices left for the second digit.
- You have 2 choices left for the third digit.

Total = $4 \times 3 \times 2 = 24$ possible numbers.

COMMON MISTAKE: Work in strict alphabetical or numerical order so you never miss a combination or write the same one twice.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Write down all the 2-digit numbers that can be made using the digits 2, 6, and 9. You can repeat digits.

STEP-BY-STEP SOLUTION:

Keep the first digit fixed, then change the second:

22, 26, 29

62, 66, 69

92, 96, 99

3 Sample Space Probabilities

□ CORE CONCEPT & METHOD

When dealing with two events (like drawing a card from two boxes), draw a quick **sample space diagram** (a grid) to list all the possible totals or products. Then count how many fit the condition!

COMMON MISTAKE: Draw a full grid so you see all $3 \times 3 = 9$ outcomes (or $6 \times 6 = 36$ for two dice) before counting winners.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Two fair 3-sided spinners, each numbered 1, 2, and 3, are spun and their scores are added together. What is the probability of getting a total of 4?

STEP-BY-STEP SOLUTION:

List all 9 outcomes (Spinner 1, Spinner 2):

$(1,1)=2$, $(1,2)=3$, $(1,3)=4$

$(2,1)=3$, $(2,2)=4$, $(2,3)=5$

$(3,1)=4$, $(3,2)=5$, $(3,3)=6$

There are 3 outcomes that give a total of 4: $(1,3)$, $(2,2)$, and $(3,1)$.

Probability = $\frac{3}{9} = \frac{1}{3}$.

4 Matches & Rankings

□ CORE CONCEPT & METHOD

Rankings: To place 4 teams in order (1st, 2nd, 3rd, 4th), it's $4 \times 3 \times 2 \times 1 = 24$ ways.

Matches: If 4 teams play each other once, Team 1 plays 3 teams, Team 2 plays 2 new teams, Team 3 plays 1 new team. Total = $3 + 2 + 1 = 6$ matches.

COMMON MISTAKE: Check if order matters: for 1st/2nd/3rd, AB and BA are DIFFERENT; for a match fixture, $A \text{ v } B$ is the SAME match as $B \text{ v } A$.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

A café offers 4 different sandwiches and 5 different drinks. How many different meal combinations of a sandwich and a drink can you choose?

STEP-BY-STEP SOLUTION:

Multiply the number of options together: $4 \text{ sandwiches} \times 5 \text{ drinks} = 20 \text{ combinations}$.

Averages

1 Mean, Median, Mode & Range

📖 CORE CONCEPT & METHOD

Mode: The most common number.

Median: The middle number (after putting them in order!).

Mean: Add them all up and divide by how many there are.

Range: Biggest number minus smallest number.

COMMON MISTAKE: PUT THE LIST IN ORDER from smallest to biggest before finding the median — never just pick the middle number of an unsorted list!

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Find the **mean, median, mode** and **range** of: 4, 9, 9, 3, 15

STEP-BY-STEP SOLUTION:

Step 1 – Order the data: 3, 4, 9, 9, 15

Mode (most common): **9** (appears twice).

Median (middle value, 5 numbers → 3rd value): **9**.

Range (largest – smallest):

$$15 - 3 = 12$$

Mean (sum ÷ count):

$$\frac{3 + 4 + 9 + 9 + 15}{5} = \frac{40}{5} = 8$$

EXAMPLE 1.2

Find the **median** of this list of 6 numbers: 5, 11, 2, 7, 4, 10

STEP-BY-STEP SOLUTION:

Step 1 – Order the data: 2, 4, 5, 7, 10, 11

Step 2 – Two middle values (6 numbers, so take the 3rd and 4th): 5 and 7.

Step 3 – Average them:

$$\frac{5 + 7}{2} = \frac{12}{2} = 6$$

Median = **6**.

2 Finding Missing Values

📖 CORE CONCEPT & METHOD

If you know the mean of 5 numbers is 10, then their **TOTAL SUM** must be $5 \times 10 = 50$.

If you know 4 of the numbers add up to 42, the missing number must be $50 - 42 = 8$.

COMMON MISTAKE: Multiply the mean by how many numbers there are **FIRST** to find the required total sum.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Five numbers have a mean of 9. Four of the numbers are 7, 12, 6 and 8. Find the missing number.

