

EDEXCEL GCSE (9-1) TUTORIALS &amp; EXAMPLES

# Grade 3 Mathematics — Tutorials & Examples

(Foundation Tier)

Plain-English Core Concepts, Visual Diagrams & Worked Examples

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- Plain-English Core Concept notes with visual diagrams
- One-line **Common Mistake** warnings on every topic step
- Model Worked Examples labelled **Calculator** or **Non-Calculator**
- Covers all 21 Grade 3 Foundation GCSE topics

Student Name: \_\_\_\_\_

Class / Teacher: \_\_\_\_\_

Target Grade: \_\_\_\_\_



SCAN FOR ONLINE PRACTICE & WORKED SOLUTIONS

**BreakingDownMaths.com**

Interactive Step-by-Step GCSE Mathematics Mastery

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Comprehensive syllabus overview of all topic tutorials and worked examples included in this booklet.

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## RATIO, PROPORTION AND RATES OF CHANGE

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## GEOMETRY AND MEASURES

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## NUMBER • APPROXIMATION &amp; BOUNDS

Target Grade 3

## TOPIC 1: Estimation

## Step 1 Estimating Calculations

## CORE CONCEPT &amp; METHOD

To estimate the answer to a calculation, you **MUST** round every single number to **1 significant figure**

**BEFORE** you do any maths! For example,  $\frac{48.7 \times 61.2}{11.3}$  becomes  $\frac{50 \times 60}{10} = 300$ .

**Square roots exception:** For a number inside a square root (such as  $\sqrt{36.1}$ ), round to the **nearest square number** ( $\sqrt{36} = 6$ ) rather than 1 significant figure (40), so the root works out cleanly without a calculator.

**COMMON MISTAKE:** Do not work out the exact answer first — round each number to 1 s.f. (or the nearest square number inside a square root) before calculating.

## WORKED EXAMPLE

## EXAMPLE 1.1

NON-CALCULATOR

Estimate the value of  $\frac{6.14 \times 29.7}{0.49}$ .

## STEP-BY-STEP SOLUTION:

Round each number to 1 significant figure first:

$$6.14 \rightarrow 6$$

$$29.7 \rightarrow 30$$

$$0.49 \rightarrow 0.5$$

Substitute into the calculation:

$$\frac{6 \times 30}{0.5} = \frac{180}{0.5}$$

Dividing by decimals can be tricky. Here are 2 alternative methods:

## Method 1: Multiply top &amp; bottom by 2

$$\frac{180 \times 2}{0.5 \times 2} = \frac{360}{1} = 360$$

## Method 2: Multiply top &amp; bottom by 10

$$\frac{180 \times 10}{0.5 \times 10} = \frac{1800}{5} \text{ (then } 1800 \div 5 = 360)$$

## Step 2 Real World Estimation

### CORE CONCEPT & METHOD

When estimating word problems, you still round every number to **1 significant figure** first.

When **adding or multiplying**, rounding both numbers **up** gives an **overestimate**, and rounding both **down** gives an **underestimate**. (For **division** or **subtraction**, check how each rounded number pushes the answer — see Step 4!)

**COMMON MISTAKE:** Do not round 0.52 to 0 or 1 — the first non-zero digit is 5, so 0.52 rounds to 0.5.

### WORKED EXAMPLE

#### EXAMPLE 2.1

NON-CALCULATOR

A college has 98 lecturers and 2,040 students. Estimate the number of students per lecturer.

#### STEP-BY-STEP SOLUTION:

Round to 1 significant figure:

$$2040 \rightarrow 2000$$

$$98 \rightarrow 100$$

$$2000 \div 100 = 20 \text{ students per lecturer.}$$

## Step 3 Estimating Shapes

### CORE CONCEPT & METHOD

Just like calculations, round the dimensions of the shape to 1 significant figure FIRST.

If estimating the area of a circle with radius 11, round 11 to 10, and round  $\pi$  to 3!

**COMMON MISTAKE:** Do not use 3.14 for  $\pi$  in an estimate — round  $\pi$  to 3 (1 significant figure).

### WORKED EXAMPLE

#### EXAMPLE 3.1

NON-CALCULATOR

Estimate the area of a circle with a radius of 6.1 cm.

#### STEP-BY-STEP SOLUTION:

Round radius to 6 cm, and use  $\pi \approx 3$ .

$$\text{Area} = \pi r^2 \approx 3 \times 6^2 = 3 \times 36 = 108 \text{ cm}^2$$

## Step 4

## Advanced Estimation

## CORE CONCEPT &amp; METHOD

In harder estimation questions, you may need to decide whether your estimate is an **overestimate** or an **underestimate** — the rule depends on the operation:

- **Adding or Multiplying ( $a + b$ ,  $a \times b$ ):** Rounding both numbers **up** gives an **overestimate**; rounding both **down** gives an **underestimate**.
- **Dividing ( $a \div b$ ):** Rounding the **top up** or the **bottom down** makes the answer **bigger** (overestimate); rounding the **top down** or the **bottom up** makes it **smaller** (underestimate).
- **Subtracting ( $a - b$ ):** Subtracting a number that was rounded **up** makes the answer **smaller**; subtracting one rounded **down** makes it **bigger**.

**COMMON MISTAKE:** Dividing by a number that was rounded down makes your answer bigger, not smaller.

## WORKED EXAMPLE

## EXAMPLE 4.1

 **NON-CALCULATOR**

(a) Work out an estimate for  $\frac{78.4}{0.43}$ .

(b) Is your estimate an overestimate or an underestimate? Give a reason for your answer.

## STEP-BY-STEP SOLUTION:

(a) Round each number to 1 significant figure:

$78.4 \rightarrow 80$  and  $0.43 \rightarrow 0.4$

$$\frac{80}{0.4} = \frac{800}{4} = 200$$

(b) **Overestimate** — the top number was rounded **up** ( $78.4 \rightarrow 80$ ) and the bottom number was rounded **down** ( $0.43 \rightarrow 0.4$ ), both of which make the answer bigger.

## NUMBER • APPROXIMATION &amp; BOUNDS

## TOPIC 2: Limits of Accuracy

Target Grade 3

## Step 1 Smallest &amp; Largest Values

## CORE CONCEPT &amp; METHOD

To find the largest or smallest possible value of a rounded number:

1. Find the **degree of accuracy** (e.g. nearest 10, nearest 1).
2. **Halve it!**
3. Add it to the number for the largest value (Upper Bound), or subtract it for the smallest value (Lower Bound).

**COMMON MISTAKE:** Never write .49 or .99 for the upper bound — halve the accuracy unit and add it cleanly (e.g. 116.5).

## WORKED EXAMPLE

## EXAMPLE 1.1

NON-CALCULATOR

The number  $x$  is 80, rounded to the nearest 10. Write down the error interval for  $x$ .

## STEP-BY-STEP SOLUTION:

The rounding unit is 10. Half of 10 is 5.

Lower bound =  $80 - 5 = 75$

Upper bound =  $80 + 5 = 85$

Error interval:  $75 \leq x < 85$

## Step 2 Error Intervals (Decimal Places)

## CORE CONCEPT &amp; METHOD

An **Error Interval** is written as an inequality:  $LB \leq y < UB$ .

If rounded to 1 decimal place, the degree of accuracy is 0.1.

Half of 0.1 is 0.05.

So  $y = 5.2$  becomes  $5.15 \leq y < 5.25$ .

**COMMON MISTAKE:** Always use  $<$  (not  $\leq$ ) at the upper bound of an error interval:  $LB \leq x < UB$ .

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

A crate has a mass  $m$  of 6.8 kg, rounded to 1 decimal place. Write down the error interval for  $m$ .

## STEP-BY-STEP SOLUTION:

The rounding unit is 0.1. Half of 0.1 is 0.05.

Lower bound =  $6.8 - 0.05 = 6.75$

Upper bound =  $6.8 + 0.05 = 6.85$

Error interval:  $6.75 \leq m < 6.85$

**Step 3 Error Intervals (Significant Figures)****CORE CONCEPT & METHOD**

For significant figures, look at the **place value** of the last significant digit.

For example, if 210 is rounded to 2 sig figs, the '1' is in the TENS column. The degree of accuracy is 10. Half is 5.

Interval:  $205 \leq x < 215$ .

**COMMON MISTAKE:** Look at the column of the last significant digit before halving (for 700 to 1 s.f., accuracy is 100, so  $\pm 50$ ).

**WORKED EXAMPLE****EXAMPLE 3.1**

 **NON-CALCULATOR**

A stadium crowd  $p$  is 70,000 rounded to 1 significant figure. Find the lower and upper limits.

**STEP-BY-STEP SOLUTION:**

The rounding unit is 10,000. Half of this is 5,000.

Lower bound =  $70,000 - 5,000 = 65,000$

Upper bound =  $70,000 + 5,000 = 75,000$

Error interval:  $65,000 \leq p < 75,000$

**Step 4 Truncation****CORE CONCEPT & METHOD**

**Truncation** means chopping off the digits without rounding up!

If  $x$  is truncated to 1 decimal place to give 6.2, the smallest it could be is exactly 6.2, and the largest it could be is 6.2999... (which is 6.3).

Interval:  $6.2 \leq x < 6.3$ .

**COMMON MISTAKE:** Do not subtract half a unit when truncating — the lower bound is the truncated number itself.

**WORKED EXAMPLE****EXAMPLE 4.1**

 **NON-CALCULATOR**

A number  $y$  is truncated to 1 decimal place to give 8.4. Write down the error interval for  $y$ .

**STEP-BY-STEP SOLUTION:**

When truncating, the number just 'cuts off'. So it must have started with 8.4...

Lower bound = 8.4

Upper bound = 8.5

Error interval:  $8.4 \leq y < 8.5$

## NUMBER • FRACTIONS &amp; DECIMALS

## TOPIC 3: Fractions (+/-)

Target Grade 3

## Step 1 Adding/Subtracting Same Denominators

## CORE CONCEPT &amp; METHOD

When fractions have the same bottom number (the **denominator**), it means the pieces are exactly the same size.

If you have 1 slice ( $\frac{1}{4}$ ) and your friend gives you 2 more slices ( $\frac{2}{4}$ ), you just count the total slices you have together!  $1 + 2 = 3$  slices. The size of the slices (quarters) doesn't change.

**COMMON MISTAKE:** Never add the bottom numbers together ( $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ , not  $\frac{3}{8}$ ).

## WORKED EXAMPLES

## EXAMPLE 1.1

NON-CALCULATOR

Example 1: Addition

What is  $\frac{2}{7} + \frac{3}{7}$ ?

## STEP-BY-STEP SOLUTION:

Both fractions are 'sevenths' (the bottom number is 7). We just add the top numbers:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

## EXAMPLE 1.2

NON-CALCULATOR

Example 2: Subtraction

What is  $\frac{8}{9} - \frac{3}{9}$ ?

## STEP-BY-STEP SOLUTION:

Both fractions are 'ninths' (the bottom number is 9). We just subtract the top numbers:

$$\frac{8}{9} - \frac{3}{9} = \frac{8-3}{9} = \frac{5}{9}$$

## Step 2 Adding/Subtracting Different Denominators

## CORE CONCEPT &amp; METHOD

When the denominators (bottom numbers) are different, we cannot add the fractions straight away. We must find a **Common Denominator** first.

For example, let's calculate:  $\frac{1}{2} + \frac{1}{3}$

Notice the bottom numbers (2 and 3) are different! Here are the two best ways to solve this problem:

### Method 1: Find the Lowest Common Multiple

List the multiples for both bottom numbers to find the smallest one they share.