STEP-BY-STEP SOLUTION:

Step 1 – Find the required total:

If the mean is 9 and there are 5 numbers:

$$\text{Total} = 5 \times 9 = 45$$

Step 2 – Add the known numbers: $7 + 12 + 6 + 8 = 33$

Step 3 – Subtract:

$$\text{Missing number} = 45 - 33 = 12$$

EXAMPLE 2.2

Six numbers have a mean of 14. The mode is 18. Two numbers are hidden. The four known numbers are 9, 12, 18, 5. Find the sum of the two hidden numbers.

STEP-BY-STEP SOLUTION:

Step 1 – Required total: $6 \times 14 = 84$

Step 2 – Sum of known numbers: $9 + 12 + 18 + 5 = 44$

Step 3 – Sum of hidden numbers:

$$84 - 44 = 40$$

(Note: since the mode is 18, at least one hidden number must be 18, making the other $40 - 18 = 22$.)

3 Combining Means

📖 CORE CONCEPT & METHOD

Always work with **TOTALS**.

If the mean of 4 people is 20, their total is 80.

If a 5th person joins who is 30, the new total is $80 + 30 = 110$.

The new mean is $110 \div 5 = 22$.

COMMON MISTAKE: Never average the two means together — find each group's **TOTAL** (mean $\times$ count), add the totals, then divide by the new count.

MODEL WORKED EXAMPLES**EXAMPLE 3.1**

A group of 5 cyclists has a mean distance of 42 km. A 6th cyclist joins who rode 54 km. Find the new mean distance.

STEP-BY-STEP SOLUTION:

Step 1 – Find the old total:

$$5 \times 42 = 210 \text{ km}$$

Step 2 – Add the new person's value:

$$210 + 54 = 264 \text{ km}$$

Step 3 – Divide by the new count (6):

$$\text{New mean} = \frac{264}{6} = 44 \text{ km}$$

EXAMPLE 3.2

Team X has 8 players with a mean score of 55. Team Y has 12 players with a mean score of 65. Find the combined mean of all 20 players.

STEP-BY-STEP SOLUTION:

Step 1 – Find each team total:

$$\text{Team X: } 8 \times 55 = 440$$

$$\text{Team Y: } 12 \times 65 = 780$$

Step 2 – Combined total: $440 + 780 = 1220$

Step 3 – Combined mean:

$$\frac{1220}{20} = 61$$

Bar Charts

1 Interpreting Bar Charts

📖 CORE CONCEPT & METHOD

A bar chart shows data using rectangular bars.

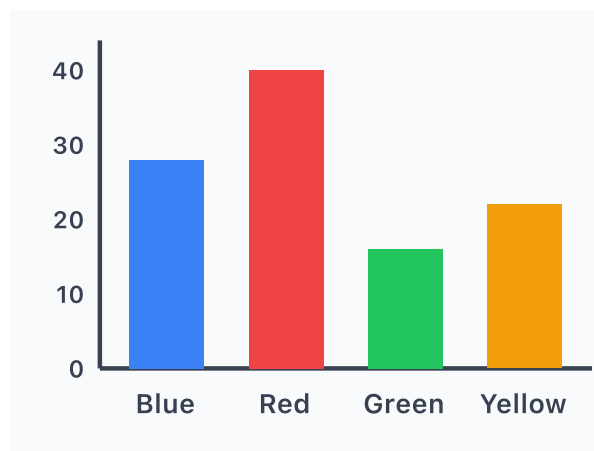
- The **height** of each bar tells you the frequency (how many).
- The **mode** is the category with the tallest bar.
- To find the **total**, you add up the heights of all the bars!

COMMON MISTAKE: Read the vertical scale carefully — one grid square often stands for 2, 5, or 10 units, not 1.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

The bar chart shows the favourite paint colours chosen by visitors at an art studio.



How many visitors chose Red? What is the mode?

STEP-BY-STEP SOLUTION:

Step 1 – Read the height of the Red bar: it reaches 40 on the vertical axis, so **40 visitors** chose Red.

Step 2 – Mode = the category with the *tallest* bar.
Red has the tallest bar, so the **mode is Red**.

Step 3 – Total visitors: $28 + 40 + 16 + 22 = 106$.

EXAMPLE 1.2

Using the same bar chart above, how many more visitors chose Red than Green?

STEP-BY-STEP SOLUTION:

Step 1 – Red = 40 visitors, **Green** = 16 visitors.

Step 2 – Difference:

$$40 - 16 = 24$$

24 more visitors chose Red than Green.

2 Dual Bar Charts

📖 CORE CONCEPT & METHOD

A dual (or multiple) bar chart shows two or more sets of data side-by-side for each category.

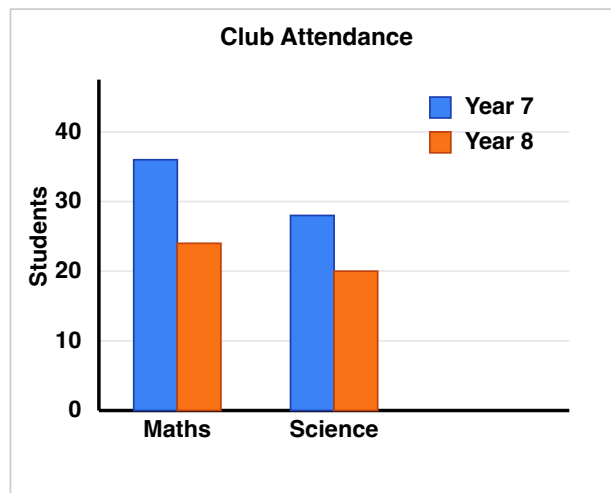
Always check the **key** to see which colour represents which group!

COMMON MISTAKE: Check the shaded Key before reading any bar so you do not read the wrong person or group.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

A dual bar chart compares Year 7 and Year 8 attendance at after-school clubs. In the Science club, 28 Year 7 students and 20 Year 8 students attended. What is the **total** number of students in the Science club?



STEP-BY-STEP SOLUTION:

Step 1 – Check the key to confirm which bar colour is Year 7 (blue) and which is Year 8 (orange).

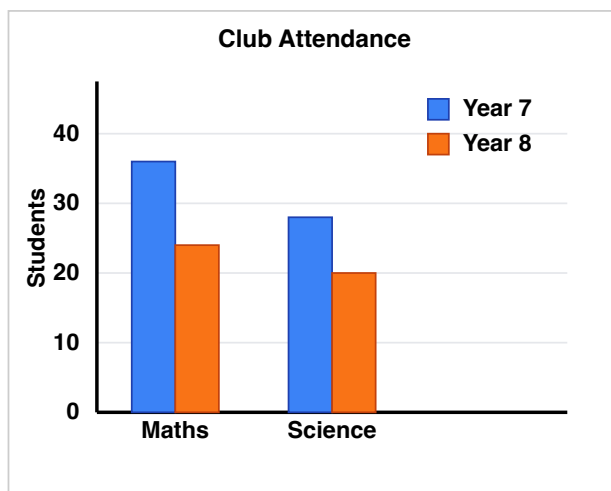
Step 2 – Read each bar height for Science: Year 7 = 28, Year 8 = 20.

Step 3 – Add them together:

$$28 + 20 = 48 \text{ students total}$$

EXAMPLE 2.2

In the same chart, 36 Year 7 students and 24 Year 8 students attended the Maths club. How many **more Year 7** than Year 8 students attended the Maths club?

**STEP-BY-STEP SOLUTION:**

Step 1 – Read Year 7 Maths bar (blue) = 36; Year 8 Maths bar (orange) = 24.

Step 2 – Difference:

$$36 - 24 = 12 \text{ more Year 7 students}$$

3 Tally Charts & Frequencies**□ CORE CONCEPT & METHOD**

A tally chart is used to record data quickly.

Each mark represents one item, and we group them in fives: **||||** for easier counting!

COMMON MISTAKE: Cross off each word in the raw list as you count it, and remember a crossed gate bundle stands for 5, not 4.

MODEL WORKED EXAMPLES**EXAMPLE 3.1**

Maya records the types of birds visiting a garden feeder.

Bird	Tally
Robin	
Sparrow	
Starling	

What is the frequency of Sparrows?

STEP-BY-STEP SOLUTION:

Step 1 – Find the row for Sparrow.

Step 2 – Count the tallies. Each **||||** represents 5.
Two groups of 5 makes 10, plus 3 single marks = 13.