Multiples of 2: 2, 4, **6**, 8...

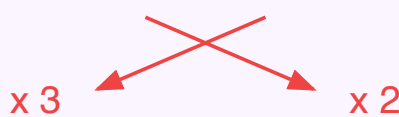
Multiples of 3: 3, **6**, 9...

The common multiple is 6. Multiply the top and bottom of each fraction to make the denominator 6:

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6} \quad \text{and} \quad \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

### Method 2: Cross Multiply

Multiply each fraction by the **other fraction's denominator** (bottom number). This creates a perfect common denominator!

$$\frac{1}{2} + \frac{1}{3}$$


x 3                      x 2

Now multiply the numbers out and add the tops:

$$\frac{3 \times 1}{3 \times 2} + \frac{2 \times 1}{2 \times 3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$

**COMMON MISTAKE:** Whatever you multiply the bottom number by, you must multiply the top number by the same value.

## WORKED EXAMPLES

### EXAMPLE 2.1

NON-CALCULATOR

Example 1: Addition

Calculate:  $\frac{1}{4} + \frac{3}{8}$

#### STEP-BY-STEP SOLUTION:

Using the Lowest Common Multiple (LCM):

$$\frac{2 \times 1}{2 \times 4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$$

Using Cross Multiply:

$$\frac{8 \times 1}{8 \times 4} + \frac{4 \times 3}{4 \times 8} = \frac{8}{32} + \frac{12}{32} = \frac{8+12}{32} = \frac{20}{32} = \frac{5}{8}$$

### EXAMPLE 2.2

NON-CALCULATOR

Example 2: Subtraction

Calculate:  $\frac{3}{4} - \frac{2}{5}$

#### STEP-BY-STEP SOLUTION:

Using Cross Multiply:

$$\frac{5 \times 3}{5 \times 4} - \frac{4 \times 2}{4 \times 5} = \frac{15}{20} - \frac{8}{20} = \frac{15-8}{20} = \frac{7}{20}$$

### Step 3 Improper to Mixed Numbers

#### CORE CONCEPT & METHOD

A **top-heavy** (or improper) fraction has a larger number on top. This means you have more than one whole!

Example:  $\frac{7}{3} \rightarrow 3 \overline{)7} = 2 \frac{1}{3}$

Annotations:   
 - **whole number**: points to the 2   
 - **remainder**: points to the 1   
 - **still divided by 3**: points to the 3 in the denominator

To convert it, see how many times the bottom number fits into the top number. The remainder is what's left over.

**COMMON MISTAKE:** Keep the same denominator for the remainder fraction (e.g.  $\frac{7}{3} = 2\frac{1}{3}$ , not  $2\frac{1}{7}$ ).

#### WORKED EXAMPLE

##### EXAMPLE 3.1

NON-CALCULATOR

Convert  $\frac{11}{4}$  into a mixed number

##### STEP-BY-STEP SOLUTION:

$$11 \div 4 = 2 \text{ remainder } 3$$

So we have 2 whole pieces and 3 pieces left over!

$$\frac{11}{4} = 2\frac{3}{4}$$

**Step 4** Mixed to Improper Fractions**CORE CONCEPT & METHOD**

Sometimes it's easier to work with **top-heavy** (improper) fractions than mixed numbers!

Example:  $2 \frac{1}{3} = \frac{3 \times 2 + 1}{3} = \frac{7}{3}$

To convert it, multiply the whole number by the bottom, then add the top! (Keep the bottom number the same).

**COMMON MISTAKE:** Multiply the whole number by the bottom first, then add the top — do not multiply by the top.

**WORKED EXAMPLE****EXAMPLE 4.1**

**NON-CALCULATOR**

Convert  $3 \frac{2}{5}$  into a top-heavy fraction

**STEP-BY-STEP SOLUTION:**

1. Multiply whole number by bottom:  $3 \times 5 = 15$
2. Add the top number:  $15 + 2 = 17$

$$3 \frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$$

**Step 5** Adding/Subtracting Mixed Numbers**CORE CONCEPT & METHOD**

When dealing with mixed numbers, we have a whole number attached to a fraction.

For example, let's calculate:  $1 \frac{2}{3} + 2 \frac{3}{5}$

It is often easiest to deal with the whole numbers first ( $1 + 2 = 3$ ), and then add the fraction parts exactly like we learned in the last step!

## WORKED EXAMPLES

**EXAMPLE 5.1**

NON-CALCULATOR

**Example 1: Addition**Calculate:  $2\frac{3}{4} + 1\frac{2}{5}$ **STEP-BY-STEP SOLUTION:**1. Add whole numbers:  $2 + 1 = 3$ 

2. Add fractions:

$$\frac{5 \times 3}{5 \times 4} + \frac{4 \times 2}{4 \times 5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$$

3. Combine everything:  $3 + 1\frac{3}{20} = 4\frac{3}{20}$ **EXAMPLE 5.2**

NON-CALCULATOR

**Example 2: Normal Subtraction**Calculate:  $4\frac{4}{5} - 2\frac{1}{4}$ **STEP-BY-STEP SOLUTION:**1. Subtract whole numbers:  $4 - 2 = 2$ 

2. Subtract fractions:

$$\frac{4 \times 4}{4 \times 5} - \frac{5 \times 1}{5 \times 4} = \frac{16}{20} - \frac{5}{20} = \frac{11}{20}$$

3. Combine everything:  $2 + \frac{11}{20} = 2\frac{11}{20}$

**EXAMPLE 5.3**

NON-CALCULATOR

**Example 3: Subtraction with Borrowing!**

Calculate:  $5\frac{1}{5} - 1\frac{3}{4}$

**STEP-BY-STEP SOLUTION:**

1. Subtract whole numbers:  $5 - 1 = 4$

2. Subtract fractions (Uh oh!):

$$\frac{4 \times 1}{4 \times 5} - \frac{5 \times 3}{5 \times 4} = \frac{4}{20} - \frac{15}{20} = -\frac{11}{20}$$

3. Borrow 1 whole from the 4:

$$4 - \frac{11}{20} = 3 + \frac{20}{20} - \frac{11}{20} = 3\frac{9}{20}$$

**EXAMPLE 5.4**

NON-CALCULATOR

**Example 4 (Alternative Method): Turning Both into Top-Heavy Fractions**

Calculate:  $5\frac{1}{5} - 1\frac{3}{4}$

**STEP-BY-STEP SOLUTION:**

*Alternative method — you can turn both mixed numbers into top-heavy fractions first so you never have to borrow!*

1. Turn both into top-heavy (improper) fractions:

$$5\frac{1}{5} = \frac{26}{5} \quad \text{and} \quad 1\frac{3}{4} = \frac{7}{4}$$

2. Find a common denominator (20) and subtract:

$$\frac{26}{5} - \frac{7}{4} = \frac{26 \times 4}{5 \times 4} - \frac{7 \times 5}{4 \times 5} = \frac{104}{20} - \frac{35}{20} = \frac{69}{20}$$

3. Convert back into a mixed number ( $69 \div 20 = 3$  remainder 9):

$$\frac{69}{20} = 3\frac{9}{20}$$

## NUMBER • FRACTIONS &amp; DECIMALS

## TOPIC 4: Fractions (x/÷)

Target Grade 3

## Step 1 Multiplying Fractions

## CORE CONCEPT &amp; METHOD

Multiplying fractions is easy! You just multiply the top numbers together, and multiply the bottom numbers together.

$$\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$$

**COMMON MISTAKE:** You do not need a common denominator to multiply fractions — just multiply tops and multiply bottoms.

## WORKED EXAMPLE

## EXAMPLE 1.1

NON-CALCULATOR

Example:  $\frac{3}{5} \times \frac{2}{7}$

## STEP-BY-STEP SOLUTION:

$$\frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

$$\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$$

## Step 2 Multiply Mixed Numbers

## CORE CONCEPT &amp; METHOD

To multiply mixed numbers, you **MUST** convert them to top-heavy (improper) fractions first! You cannot just multiply the whole numbers and fractions separately.

**COMMON MISTAKE:** Never multiply whole numbers and fractions separately ( $2\frac{1}{4} \times 3\frac{1}{3} \neq 6\frac{1}{12}$ ) — turn into top-heavy fractions first.

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

Example:  $1\frac{3}{4} \times 2\frac{1}{3}$

## STEP-BY-STEP SOLUTION:

1. Convert:  $\frac{7}{4} \times \frac{7}{3}$

2. Multiply:  $\frac{49}{12}$

3. Mixed:  $4\frac{1}{12}$

$$1\frac{3}{4} \times 2\frac{1}{3} = 4\frac{1}{12}$$

## Step 3 Dividing Fractions

## CORE CONCEPT &amp; METHOD

To divide fractions, remember **KFC** (Keep, Flip, Change)!

For example:  $\frac{2}{3} \div \frac{4}{5}$

KEEP

$$\frac{2}{3} \downarrow \frac{2}{3}$$

CHANGE

$$\div \downarrow \times$$

FLIP

$$\frac{4}{5} \downarrow \frac{5}{4}$$

$$\frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$$

**COMMON MISTAKE:** Only flip the second fraction (the one you divide by), never the first fraction.

## WORKED EXAMPLE

## EXAMPLE 3.1

NON-CALCULATOR

Example:  $\frac{4}{5} \div \frac{3}{7}$

## STEP-BY-STEP SOLUTION:

$$\begin{array}{ccccc} \frac{4}{5} & \div & \frac{3}{7} & & \\ \text{K} & \text{C} & \text{F} & & \\ \downarrow & \downarrow & \downarrow & & \\ \frac{4}{5} & \times & \frac{7}{3} & = & \frac{28}{15} = 1\frac{13}{15} \end{array}$$

$$\frac{4}{5} \div \frac{3}{7} = 1\frac{13}{15}$$

## Step 4 Divide Mixed Numbers

## CORE CONCEPT &amp; METHOD

Combine the rules! Convert to improper fractions first, then use Keep-Flip-Change.

**COMMON MISTAKE:** Change both mixed numbers into improper fractions before you flip the second fraction.

## WORKED EXAMPLE

## EXAMPLE 4.1

NON-CALCULATOR

Example:  $3\frac{1}{3} \div 1\frac{2}{3}$

## STEP-BY-STEP SOLUTION:

1. Improper:  $\frac{10}{3} \div \frac{5}{3}$

2. KFC:  $\frac{10}{3} \times \frac{3}{5} = \frac{30}{15} = 2$

$$3\frac{1}{3} \div 1\frac{2}{3} = 2$$

## Step 5 Fractions &amp; Whole Numbers

## CORE CONCEPT &amp; METHOD

When dealing with a whole number, just write it as a fraction over 1. For example, 5 becomes  $\frac{5}{1}$ . Then follow the normal rules!

**COMMON MISTAKE:** Write a whole number  $n$  as  $\frac{n}{1}$ , not  $\frac{n}{n}$  — multiplying only scales the top number.

## WORKED EXAMPLES

## EXAMPLE 5.1

NON-CALCULATOR

Example (Multiplying):  $4 \times \frac{2}{9}$

## STEP-BY-STEP SOLUTION:

$$\frac{4}{1} \times \frac{2}{9} = \frac{8}{9}$$

$$4 \times \frac{2}{9} = \frac{8}{9}$$

## EXAMPLE 5.2

NON-CALCULATOR

Example (Dividing):  $\frac{6}{7} \div 3$

## STEP-BY-STEP SOLUTION:

1. Write the whole number as a fraction:  $\frac{6}{7} \div \frac{3}{1}$

2. KFC:  $\frac{6}{7} \times \frac{1}{3}$

3. Multiply:  $\frac{6}{21}$

4. Simplify:  $\frac{2}{7}$

$$\frac{6}{7} \div 3 = \frac{2}{7}$$

## Step 6 Mixed &amp; Proper Fractions

## CORE CONCEPT &amp; METHOD

To multiply or divide a mixed number and a proper fraction, convert the mixed number to an improper fraction first! Then use the standard rules for multiplying and dividing.

**COMMON MISTAKE:** Convert the mixed number to an improper fraction before cancelling or multiplying.

## WORKED EXAMPLE

**EXAMPLE 6.1** **NON-CALCULATOR**

Work out  $2\frac{1}{3} \times \frac{2}{5}$

**STEP-BY-STEP SOLUTION:**

First, convert  $2\frac{1}{3}$  into an improper fraction:  $\frac{7}{3}$ .

Now multiply:  $\frac{7}{3} \times \frac{2}{5}$ .

Multiply the numerators:  $7 \times 2 = 14$ .

Multiply the denominators:  $3 \times 5 = 15$ .

Answer:  $\frac{14}{15}$ .

## NUMBER • FRACTIONS &amp; DECIMALS

Target Grade 3

## TOPIC 5: Reciprocals

## CORE CONCEPT &amp; METHOD

The **reciprocal** of a number is simply 1 divided by that number.

The easiest way to think of it is flipping the number upside down!

- For a fraction like  $\frac{3}{4}$ , flip it to get  $\frac{4}{3}$ .
- For a whole number like 5, imagine it as  $\frac{5}{1}$ . Flip it to get  $\frac{1}{5}$ .
- For a decimal like 0.2, write it as a fraction (which is  $\frac{2}{10}$ ) and then flip it to get  $\frac{10}{2} = 5$ .

**COMMON MISTAKE:** For a decimal or mixed number, write it as a single fraction first (e.g.  $0.25 = \frac{1}{4}$  or  $1\frac{2}{3} = \frac{5}{3}$ ) before flipping.

## WORKED EXAMPLES

## EXAMPLE 1

NON-CALCULATOR

What is the reciprocal of  $\frac{3}{8}$ ?

## STEP-BY-STEP SOLUTION:

Just flip the fraction upside down!

$$\frac{8}{3}$$

## EXAMPLE 2

NON-CALCULATOR

What is the reciprocal of 6?

## STEP-BY-STEP SOLUTION:

Write 6 as  $\frac{6}{1}$ , and then flip it!

$$\frac{1}{6}$$

**EXAMPLE 3** **NON-CALCULATOR**

What is the reciprocal of 0.25?

**STEP-BY-STEP SOLUTION:**

First, write 0.25 as a fraction:  $\frac{1}{4}$  (or  $\frac{25}{100}$ ).

Now flip it:  $\frac{4}{1}$ , which is just 4 (or  $\frac{100}{25} = 4$ ).

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## NUMBER • PERCENTAGES

Target Grade 3

## TOPIC 6: Percentages of an Amount

## CORE CONCEPT &amp; METHOD

There are two main ways to find a percentage of an amount:

## 1. Non-Calculator: Building Blocks

Find 10% by dividing by 10.

Find 1% by dividing by 100.

Find 5% by halving 10%.

Then just add them together!

## 2. Calculator: The Fraction Button

Type the percentage as a fraction over 100 using your calculator's fraction button, and then multiply by the amount.

**COMMON MISTAKE:** Don't forget to divide by 100 when turning a percentage into a decimal or fraction multiplier.

## WORKED EXAMPLES

## EXAMPLE 1

NON-CALCULATOR

Find 15% of 60 (Non-Calculator)

STEP-BY-STEP SOLUTION:

$$\begin{array}{rcl}
 10\% \text{ of } 60 & = & 60 \div 10 = 6 \\
 \div 2 \quad \swarrow & & \searrow \quad \div 2 \\
 5\% \text{ of } 60 & = & 3 \\
 \hline
 15\% & = & 9
 \end{array}$$

## EXAMPLE 2

CALCULATOR

Find 38% of 250 (Calculator)

STEP-BY-STEP SOLUTION:

*use fraction button*  $\rightarrow$

$$\frac{\boxed{38}}{\boxed{100}} \times 250 = 95$$

## NUMBER • PERCENTAGES

## TOPIC 7: Percentage Change

Target Grade 3

## Step 1 Percentage Change

## CORE CONCEPT &amp; METHOD

To find the percentage change (like a profit or a discount), use this golden formula:

$$\frac{\text{Difference}}{\text{Original}} \times 100$$

 **Exam Tip:** Exams use lots of different words for this! They might ask you to calculate the **percentage change**, **percentage profit**, **percentage loss**, **percentage increase**, or **percentage decrease**. Don't panic — they all use this exact same formula!

**Trap Warning:** Always divide by the *original* starting amount, never the new amount!

**COMMON MISTAKE:** Don't forget to multiply your decimal by 100 at the end — e.g.  $\frac{3}{20} = 0.15 = 15\%$ , NOT 0.15%.

## WORKED EXAMPLE

## EXAMPLE 1.1

 CALCULATOR

A watch is bought for £160 and sold for £208. What is the percentage profit?

## STEP-BY-STEP SOLUTION:

1. Find the difference (profit):  $208 - 160 = 48$
2. Put it into the formula:  $\frac{48}{160} \times 100 = 30\%$

30%

## Step 2 Complex Percentage Change

## CORE CONCEPT &amp; METHOD

The formula is exactly the same: **Difference ÷ Original × 100**. Just be careful to round your final answer to the requested number of decimal places or significant figures!

**COMMON MISTAKE:** Do not round halfway through — round only at the end to the asked number of decimal places.

## WORKED EXAMPLE

## EXAMPLE 2.1

 CALCULATOR

The population of a town was 52,400 in 2021. In 2022, the population grew to 55,180.  
Calculate the percentage increase.  
Give your answer to 1 decimal place.

## STEP-BY-STEP SOLUTION:

1. Put the values into the formula  $\frac{\text{Difference}}{\text{Original}} \times 100$ :

$$\frac{55,180 - 52,400}{52,400} \times 100 = 5.3053 \dots \%$$

2. Round to 1 decimal place: 5.3%.

## Step 3 Contextual Profit

## CORE CONCEPT &amp; METHOD

Sometimes you aren't given the selling price directly. You have to calculate the **Total Revenue** first (e.g. by multiplying the number of items sold by the price per item) before you can calculate the profit.

**COMMON MISTAKE:** Check pounds vs pence carefully — convert 37p to £0.37 so all money is in the same unit.

## WORKED EXAMPLE

## EXAMPLE 3.1

 CALCULATOR

Tariq buys a crate of 80 mangoes for £20.  
He finds that 15% of the mangoes are bruised and throws them away.  
He sells the rest of the mangoes for 40p each.

Calculate Tariq's percentage profit.

## STEP-BY-STEP SOLUTION:

1. Find how many mangoes he sells: 15% of 80 is 12 mangoes. He sells  $80 - 12 = 68$  mangoes.

2. Find his total revenue:  $68 \times 40\text{p} = 2720\text{p} = £27.20$ .

3. Calculate the actual profit:  $£27.20 - £20.00 = £7.20$ .

4. Percentage profit:  $\frac{7.20}{20.00} \times 100 = 36\%$ .

## NUMBER • PERCENTAGES

## TOPIC 8: Simple Interest

Target Grade 3

## CORE CONCEPT &amp; METHOD

**Simple Interest** is fixed interest calculated only on the **original amount** you invest or borrow.

Because the interest is always calculated on the starting amount, the interest earned or paid is **the exact same every single year!**

**The 3 Steps to Calculate Simple Interest:****1. Find amount of interest paid (for 1 year):**

$$\text{Interest for 1 year} = \text{Original amount} \times \text{Interest \%}$$

**2. Multiply by the number of years:**

$$\text{Total interest earned} = \text{Interest for 1 year} \times \text{Number of years}$$

**3. Add to the original amount:**

$$\text{Total amount} = \text{Original amount} + \text{Total interest earned}$$

*(Only do Step 3 if the question asks for the "total amount" or "total to pay back"!)*

**Key Distinction:** Unlike *Compound Interest* (where you earn interest on top of previous interest), Simple Interest never increases from year to year — it stays flat!

**COMMON MISTAKE:** Read the last line carefully: check whether it asks for just the interest earned or the total final amount.

## WORKED EXAMPLES

**EXAMPLE 1**

NON-CALCULATOR

Find the simple interest on £800 invested at 3% per year for 4 years.

**STEP-BY-STEP SOLUTION:**

**Step 1:** Find the interest for 1 year:

$$3\% \text{ of } £800 = \frac{3}{100} \times 800 = £24$$

**Step 2:** Multiply by 4 years to get total interest earned:

$$24 \times 4 = £96$$

---

**£96**

**EXAMPLE 2**

NON-CALCULATOR

James deposits £1200 into a savings account paying 4.5% simple interest per year. How much total money is in the account after 3 years?

**STEP-BY-STEP SOLUTION:**

**Step 1:** Find the interest for 1 year:

$$4.5\% \text{ of } £1200 = 0.045 \times 1200 = £54$$

**Step 2:** Multiply by 3 years to get total interest earned:

$$54 \times 3 = £162$$

**Step 3:** Add to the original amount to get the total amount:

$$£1200 + £162 = £1362$$

---

**£1362**

**EXAMPLE 3**

NON-CALCULATOR

Elena borrows £1800 for a car loan at a simple interest rate of 6% per year for 3 years. Calculate the total amount she must pay back.

**STEP-BY-STEP SOLUTION:**

**Step 1:** Find the interest for 1 year:

$$6\% \text{ of } £1800 = £108$$

**Step 2:** Multiply by 3 years to get total interest paid:

$$108 \times 3 = £324$$

**Step 3:** Add to the original amount to get the total repayment:

$$£1800 + £324 = £2124$$

---

**£2124**

## RATIO, PROPORTION AND RATES OF CHANGE • RATIO

Target Grade 3

## TOPIC 9: Simplifying Ratios

## Step 1 Simplifying Basic Ratios

## CORE CONCEPT &amp; METHOD

A ratio compares the size of different parts. To simplify a ratio, you divide both sides by their **Highest Common Factor (HCF)**.

This is exactly the same process as simplifying a fraction!

**COMMON MISTAKE:** Keep the ratio parts in the exact same order given in the question ( $7 : 12$  is different from  $12 : 7$ ).

## WORKED EXAMPLES

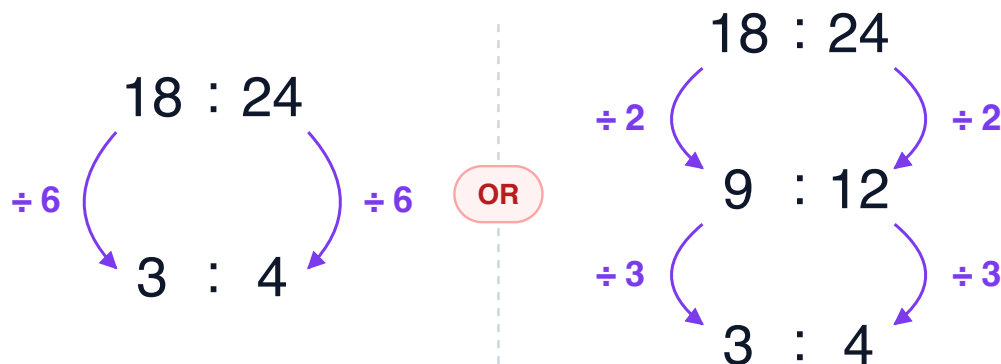
## EXAMPLE 1.1

NON-CALCULATOR

Simplify the ratio  $18 : 24$

## STEP-BY-STEP SOLUTION:

Find the biggest number that divides into both 18 and 24 ( $\text{HCF} = 6$ ), **or** divide in smaller steps until you can't divide any further:



*Can't divide both by any other number (apart from 1), so stop.*

**3 : 4**

**EXAMPLE 1.2****NON-CALCULATOR****Simplify the ratio 35 : 14****STEP-BY-STEP SOLUTION:**

Find the Highest Common Factor of 35 and 14.