Frequency = 13

4 Advanced Bar Charts

□ CORE CONCEPT & METHOD

To find the **total** of a numerical category (like 'total goals'), you must multiply the category number by its frequency, then add them all together!

When drawing a bar chart, you must always include:

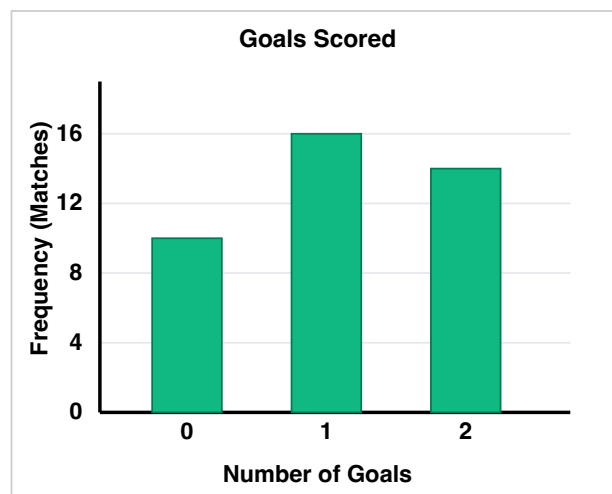
1. Axis labels (e.g. 'Frequency')
2. Equal gaps between the bars
3. A consistent, even scale on the vertical axis.

COMMON MISTAKE: When spotting mistakes on a bar chart, check THREE things: uneven scale jumps, missing axis labels, and unequal bar gaps.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

A bar chart shows the number of goals scored in 40 matches.



Work out the **total number of goals** scored across all matches.

STEP-BY-STEP SOLUTION:

Step 1 – Find the frequency (number of matches) for each goal amount:

0 goals: 10 matches

1 goal: 16 matches

2 goals: 14 matches

Step 2 – Multiply each goal amount by its frequency:

$$0 \times 10 = 0$$

$$1 \times 16 = 16$$

$$2 \times 14 = 28$$

Step 3 – Add them together to find total goals:

$$0 + 16 + 28 = 44 \text{ goals total}$$

Pie Charts

1 Interpreting Pie Charts

📖 CORE CONCEPT & METHOD

A full pie chart represents **360°**.

If a sector has an angle of 90°, it represents $\frac{90}{360} = \frac{1}{4}$ of the total.

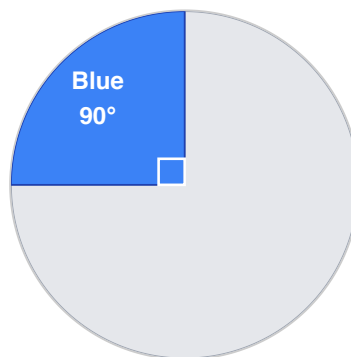
If that 90° sector represents £20, then the whole pie chart must be $4 \times £20 = £80$.

COMMON MISTAKE: A full pie chart is 360° (NOT 100°), so fraction of total = $\frac{\text{angle}}{360}$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

A pie chart represents 160 pupils. The 'Blue' sector has an angle of 90°.



How many pupils chose Blue?

STEP-BY-STEP SOLUTION:

A full circle is 360°.

$$\text{Fraction for Blue} = \frac{90}{360} = \frac{1}{4}.$$

$$\text{Number of pupils} = \frac{1}{4} \text{ of } 160 = 40 \text{ pupils.}$$

2 Calculating Angles

📖 CORE CONCEPT & METHOD

To find the angle for a pie chart:

1. Find the **Total Frequency** of all items.
2. Find the **multiplier**: $360^\circ \div \text{Total Frequency}$. (This tells you how many degrees each 1 item is worth).
3. Multiply each frequency by this multiplier to get the angle!

COMMON MISTAKE: Don't forget to divide 360° by the TOTAL frequency — check your finished angles add up to 360°.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

You are drawing a pie chart for 60 people. 18 people chose 'Dog'.

Pet	Frequency	Angle
Dog	18	?
Cat	...	...
Total	60	360°

What angle should the 'Dog' sector be?

STEP-BY-STEP SOLUTION:

Step 1: Find the multiplier from the **Total** row: $360^\circ \div 60 = 6$ (multiply by 6).