The HCF is 7.

Divide both sides by 7:

$$35 \div 7 = 5$$

$$14 \div 7 = 2$$

$$\begin{array}{ccc} & 35 : 14 & \\ \div 7 \swarrow & & \searrow \div 7 \\ & 5 : 2 & \\ & 5 : 2 & \end{array}$$

**Step 2 Three-Part Ratios****CORE CONCEPT & METHOD**

Ratios can have more than two parts! The rule is exactly the same: find a number that goes into **all** of the parts, and divide.

**COMMON MISTAKE:** You must divide all three parts by the same common factor, not just two of them.

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

Simplify the ratio  $12 : 18 : 30$ 

## STEP-BY-STEP SOLUTION:

Find a number that divides into all three parts.  
They are all in the 6 times table!

Divide all parts by 6:

$$12 \div 6 = 2$$

$$18 \div 6 = 3$$

$$30 \div 6 = 5$$

$$\begin{array}{ccc}
 12 & : & 18 & : & 30 \\
 \div 6 & \swarrow & \downarrow & \searrow & \div 6 \\
 2 & : & 3 & : & 5 \\
 \hline
 2 & : & 3 & : & 5
 \end{array}$$

## Step 3 Form 1:n and n:1

## CORE CONCEPT &amp; METHOD

Sometimes a question asks you to write a ratio in the form  $1 : n$ .

This means the left side **must** be 1. To make the left side 1, you simply divide both sides by whatever the number on the left is!

In this topic,  $n$  might be a decimal or fraction. That is perfectly fine!

Similarly, to write a ratio in the form  $n : 1$ , divide both sides by whatever the number on the right is so it becomes 1!

**COMMON MISTAKE:** For  $1 : n$ , divide both sides by the left number (even if  $n$  becomes a decimal like  $1 : 1.25$ ).

## WORKED EXAMPLES

**EXAMPLE 3.1**

NON-CALCULATOR

Write  $6 : 42$  in the form  $1 : n$ **STEP-BY-STEP SOLUTION:**Line up  $1 : n$  underneath  $6 : 42$ . We need the 6 to become 1, so divide both sides by 6:

$$\begin{array}{ccc}
 6 & : & 42 \\
 \div 6 & \swarrow & \searrow \div 6 \\
 \underline{1} & : & n \\
 \underline{1} & : & 7 \\
 1 & : & 7
 \end{array}$$

**EXAMPLE 3.2**

NON-CALCULATOR

Write  $4 : 15$  in the form  $1 : n$ **STEP-BY-STEP SOLUTION:**Line up  $1 : n$  underneath  $4 : 15$ . We need the 4 to become 1, so divide both sides by 4:

$$\begin{array}{ccc}
 4 & : & 15 \\
 \div 4 & \swarrow & \searrow \div 4 \\
 \underline{1} & : & n \\
 \underline{1} & : & 3.75 \\
 1 & : & 3.75
 \end{array}$$

**EXAMPLE 3.3**

NON-CALCULATOR

Write  $15 : 4$  in the form  $n : 1$ **STEP-BY-STEP SOLUTION:**Line up  $n : 1$  underneath  $15 : 4$ . We need the 4 on the right to become 1, so divide both sides by 4:

$$\begin{array}{ccc}
 15 & : & 4 \\
 \div 4 & \swarrow & \searrow \div 4 \\
 n & : & \underline{1} \\
 3.75 & : & \underline{1} \\
 \hline
 3.75 & : & 1
 \end{array}$$

**Step 4 Different Units****CORE CONCEPT & METHOD**

You cannot simplify a ratio if the units are different! A ratio must compare identical things.

**Rule:** Always convert the larger unit (e.g. £) into the smaller unit (e.g. p) before simplifying.**COMMON MISTAKE:** Never simplify before matching the units — change £5 to 500p or 3 kg to 3000 g first.**WORKED EXAMPLES****EXAMPLE 4.1**

NON-CALCULATOR

Simplify the ratio  $60\text{p} : £3$ **STEP-BY-STEP SOLUTION:**

First, make the units the same. Convert the pounds into pence.

$$£3 = 300\text{p}$$

Now we simplify the ratio  $60 : 300$ .

Divide both sides by 60!

$$\begin{array}{ccc}
 60 & : & 300 \\
 \div 60 & \swarrow & \searrow \div 60 \\
 1 & : & 5 \\
 \hline
 1 & : & 5
 \end{array}$$

**EXAMPLE 4.2**

NON-CALCULATOR

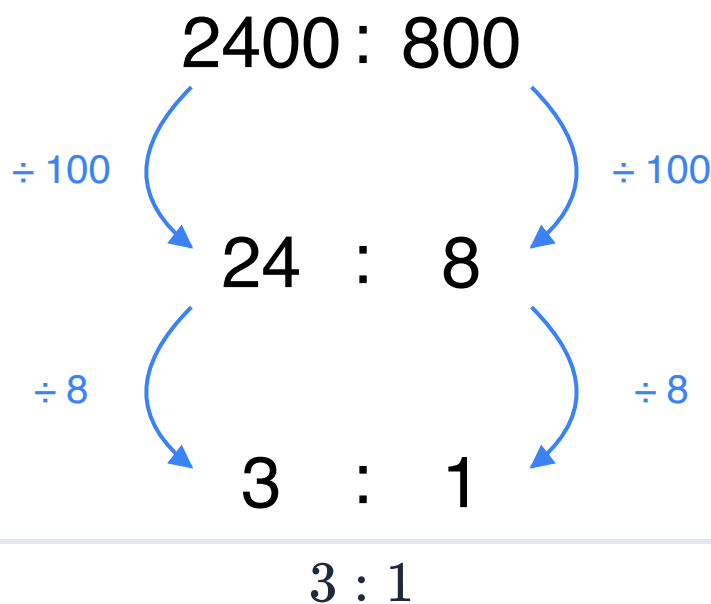
Simplify the ratio  $2.4\text{kg} : 800\text{g}$ **STEP-BY-STEP SOLUTION:**

First, convert the kilograms into grams. Multiply by 1000.

$$2.4\text{kg} = 2400\text{g}$$

Now simplify  $2400 : 800$ .

You can simplify in steps! First divide by 100, then divide by 8.

**Step 5 Worded Multiplier Ratios****CORE CONCEPT & METHOD**When a question tells you there are 'twice as many' or 'half as many', you should assign **parts** to each item.**Top Tip:** Let the smallest item be 1 or 2 parts to avoid nasty fractions!**COMMON MISTAKE:** "Four times as many blue as black" means Blue : Black = 4 : 1, not 1 : 4.**WORKED EXAMPLES****EXAMPLE 5.1**

NON-CALCULATOR

In a drawer, there are three times as many black socks as white socks.

Write down the ratio of black socks to white socks.

**STEP-BY-STEP SOLUTION:**

Let the smallest item (white socks) be 1 part.

Because there are **three times** as many black socks, they get 3 parts.

Black : White

$$3 : 1$$

$$3 : 1$$

**EXAMPLE 5.2** **NON-CALCULATOR**

In a harbour, there are five times as many sailboats as motorboats.  
Write down the ratio of motorboats to sailboats.

---

**STEP-BY-STEP SOLUTION:**

Let the smallest item (motorboats) be 1 part.

Because there are **five times** as many sailboats, they get 5 parts.

Careful with the order! The question asks for Motorboats : Sailboats.

Motorboats : Sailboats

1 : 5

---

**1 : 5**

## RATIO, PROPORTION AND RATES OF CHANGE • RATIO

## TOPIC 10: Sharing in a Ratio

### Target Grade 3

## Step 1 Basic Sharing

## CORE CONCEPT & METHOD

To share an amount in a ratio, use the **ADM** method:

- **Add** the parts together.
- **Divide** the total amount by the total parts (to find 1 part).
- **Multiply** to find the final amounts!

**COMMON MISTAKE:** Check which person's share the question asks for before writing your final answer.

### WORKED EXAMPLE

### EXAMPLE 1.1

☒ NON-CALCULATOR

Share £63 in the ratio 4 : 3. How much is the larger share?

### STEP-BY-STEP SOLUTION:

You can set this up as a table to find the multiplier!

Start  $4 : 3 = 7$  Total

New  $36 : 27 = 63$

Diagram illustrating the scaling of a division problem. The original problem is  $4 : 3 = 7$ . The new problem is  $36 : 27 = 63$ . The numbers 36, 27, and 63 are shown in dashed red boxes, indicating they are scaled versions of 4, 3, and 7 respectively, each multiplied by 9.

The larger share is the 4 parts, which became £36.

£36

## Step 2 Three-Part Sharing

## CORE CONCEPT & METHOD

The method is exactly the same for three parts! Just add all three numbers to get the total parts.

**COMMON MISTAKE:** Add all three ratio numbers together to get the total number of parts before dividing.

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

Amira, Ben and Chloe share £84 in the ratio 1 : 2 : 4. How much does Ben get?

## STEP-BY-STEP SOLUTION:

Set this up as a three-part table to find the multiplier!

|       |        |   |        |   |        |   |        |
|-------|--------|---|--------|---|--------|---|--------|
|       | A      | : | B      | : | C      | = | Total  |
| Start | 1      | : | 2      | : | 4      | = | 7      |
|       | ↓ × 12 |   | ↓ × 12 |   | ↓ × 12 |   | ↓ × 12 |
| New   | 12     | : | 24     | : | 48     | = | 84     |

Ben is the middle person (B). His share is £24.

**£24**

## Step 3 Reverse Sharing

## CORE CONCEPT &amp; METHOD

**Trap Warning!** Sometimes a question will give you **one person's amount** instead of the total.

If Ben gets £20 and his ratio part is 4, you find 1 part by dividing Ben's money by Ben's parts:  
 $£20 \div 4 = £5$ . Do NOT divide by the total parts!

**COMMON MISTAKE:** If you are given one person's share (not the total), divide by that person's parts — do not add all the parts first.

## WORKED EXAMPLE

## EXAMPLE 3.1

NON-CALCULATOR

Amira and Ben share money in the ratio 3 : 5.

Ben gets £45.

How much does Amira get?

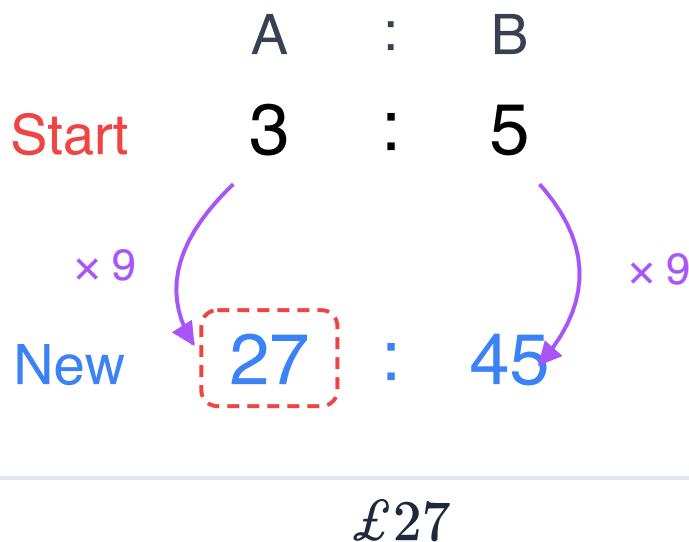
## STEP-BY-STEP SOLUTION:

We know Ben gets £45, and Ben has 5 parts.

Value of 1 part =  $£45 \div 5 = £9$

Amira has 3 parts.

Amira gets  $3 \times £9 = £27$



## Step 4 Difference Sharing

## CORE CONCEPT &amp; METHOD

If a question tells you someone gets '£20 **more than**' someone else, it's giving you the **difference**.

Step 1: Find the difference in the ratio parts.

Step 2: Divide the money difference by the parts difference to find 1 part.

**COMMON MISTAKE:** "£6 more than" is the difference — subtract the ratio parts to find the gap, do not add them.

## WORKED EXAMPLE

## EXAMPLE 4.1

NON-CALCULATOR

Amira and Ben share money in the ratio 7 : 3.

Amira gets £24 more than Ben. How much do they share in total?

## STEP-BY-STEP SOLUTION:

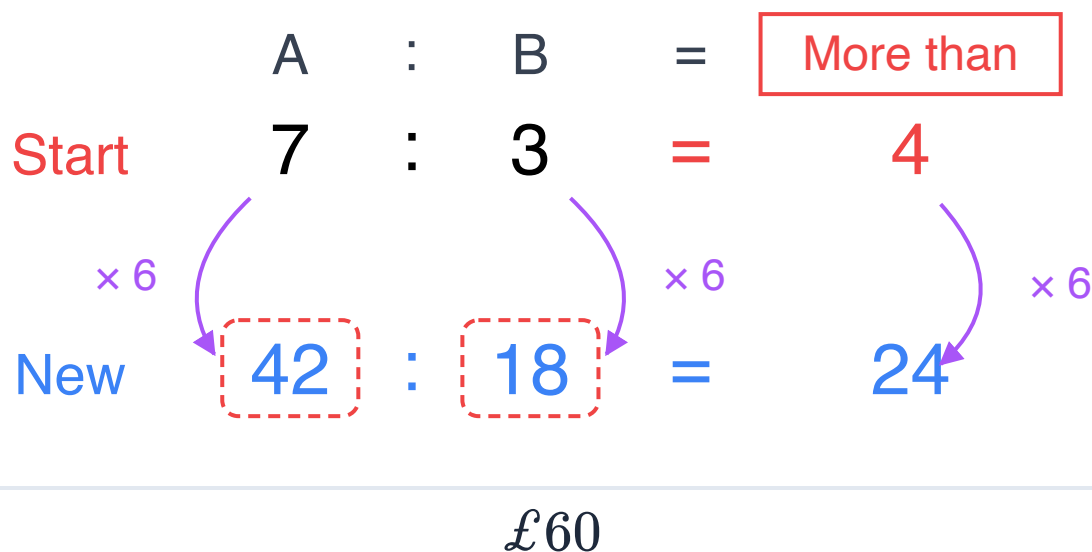
The difference in money is £24.

The difference in parts is  $7 - 3 = 4$  parts.

Value of 1 part =  $£24 \div 4 = £6$

We want the total. Total parts =  $7 + 3 = 10$ .

Total money =  $10 \times £6 = £60$



## RATIO, PROPORTION AND RATES OF CHANGE • PROPORTION

Target Grade 3

## TOPIC 11: Recipes

## Step 1 Scaling Up Recipes

## CORE CONCEPT &amp; METHOD

To find out how much of an ingredient you need, find the amount for **1 person**, then multiply by the new amount of people.

**Example:** Recipe for 8 brownies needs 120g of cocoa. How much for 12?

| Brownies               | Cocoa                  |
|------------------------|------------------------|
| 8                      | 120g                   |
| $\downarrow \div 8$    | $\downarrow \div 8$    |
| 1                      | 15g                    |
| $\downarrow \times 12$ | $\downarrow \times 12$ |
| 12                     | 180g                   |

For 1 brownie:  $120 \div 8 = 15\text{g}$ .

For 12 brownies:  $15 \times 12 = 180\text{g}$ .

*Alternatively, notice that 12 is 1.5 times bigger than 8, so  $120 \times 1.5 = 180\text{g}$ .*

**COMMON MISTAKE:** Divide by the recipe's serving number to find 1 serving first, then multiply up.

## WORKED EXAMPLES

**EXAMPLE 1.1**
 **NON-CALCULATOR**

A scone recipe for 8 people needs 320 g of flour, 80 g of butter, and 4 eggs. How much of each ingredient is needed for 14 people?

**STEP-BY-STEP SOLUTION:**

Find the amount for 1 person by dividing by 8, then multiply by 14.

| 8 people      | $\div 8 \rightarrow$ | 1 person | $\times 14 \rightarrow$ | 14 people |
|---------------|----------------------|----------|-------------------------|-----------|
| Flour<br>320g | $\div 8 \rightarrow$ | 40g      | $\times 14 \rightarrow$ | 560g      |
| Butter<br>80g | $\div 8 \rightarrow$ | 10g      | $\times 14 \rightarrow$ | 140g      |
| Eggs<br>4     | $\div 8 \rightarrow$ | 0.5      | $\times 14 \rightarrow$ | 7         |

**EXAMPLE 1.2**
 **NON-CALCULATOR**

A pancake recipe for 6 people needs 240 g of flour, 60 g of caster sugar, and 2 eggs. How much of each ingredient is needed for 15 people?

**STEP-BY-STEP SOLUTION:**

If you don't have a calculator, you can build up to the target amount using chunks you already know! For example, 15 people is two groups of 6, plus a group of 3 (which is half of 6).

| 6 people + 6 people + 3 people = |      |   |      |   |                           | 15 people |
|----------------------------------|------|---|------|---|---------------------------|-----------|
| Flour                            | 240g | + | 240g | + | <sup>(half)</sup><br>120g | = 600g    |
| Sugar                            | 60g  | + | 60g  | + | 30g                       | = 150g    |
| Eggs                             | 2    | + | 2    | + | 1                         | = 5       |

**Step 2** Finding the Yield**CORE CONCEPT & METHOD**

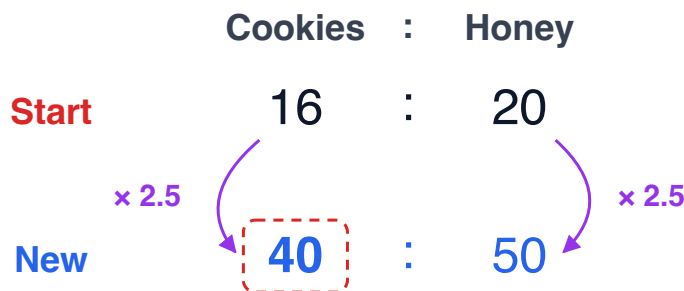
Work backwards! See how many times the recipe's ingredient fits into the amount you used.

**Example:** Recipe makes 16 oat cookies with 20ml of honey. Nadia uses 50ml of honey. How many did she make?

Find the multiplier by dividing what you **used** by what the **recipe** needs:

$$\text{Multiplier} = 50 \div 20 = \mathbf{2.5}.$$

Since she used 2.5 times as much honey, she must have made 2.5 times as many cookies!



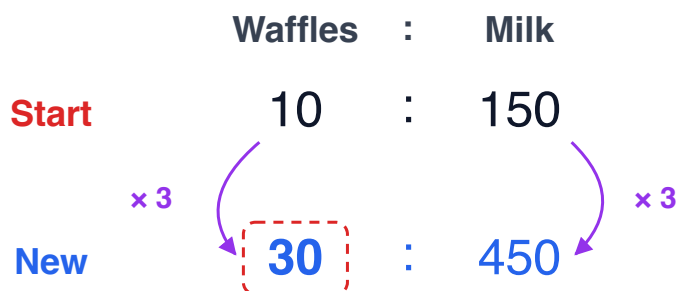
**COMMON MISTAKE:** Divide the amount used by the recipe amount (not the other way round) to find the scale factor.

**WORKED EXAMPLE****EXAMPLE 2.1**
**NON-CALCULATOR**

A recipe for 10 waffles uses 150ml of milk. Nadia uses 450ml of milk. How many waffles does she make?

**STEP-BY-STEP SOLUTION:**

1. Find the multiplier by dividing the milk used by the recipe amount:  $450 \div 150 = \mathbf{3}$ .
2. Multiply the number of waffles in the recipe by 3:  $10 \times 3 = \mathbf{30}$  waffles.



### Step 3 Maximum Yield

#### CORE CONCEPT & METHOD

Calculate how many times you can make the recipe for **each** ingredient separately. The smallest number is your limit!

**Example:** A loaf of bread needs 250g flour, 150ml water, and 2 spoons of yeast. You have 1200g flour, 600ml water, and 12 spoons of yeast. How many loaves can you make?

|       | Need  | Have  | How many Loaves          |
|-------|-------|-------|--------------------------|
| Flour | 250g  | 1200g | $\frac{1200}{250} = 4.8$ |
| Water | 150ml | 600ml | $\frac{600}{150} = 4$    |
| Yeast | 2     | 12    | $\frac{12}{2} = 6$       |

Water gives the smallest yield (4). The water will run out first!

The maximum you can make is **4 loaves**.

**COMMON MISTAKE:** Always pick the smallest number of batches (and round down if needed) — when one ingredient runs out, you must stop.

#### WORKED EXAMPLE

##### EXAMPLE 3.1

 **NON-CALCULATOR**

A batch of 8 muffins needs 200g of flour and 100g of blueberries. Imran has 700g of flour and 300g of blueberries. What is the maximum number of muffins he can make?

#### STEP-BY-STEP SOLUTION:

|             | Need | Have | How many Batches        |
|-------------|------|------|-------------------------|
| Flour       | 200g | 700g | $\frac{700}{200} = 3.5$ |
| Blueberries | 100g | 300g | $\frac{300}{100} = 3$   |

Blueberries give the smallest yield (**3 batches**). The blueberries will run out first!

Each batch makes 8 muffins, so the maximum is  $3 \times 8 = \mathbf{24 \text{ muffins}}$ .

## Step 4 Ingredient Shortfall

### CORE CONCEPT & METHOD

First, calculate exactly how much you need. Then, subtract what you already have.

**COMMON MISTAKE:** Don't stop after finding the total needed — subtract what is already in the cupboard to find how much more to buy.

### WORKED EXAMPLE

#### EXAMPLE 4.1

NON-CALCULATOR

A pasta recipe for 5 people requires 400g of penne.

Tariq wants to cook for 12 people.

He currently has 650g of penne in his cupboard.

How many more grams of penne does he need to buy?

#### STEP-BY-STEP SOLUTION:

1. Find out how much he needs for 12 people.

|               |                      |          |                         |           |
|---------------|----------------------|----------|-------------------------|-----------|
| 5 people      | $\div 5 \rightarrow$ | 1 person | $\times 12 \rightarrow$ | 12 people |
| Penne<br>400g | $\div 5 \rightarrow$ | 80g      | $\times 12 \rightarrow$ | 960g      |

2. He needs **960g**, but he only has **650g**.

3. Subtract what he has from what he needs:

$$960 - 650 = 310\text{g}$$

## Step 5 Do you have enough?

### CORE CONCEPT & METHOD

Calculate the required amount and compare it to what you have. Always state your final conclusion clearly (Yes or No).