Step 2: Multiply the **Dog** frequency by the same multiplier: $18 \times 6 = 108^\circ$.

Pet	Frequency	Angle
Dog	18	108°
Cat	...	...
Total	60	360°

3 Missing Frequencies & Angles**CORE CONCEPT & METHOD**

If you are given an angle and its matching frequency, you can find the multiplier just from that row!

$$\text{Multiplier} = \text{Angle} \div \text{Frequency}$$

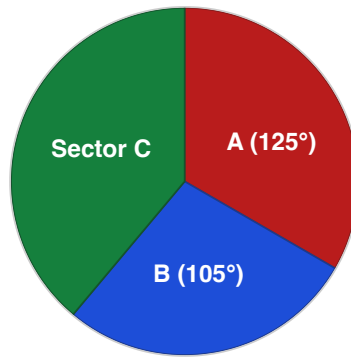
Then you can use this multiplier to fill in the rest of the table.

COMMON MISTAKE: Find the angle per person using the complete row ($\text{Angle} \div \text{Frequency}$) before filling in blanks.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

A pie chart has three sectors. Sector A is 125° and Sector B is 105° .



What is the angle for Sector C?

STEP-BY-STEP SOLUTION:

Angles in a pie chart add up to 360° .

$$\text{Sector C} = 360 - (125 + 105) = 360 - 230 = 130^\circ.$$

4 Visual Pie Charts

CORE CONCEPT & METHOD

When looking at a drawn pie chart, look out for special markers:

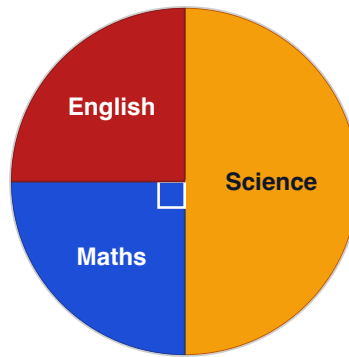
1. A **Right Angle** square means the sector is exactly 90° (which is $\frac{1}{4}$ of the pie).
2. A **straight line** across the circle splits it exactly in half (180°).

Remember: A pie chart only shows *proportions*, not actual amounts! You cannot compare the total number of people between two pie charts unless you know how many people each represents.

COMMON MISTAKE: Spot hidden angles: a square corner = 90° ($\frac{1}{4}$ of the pie) and a straight diameter line = 180° ($\frac{1}{2}$ of the pie).

MODEL WORKED EXAMPLES**EXAMPLE 4.1**

A pie chart shows the favourite subjects of 84 students.



How many students chose Maths?

STEP-BY-STEP SOLUTION:

1. The square marker for Maths means it is exactly 90° .
2. 90° is exactly $\frac{1}{4}$ of the pie chart.
3. Total students = 84.
4. $\frac{1}{4}$ of 84 = $84 \div 4 = 21$ students.

STATISTICS — DATA REPRESENTATION

Stem and Leaf Diagrams

1 Averages & Spread

CORE CONCEPT & METHOD

A stem and leaf diagram is an ordered list of numbers. Count the total number of leaves (n) and count from the top left to find the **median**:

- **Odd number of values:** Use the $\frac{n+1}{2}$ th value.
- **Even number of values:** Find the two middle values and calculate their mean (add them together and divide by 2).

COMMON MISTAKE: Use the Key! In $2 \mid 6 = 26$, the leaf 6 stands for 26, NOT 6 — and for Range, subtract the whole numbers ($\text{max} - \text{min}$), not just the leaves.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

The stem-and-leaf diagram shows the scores of 9 contestants in a quiz.

2	1	4	8	
3	0	5	7	9
4	2	6		

Key: $2 \mid 4$ means 24

Find the **median** and the **range**.

STEP-BY-STEP SOLUTION:

Step 1 — List the values in order:

21, 24, 28, 30, 35, 37, 39, 42, 46

Step 2 — Find the median:

There are $n = 9$ values. The median is the $\frac{9+1}{2} = 5$ th value.

Counting along: 21, 24, 28, 30, **35**, ... → **Median = 35**

Step 3 — Find the range:

Range = Maximum – Minimum = $46 - 21 = 25$

2 Probabilities from Diagrams

CORE CONCEPT & METHOD

To find a probability:

1. Count how many leaves fit the condition (e.g. 'more than 80').
2. Count the TOTAL number of leaves.
3. Write it as a fraction: $\frac{\text{Condition}}{\text{Total}}$.