**COMMON MISTAKE:** You get zero marks for writing "Yes" or "No" alone — you must show the exact numbers you are comparing.

## WORKED EXAMPLE

**EXAMPLE 5.1** **NON-CALCULATOR**

A recipe for 8 muffins requires 240g of oats.

Clara wants to make 18 muffins.

She has a 500g bag of oats.

Does Clara have enough oats to make 18 muffins?

**STEP-BY-STEP SOLUTION:**

1. Find out how much she needs for 18 muffins.

|                     |                      |                 |                         |                      |
|---------------------|----------------------|-----------------|-------------------------|----------------------|
| <b>8</b><br>muffins | $\div 8 \rightarrow$ | <b>1 muffin</b> | $\times 18 \rightarrow$ | <b>18</b><br>muffins |
| <b>Oats</b><br>240g | $\div 8 \rightarrow$ | 30g             | $\times 18 \rightarrow$ | <b>540g</b>          |

2. She needs **540g**, but she only has **500g**.

3. 540g is more than 500g, so she does **not** have enough.

Conclusion: **No**

## RATIO, PROPORTION AND RATES OF CHANGE • PROPORTION

Target Grade 3

## TOPIC 12: Exchange Rates

## Step 1 Converting Currency

## CORE CONCEPT &amp; METHOD

To convert **from Pounds (£) to Foreign Currency**, you MULTIPLY by the exchange rate.

To convert **from Foreign Currency back to Pounds (£)**, you DIVIDE by the exchange rate.

*Always round money to 2 decimal places unless told otherwise!*

**COMMON MISTAKE:** Multiply when going from £1 to foreign currency, and divide when changing foreign currency back into £.

## WORKED EXAMPLES

## EXAMPLE 1.1

CALCULATOR

The exchange rate is £1 = \$1.35. Convert £60 into Dollars.

## STEP-BY-STEP SOLUTION:

Multiply by the exchange rate.

$$\begin{array}{ccc}
 \text{£1} & : & \$1.35 \\
 \downarrow \times 60 & & \downarrow \times 60 \\
 \text{£60} & : & \$81
 \end{array}$$

$$60 \times 1.35 = 81$$

So, \$81

## EXAMPLE 1.2

CALCULATOR

You change £300 into Euros at £1 = €1.18. On holiday you spend €114, then change the rest back into Pounds at £1 = €1.20. How many Pounds do you get back?

## STEP-BY-STEP SOLUTION:

|        |      |                           |      |          |
|--------|------|---------------------------|------|----------|
| STEP 1 | £300 | $\times 1.18 \rightarrow$ | €354 | £ to €   |
| STEP 2 | €354 | $- €114 \rightarrow$      | €240 | Spending |
| STEP 3 | €240 | $\div 1.20 \rightarrow$   | £200 | € to £   |

You get back £200.

## Step 2 Comparing Costs Abroad

### CORE CONCEPT & METHOD

To compare the cost of an item in two different countries, you must convert them so they are both in the **same currency** (usually Pounds).

Divide the foreign price by the exchange rate to get the price in Pounds, then compare!

**COMMON MISTAKE:** Convert both prices into the same currency before finding the difference.

### WORKED EXAMPLE

#### EXAMPLE 2.1

[CALCULATOR](#)

A digital camera costs £240 in the UK and \$280 in the US. The exchange rate is £1 = \$1.25. Which is cheaper?

#### STEP-BY-STEP SOLUTION:

Divide the US price by the exchange rate (1.25) to convert it into Pounds:

$$\begin{array}{rcccl}
 & & \div 1.25 & & \\
 \text{Change \$ to £} & \text{£1} & : & \text{\$1.25} & \\
 & & & & \\
 \text{US Cost in £} & \text{£224} & : & \text{\$280} & \text{Camera Cost} \\
 & & \div 1.25 & & 
 \end{array}$$

$\$280 \div 1.25 = \text{£}224$ , which is cheaper than the UK price (**£240**), so the **US** is cheaper.  
(Alternatively, converting the UK price to Dollars gives  $\text{£}240 \times 1.25 = \$300$ , which is dearer than \$280.)

## Step 3 Best Value & Banknotes

### CORE CONCEPT & METHOD

**Banknotes:** If you are buying currency and want to know how many notes you can get, you must round **DOWN** to the nearest whole note, because you can't buy a fraction of a note!

**Best Exchange Rate:** The best exchange rate is the one that gives you the **MOST** foreign currency for every £1.

**COMMON MISTAKE:** Round down to the whole number of banknotes — a bank cannot hand out part of a note.

### WORKED EXAMPLES

#### EXAMPLE 3.1

[CALCULATOR](#)

Sam changes £250 into Euros at a rate of £1 = €1.18. The bank only gives €20 notes. Work out the greatest number of €20 notes Sam can get.

#### STEP-BY-STEP SOLUTION:

1. Convert £250 into Euros:  $250 \times 1.18 = \text{€}295$
2. Divide by the note value:  $295 \div 20 = 14.75$
3. Round **down** to a whole number of notes: Sam can get **14** notes ( $14 \times \text{€}20 = \text{€}280$ ).

**EXAMPLE 3.2** **CALCULATOR**

In London, the exchange rate is £1 = \$1.28. In New York, the exchange rate is \$1 = £0.80. Which city gives more Dollars for £1?

**STEP-BY-STEP SOLUTION:**

1. London already shows £1 = **\$1.28**.
2. Convert the New York rate to £1 by dividing by 0.80:  $1 \div 0.80 = \textbf{\$1.25}$  per £1.
3. Compare: \$1.28 is greater than \$1.25, so **London** gives the better exchange rate.

**Step 4 Unit Conversions & Currency****CORE CONCEPT & METHOD**

Make sure everything is in the **same currency AND the same units** before comparing!

**COMMON MISTAKE:** Convert both the currency and the weight unit (e.g. per kg) before deciding which shop is cheaper.

**WORKED EXAMPLE****EXAMPLE 4.1** **CALCULATOR**

Coffee beans cost £8.40 per kg in the UK.

In Spain, they cost €4.50 for 500g.

The exchange rate is £1 = €1.20.

Which country is cheaper?

**STEP-BY-STEP SOLUTION:**

1. Make the **units** the same (find 1kg in Spain):

|               |              |                                          |                              |
|---------------|--------------|------------------------------------------|------------------------------|
| <b>Spain:</b> | <b>500g</b>  | <b><math>\times 2 \rightarrow</math></b> | <b>1kg</b><br><b>(1000g)</b> |
|               | <b>€4.50</b> | <b><math>\times 2 \rightarrow</math></b> | <b>€9.00</b>                 |

2. Make the **currency** the same (Convert Spain to £):

$$\textbf{€9.00} \div \textbf{1.20} = \textbf{£7.50}$$

3. Compare per kg:

- UK: **£8.40**
- Spain: **£7.50**

**Spain** is cheaper.

**Step 5 Multi-Currency Budgets****CORE CONCEPT & METHOD**

Convert all of the expenses and your total budget into a **single common currency** so you can compare them fairly.

**COMMON MISTAKE:** Multiply the hotel nightly cost by the number of nights before converting currency.

## WORKED EXAMPLE

**EXAMPLE 5.1** **CALCULATOR**

Maya has a budget of £1200 for a trip to Spain.

She books flights for £220.

She books a hotel for 5 nights at €108 per night.

She books excursions for €180.

The exchange rate is £1 = €1.20.

How much of her budget does she have left in Pounds (£)?

**STEP-BY-STEP SOLUTION:**

1. Find the total hotel cost, add her Euro costs, and convert to Pounds:

$$\text{Hotel: } 5 \times \text{€}108 = \text{€}540$$

$$\text{€}540 + \text{€}180 = \text{€}720 \rightarrow \text{€}720 \div 1.20 = \text{£}600$$

2. Add her Pound costs:

$$\text{£}220 \text{ (flights)} + \text{£}600 = \text{£}820 \text{ total spent}$$

3. Subtract from her budget:

$$\text{£}1200 - \text{£}820 = \text{£}380 \text{ left}$$

## RATIO, PROPORTION AND RATES OF CHANGE • PROPORTION

Target Grade 3

## TOPIC 13: Best Buys

## Step 1 Comparing Two Items

## CORE CONCEPT &amp; METHOD

To compare prices, you must find a common amount. The easiest way is to find the cost of **1 unit** (e.g. 1 pen, 100g, or 1kg).

Example: 8 pens for £2.80 or 5 pens for £1.65

8 pens

£2.80

 $\downarrow \div 8$ 

1 pen

£0.35

5 pens

£1.65

 $\downarrow \div 5$ 

1 pen

£0.33

Pack 2 is cheaper per pen, so it is the best value.

**COMMON MISTAKE:** Always divide Money  $\div$  Quantity to get the cost per 1 unit, and write a final sentence naming the cheaper option.

## WORKED EXAMPLES

## EXAMPLE 1.1

CALCULATOR

A small box of teabags contains 15 teabags and costs £2.10. A large box contains 25 teabags and costs £3.25. Which box is better value?

## STEP-BY-STEP SOLUTION:

Find the cost per teabag for each box:

## SMALL BOX

15  
teabags

£2.10

 $\downarrow \div 15$ 

1 teabag

£0.14

## LARGE BOX

25  
teabags

£3.25

 $\downarrow \div 25$ 

1 teabag

£0.13

The large box is cheaper per teabag, so it is **better value**.

**EXAMPLE 1.2** **CALCULATOR**

Apple juice: Carton A is 400 ml for 76p. Carton B is 600 ml for £1.08. Which is the better buy?

**STEP-BY-STEP SOLUTION:**

Convert to cost per 100 ml:

| CARTON A |       | CARTON B |       |
|----------|-------|----------|-------|
| 400 ml   | 76p   | 600 ml   | £1.08 |
| ↓ ÷ 4    | ↓ ÷ 4 | ↓ ÷ 6    | ↓ ÷ 6 |
| 100 ml   | 19p   | 100 ml   | 18p   |

Carton B costs less per 100 ml, so it is the **better value**.

**Step 2 Comparing Three Items****CORE CONCEPT & METHOD**

The method is exactly the same, you just have to do the division three times! Always clearly state which one is best at the end.

**COMMON MISTAKE:** Convert mixed weights into the same unit first (e.g. 1.5 kg = 1500 g), and if a calculator gives a decimal in pounds (like £1.508), do not round it too early before comparing.

## WORKED EXAMPLE

## EXAMPLE 2.1

 CALCULATOR

Porridge oats are sold in three sizes: Small (500 g for £1.85), Medium (800 g for £2.80), Large (1.5 kg for £4.95). Which is the best value?

## STEP-BY-STEP SOLUTION:

First match the units ( $1.5 \text{ kg} = 1500 \text{ g}$ ), then find the cost per 100 g for each:

| SMALL |       | MEDIUM |       |
|-------|-------|--------|-------|
| 500 g | £1.85 | 800 g  | £2.80 |
| ↓ ÷ 5 | ↓ ÷ 5 | ↓ ÷ 8  | ↓ ÷ 8 |
| 100 g | 37p   | 100 g  | 35p   |

| LARGE           |        |
|-----------------|--------|
| 1.5 kg (1500 g) | £4.95  |
| ↓ ÷ 15          | ↓ ÷ 15 |
| 100 g           | 33p    |

The large bag is cheapest per 100 g, so it is the **best value**.