COMMON MISTAKE: "More than 70" means 71 or above — do NOT count 70 itself unless it says "70 or more".

MODEL WORKED EXAMPLES

EXAMPLE 2.1

The stem-and-leaf diagram shows the distances (km) cycled by 10 riders.

2	3	6		
3	1	4	7	8
4	0	2	5	9

Key: 3 | 1 means 31 km

What is the probability that a rider picked at random cycled **more than 32 km**?

STEP-BY-STEP SOLUTION:

1. Identify the distances strictly greater than 32 km: **34, 37, 38, 40, 42, 45, 49**.
2. Count them: There are **7** qualifying distances.
3. Total riders = **10**.
4. Probability = $\frac{7}{10}$.

3 Decimals, 3-Digits & Changes**□ CORE CONCEPT & METHOD**

Decimals: Sometimes the stem is the whole number and the leaf is the decimal. Look at the key!

3-Digits: If a number is 156, the stem is '15' and the leaf is '6'.

Adding a value: If you add a new number to the data, the total count n increases by 1, which shifts the middle position of the median.

COMMON MISTAKE: Check the Key for decimals (e.g. $14 \mid 5 = 14.5$, not 145) before working out the median or range.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

The stem-and-leaf diagram shows the masses (kg) of 7 parcels.

15		2	7	
16		1	4	8
17		3	6	

Key: 15 | 2 means 15.2 kg

Find the **median** mass.

STEP-BY-STEP SOLUTION:

1. There are $n = 7$ values. The median is the $\frac{7+1}{2} = 4$ th value.
2. Count along the leaves in order: 15.2, 15.7, 16.1, **16.4**.
3. The median mass is **16.4 kg**.

Frequency Polygons

CORE CONCEPT & METHOD

To draw a frequency polygon from a grouped table, plot a cross (x) for each row and join consecutive crosses with straight ruled lines.

The coordinates for each point are: **(Midpoint, Frequency)**.

- The **x-coordinate** is the midpoint of the class interval.
- The **y-coordinate** is the frequency.

COMMON MISTAKE: Plot at the MIDPOINT (halfway across each group), join with a ruler, and NEVER join the first and last points together.

MODEL WORKED EXAMPLES

EXAMPLE 1

The table shows information about the lengths (mm) of pine cones.

Length (mm)	Frequency
$0 < l \leq 20$	16
$20 < l \leq 40$	30
$40 < l \leq 60$	22

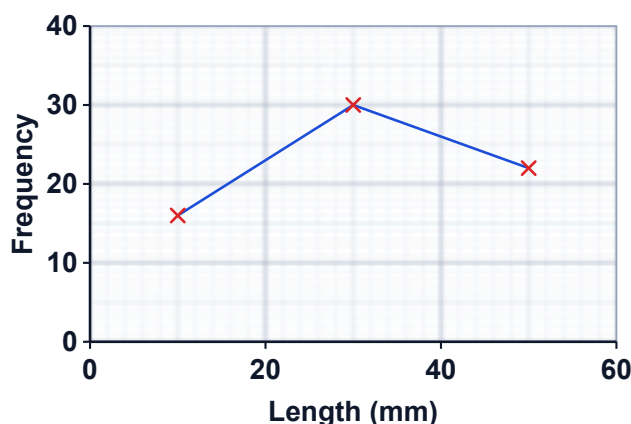
Draw a frequency polygon for this information.

STEP-BY-STEP SOLUTION:

Step 1 — Find the midpoint of each interval to get (Midpoint, Frequency):

- $0 < l \leq 20$: midpoint = $\frac{0 + 20}{2} = 10 \rightarrow$ plot **(10, 16)**
- $20 < l \leq 40$: midpoint = $\frac{20 + 40}{2} = 30 \rightarrow$ plot **(30, 30)**
- $40 < l \leq 60$: midpoint = $\frac{40 + 60}{2} = 50 \rightarrow$ plot **(50, 22)**

Step 2 — Plot each cross and join with straight ruled lines:



Note: Points are plotted at the midpoints and joined by straight lines. Do not join the ends together or down to the axis!