## Step 3 Best Buys with Offers

## CORE CONCEPT &amp; METHOD

First apply the special offer to find the **new price** or **new amount**. Then calculate the cost per unit.

**Example:** Box A is 200 tissues + 25% extra free for £3. Box B is 200 tissues with 15% off £3.

**Box A:** 25% of 200 is 50. New amount = 250 tissues. Price = £3.00.

Cost per tissue =  $£3.00 \div 250 = £0.012$

**Box B:** 15% of £3 is 45p. New price = £2.55. Amount = 200 tissues.

Cost per tissue =  $£2.55 \div 200 = £0.01275$

Box A is cheaper per tissue!

**COMMON MISTAKE:** "25% extra free" increases the amount inside the pack — it does not reduce the price by 25%.

## WORKED EXAMPLE

## EXAMPLE 3.1

 CALCULATOR

Jar A: 400 g of honey for £6.40 with '25% extra free'. Jar B: 400 g of honey for £6.40 with '15% off'. Which is better value?

## STEP-BY-STEP SOLUTION:

## JAR A: 25% EXTRA FREE

400 g    + 100 g →    500 g  
£6.40    stays →    £6.40

500 g                    £6.40

↓ ÷ 5

↓ ÷ 5

100 g

£1.28

## JAR B: 15% OFF

400 g    stays →    400 g  
£6.40    - 96p →    £5.44

400 g                    £5.44

↓ ÷ 4

↓ ÷ 4

100 g

£1.36

Jar A costs less per 100 g, so **Jar A** is better value.

## RATIO, PROPORTION AND RATES OF CHANGE • SCALE &amp; CONVERSION

Target Grade 3

## TOPIC 14: Unit Conversions

## Step 1 Basic Metric Conversions

## CORE CONCEPT &amp; METHOD

To convert from a larger unit to a smaller unit, **multiply**. To convert from a smaller unit to a larger unit, **divide**.

- 1 km = 1000 m
- 1 m = 100 cm
- 1 cm = 10 mm
- 1 kg = 1000 g
- 1 litre = 1000 ml

**COMMON MISTAKE:** Big unit to small unit → multiply (more of them); small unit to big unit → divide.

## WORKED EXAMPLES

## EXAMPLE 1.1

NON-CALCULATOR

Convert 7.2 km into metres.

## STEP-BY-STEP SOLUTION:

Kilometres → metres: **multiply by 1000** (larger unit to smaller unit).

$$7.2 \times 1000 = 7200 \text{ m}$$

## EXAMPLE 1.2

NON-CALCULATOR

Convert 6800 g into kilograms.

## STEP-BY-STEP SOLUTION:

Grams → kilograms: **divide by 1000** (smaller unit to larger unit).

$$6800 \div 1000 = 6.8 \text{ kg}$$

## Step 2 Metric to Imperial

## CORE CONCEPT &amp; METHOD

Use standard conversion facts like a proportion problem.

- 5 miles  $\approx$  8 km
- 1 inch  $\approx$  2.5 cm
- 1 kg  $\approx$  2.2 lbs
- 1 gallon  $\approx$  4.5 litres

**COMMON MISTAKE:** Check whether the answer should be bigger or smaller (8 km  $\approx$  5 miles, so km numbers are larger than miles).

## WORKED EXAMPLES

**EXAMPLE 2.1**

NON-CALCULATOR

Convert 55 miles into kilometres. (Use 5 miles  $\approx$  8 km.)**STEP-BY-STEP SOLUTION:**

Set up the proportion and scale up both top and bottom by 11:

$$\frac{8 \text{ km}}{5 \text{ miles}} = \frac{? \text{ km}}{55 \text{ miles}}$$

$\times 11$   
 $\times 11$

$$? = 8 \times 11 = 88 \text{ km}$$

OR — Multiply both sides by 55:

$$? = \frac{8}{5} \times 55 = 1.6 \times 55 = 88 \text{ km}$$

**EXAMPLE 2.2**

NON-CALCULATOR

A suitcase weighs 6.5 kg. Convert this to pounds. (Use 1 kg  $\approx$  2.2 lbs.)**STEP-BY-STEP SOLUTION:**

$$6.5 \text{ kg} \times 2.2 = 14.3 \text{ lbs}$$

**Step 3 Area & Volume Conversions****CORE CONCEPT & METHOD**

When converting area ( $^2$ ), you must square the conversion factor. For volume ( $^3$ ), you must cube the conversion factor.

**Example: Convert 5 m<sup>2</sup> to cm<sup>2</sup>.**

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^2 = 100^2 \text{ cm}^2 = 10,000 \text{ cm}^2$$

$$\text{So } 5 \text{ m}^2 = 5 \times 10,000 = 50,000 \text{ cm}^2.$$

**COMMON MISTAKE:** 1 m<sup>2</sup> is NOT 100 cm<sup>2</sup> — square the length factor: 100<sup>2</sup> = 10,000 cm<sup>2</sup>.

## WORKED EXAMPLES

**EXAMPLE 3.1**

NON-CALCULATOR

Convert 4.2 m<sup>2</sup> into cm<sup>2</sup>.**STEP-BY-STEP SOLUTION:**

Since 1 m = 100 cm, we square the scale factor for area: 1 m<sup>2</sup> = 100<sup>2</sup> cm<sup>2</sup> = 10,000 cm<sup>2</sup>.

$$4.2 \times 10,000 = 42,000 \text{ cm}^2$$

**EXAMPLE 3.2**

NON-CALCULATOR

Convert 8,000,000 cm<sup>3</sup> into m<sup>3</sup>.

**STEP-BY-STEP SOLUTION:**

Since 1 m = 100 cm, we cube the scale factor for volume:  $1 \text{ m}^3 = 100^3 \text{ cm}^3 = 1,000,000 \text{ cm}^3$ .

$$8,000,000 \div 1,000,000 = 8 \text{ m}^3$$

**Step 4 Compound Units****CORE CONCEPT & METHOD**

Convert the distance unit first, then convert the time unit. Finally, divide distance by time.

**Example: Convert 90 km/h to m/s.**

1. Change km to m:  $90 \times 1000 = 90,000 \text{ m}$  (per hour).
2. Change hours to seconds:  $1 \text{ hour} = 60 \times 60 = 3600 \text{ seconds}$ .
3. Divide:  $90,000 \div 3600 = 25 \text{ m/s}$ .

**COMMON MISTAKE:** There are 3600 seconds in 1 hour ( $60 \times 60$ ), not 60 or 100.

**WORKED EXAMPLE****EXAMPLE 4.1**

NON-CALCULATOR

Convert a speed of 54 km/h into m/s.

**STEP-BY-STEP SOLUTION:**

**Step 1:** Convert kilometres into metres:  $54 \times 1000 = 54,000 \text{ m}$  per hour.

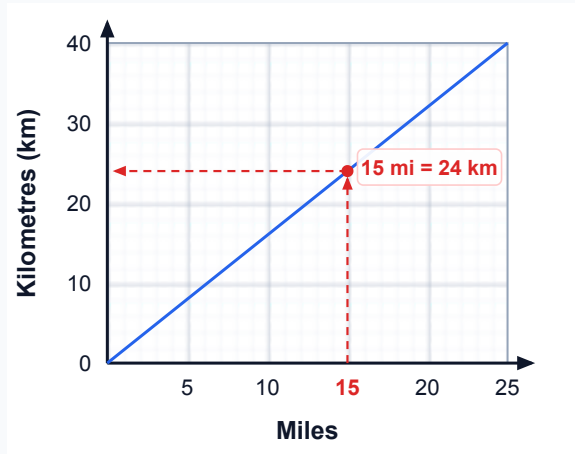
**Step 2:** Divide by 3600 seconds in an hour:

$$54,000 \div 3600 = 15 \text{ m/s}$$

## Step 5 Conversion Graphs

### CORE CONCEPT & METHOD

To use a conversion graph, draw straight guide lines with a ruler: read **up from the  $x$ -axis to the line**, then **across to the  $y$ -axis** (or start on the  $y$ -axis, go across to the line, and read down).



If the value is **off the graph**, read off a smaller value on the graph and multiply! For example, to convert 150 miles, read off 15 miles = 24 km and multiply by 10 to get 240 km.

**COMMON MISTAKE:** Check the value of 1 small grid square on each axis before reading off your answer.

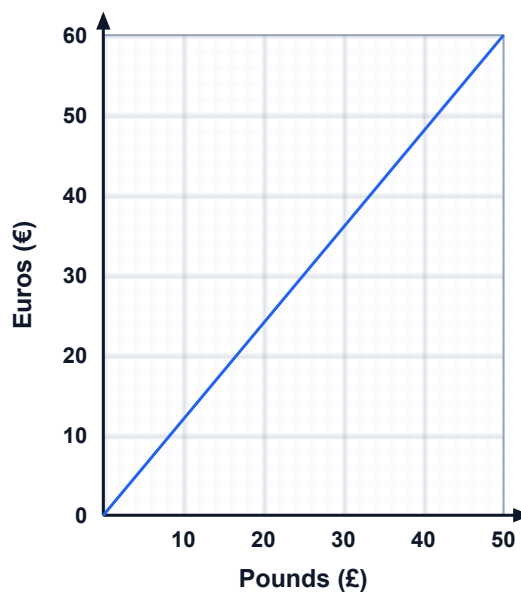
### WORKED EXAMPLES

#### EXAMPLE 5.1

NON-CALCULATOR

You can use this conversion graph to change between Pounds (£) and Euros (€).

- (a) Change £25 into Euros (€).
- (b) Change €48 into Pounds (£).

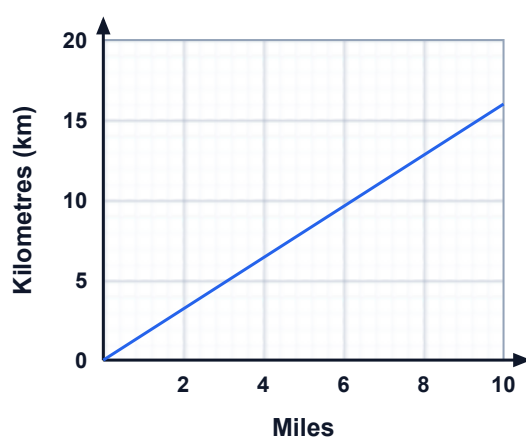


#### STEP-BY-STEP SOLUTION:

- (a) Find 25 halfway between 20 and 30 on the Pounds ( $x$ ) axis. Follow the grid line up to the blue line and across to the Euros ( $y$ ) axis: **€30**.
- (b) Each small square on the Euros ( $y$ ) axis is worth  $10 \div 5 = 2$ . Find 48 (one small square below 50), go across to the blue line, and read straight down to the Pounds ( $x$ ) axis: **£40**.

**EXAMPLE 5.2****NON-CALCULATOR**

Use this conversion graph between miles and kilometres to convert 70 miles into kilometres.

**STEP-BY-STEP SOLUTION:**

Since 70 miles is off the graph, read off 10 miles first (each small vertical square is  $5 \div 5 = 1$  km, so 10 miles = 16 km).

Multiply by 7 to find 70 miles:

$$16 \times 7 = 112 \text{ km}$$

## RATIO, PROPORTION AND RATES OF CHANGE • SCALE &amp; CONVERSION

Target Grade 3

## TOPIC 15: Scale Drawings &amp; Maps

## Step 1 Using Scale Drawings

## CORE CONCEPT &amp; METHOD

To find real-life sizes from a drawing, first find the **scale multiplier** between the drawing and real life for one known measurement. Then apply it to the unknown one.

**Example:** A drawing shows a van. Drawing height = 3cm. Drawing length = 9cm. Real height = 1.8m. Find real length.

|         | Height | Length |
|---------|--------|--------|
| Drawing | 3 cm   | 9 cm   |
|         | ↓ ×0.6 | ↓ ×0.6 |
| Real    | 1.8 m  | 5.4 m  |

- Multiplier:  $1.8 \text{ m} \div 3 \text{ cm} = 0.6 \text{ m per cm of drawing.}$
- Real length:  $9 \text{ cm} \times 0.6 = 5.4 \text{ m.}$

**Perimeter tip:** If asked for a real-life **perimeter**, find the real length and real width first, then add all four sides together.

**COMMON MISTAKE:** Watch the units in your multiplier: if 1 cm on the drawing already represents metres in real life (e.g. 2.5 m per cm), do not divide by 100 at the end.

## WORKED EXAMPLE

## EXAMPLE 1.1

NON-CALCULATOR

A scale drawing of a studio shows a wall that is 4 cm long. The real wall is 10 m long. Another wall on the drawing is 7 cm long. What is the real length of the second wall?

## STEP-BY-STEP SOLUTION:

|         | Wall 1 | Wall 2 |
|---------|--------|--------|
| Drawing | 4 cm   | 7 cm   |
|         | ↓ ×2.5 | ↓ ×2.5 |
| Real    | 10 m   | 17.5 m |

**Step 1:** Find the scale multiplier using the known measurement.

$$\text{Multiplier} = \frac{10 \text{ m}}{4 \text{ cm}} = 2.5 \text{ m per cm}$$

**Step 2:** Apply the multiplier to the unknown wall.

$$7 \text{ cm} \times 2.5 = 17.5 \text{ m}$$

The real length of the second wall is **17.5 m**.

## Step 2 Scale Models

## CORE CONCEPT &amp; METHOD

A scale of 1:50 means that 1 unit on the model is 50 units in real life. Multiply by 50 to get the real length in **centimetres**, then divide by 100 to convert to **metres**!

**COMMON MISTAKE:** Multiplying by the scale gives centimetres if the model is in **cm** — remember to divide by 100 for metres.

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

A model of a tower is made using a scale of 1:250. The model is 18 cm tall. How tall is the real tower in metres?

## STEP-BY-STEP SOLUTION:

|       | Scale  | Tower   |
|-------|--------|---------|
| Model | 1      | 18 cm   |
|       | ↓ ×250 | ↓ ×250  |
| Real  | 250    | 4500 cm |

**Step 1:** Multiply the model length by the scale factor to get the real length in cm.

$$18 \times 250 = 4500 \text{ cm}$$

**Step 2:** Convert cm to metres by dividing by 100.

$$4500 \div 100 = 45 \text{ m}$$

The real tower is **45 m** tall.

## Step 3 Map Scales

## CORE CONCEPT &amp; METHOD

A map scale like 1:50,000 means 1cm on the map is 50,000cm in real life.

**Trick:** To convert cm to km quickly, divide by 100,000 (cross off 5 zeros)!

So 50,000cm = 0.5km.

**COMMON MISTAKE:** Remember 1 km = 100,000 cm (divide centimetres by 100 for metres, then by 1000 for kilometres).

## WORKED EXAMPLE

## EXAMPLE 3.1

NON-CALCULATOR

A map has a scale of 1:50,000. Two villages are 7 cm apart on the map. What is the real distance between the villages in kilometres?

## STEP-BY-STEP SOLUTION:

**Step 1:** Multiply the map distance by the scale factor to get the real distance in cm.

$$7 \times 50,000 = 350,000 \text{ cm}$$

**Step 2:** Convert cm to km by dividing by 100,000.

$$350,000 \div 100,000 = 3.5 \text{ km}$$

The real distance is **3.5 km**.

## GEOMETRY AND MEASURES • CIRCLES

## TOPIC 16: Area &amp; Circumference

Target Grade 3

## Step 1 Basic Circumference

## CORE CONCEPT &amp; METHOD

The circumference is the distance around the outer edge of a circle.

Use  $C = \pi d$  when given the diameter  $d$ , or  $C = 2\pi r$  when given the radius  $r$ .

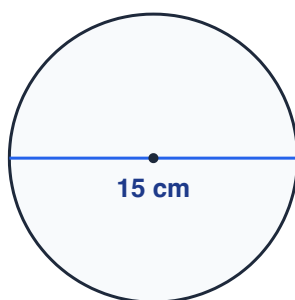
**COMMON MISTAKE:** Check if the diagram gives the radius  $r$  or the full diameter  $d$  — if given  $r$ , double it first to get  $d$ .

## WORKED EXAMPLES

## EXAMPLE 1.1

CALCULATOR

Work out the circumference of this circle. Give your answer to 1 decimal place.



## STEP-BY-STEP SOLUTION:

Substitute  $d = 15$  into  $C = \pi d$ :

$$C = \pi \times 15 \approx 47.12 = 47.1 \text{ cm.}$$

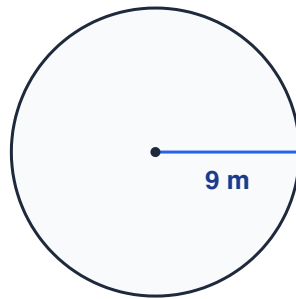
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47.1

**EXAMPLE 1.2**

CALCULATOR

Work out the circumference of a circle with radius 9 m to 1 decimal place.

**STEP-BY-STEP SOLUTION:**

Double the radius to get the diameter ( $d = 18$  m), then multiply by  $\pi$ :

$$C = 2 \times \pi \times 9 = 18\pi \approx 56.5 \text{ m.}$$

---

56.5

**Step 2 Basic Area****CORE CONCEPT & METHOD**

The area is the space inside the circle. The formula is  $A = \pi r^2$ .

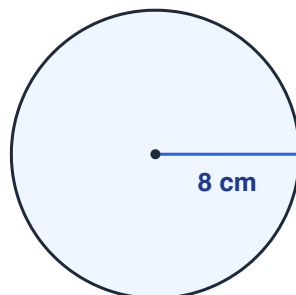
Remember to square the radius FIRST before multiplying by  $\pi$  — and if you are given the diameter, halve it first!

**COMMON MISTAKE:** In  $A = \pi r^2$ , square the radius only ( $r \times r$ , not  $r \times 2$ ), and halve the diameter first if given  $d$ .

**WORKED EXAMPLES****EXAMPLE 2.1**

CALCULATOR

Work out the area of this circle to 1 decimal place.

**STEP-BY-STEP SOLUTION:**

$$A = \pi r^2 = \pi \times 8^2 = 64\pi \approx 201.1 \text{ cm}^2.$$

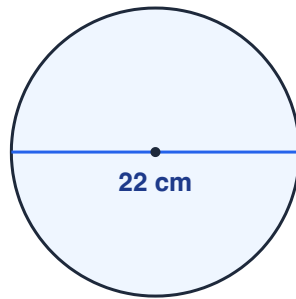
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201.1

**EXAMPLE 2.2**

CALCULATOR

Work out the area of this circle with diameter 22 cm to 1 decimal place.

**STEP-BY-STEP SOLUTION:**

First halve the diameter:  $r = 22 \div 2 = 11$  cm.

$$A = \pi \times 11^2 = 121\pi \approx 380.1 \text{ cm}^2.$$

---

380.1

**Step 3 Semi Circles****CORE CONCEPT & METHOD**

A semi-circle's area is half of a full circle ( $\frac{1}{2}\pi r^2$ ).

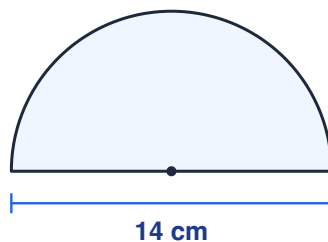
Its **perimeter** is half the circumference PLUS the straight diameter line across the base!

**COMMON MISTAKE:** For the perimeter of a semi-circle, halve  $\pi d$  for the curved edge and then ADD the straight diameter  $d$ .

**WORKED EXAMPLE****EXAMPLE 3.1**

CALCULATOR

Work out the total perimeter of this semi-circle to 1 decimal place.

**STEP-BY-STEP SOLUTION:**

$$\text{Curved arc} = \frac{1}{2} \times \pi \times 14 \approx 21.99 \text{ cm.}$$

Add the straight base edge (14 cm):

$$\text{Perimeter} = 21.99 + 14 = 36.0 \text{ cm.}$$

---

36.0

## Step 4 Compound Shapes

### CORE CONCEPT & METHOD

Break compound shapes into simple parts: either **subtract** the area of a cut-out hole from the outer shape, or **add** the area of a semi-circle to a rectangle.

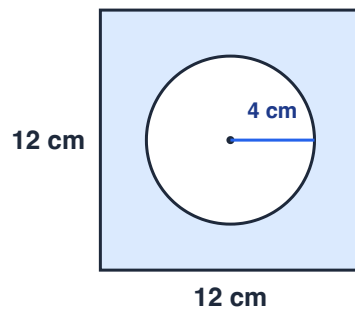
**COMMON MISTAKE:** Subtract areas when a hole is cut out, and add areas when shapes are joined together.

### WORKED EXAMPLE

#### EXAMPLE 4.1

 **CALCULATOR**

A circle of radius 4 cm is cut out from a 12 cm by 12 cm square. Find the shaded area to 1 decimal place.



#### STEP-BY-STEP SOLUTION:

$$\text{Square area} = 12 \times 12 = 144 \text{ cm}^2.$$

$$\text{Circle area} = \pi \times 4^2 \approx 50.27 \text{ cm}^2.$$

$$\text{Shaded area} = 144 - 50.27 = 93.7 \text{ cm}^2.$$

**93.7**

## Step 5 In Terms of Pi

### CORE CONCEPT & METHOD

When asked for an answer **in terms of  $\pi$** , work out the numerical part and leave the  $\pi$  symbol attached in your final answer.

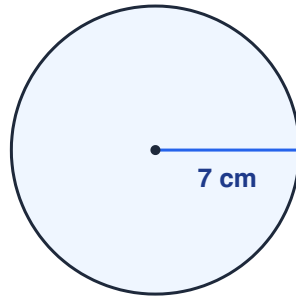
**COMMON MISTAKE:** Do not press the SD decimal button — leave  $\pi$  written next to your number (e.g.  $18\pi$ ).

## WORKED EXAMPLE

## EXAMPLE 5.1

NON-CALCULATOR

Calculate the exact area of this circle in terms of  $\pi$ .



## STEP-BY-STEP SOLUTION:

Use  $A = \pi r^2$ :

$$A = \pi \times 7^2 = 49\pi \text{ cm}^2.$$

$$49\pi$$

## Step 6 Working Backwards

## CORE CONCEPT &amp; METHOD

Set up an equation using the area or circumference formula and solve backwards to find the missing radius or diameter first.

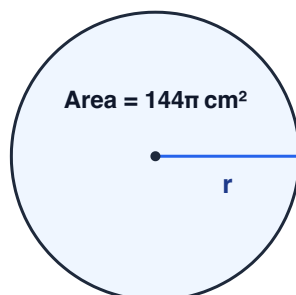
**COMMON MISTAKE:** If  $C = \pi d$ , divide the circumference by  $\pi$  to get the diameter  $d$  (halve afterwards if asked for radius).

## WORKED EXAMPLE

## EXAMPLE 6.1

CALCULATOR

The area of this circle is  $144\pi \text{ cm}^2$ . Find its radius  $r$ .



## STEP-BY-STEP SOLUTION:

Set up the equation:  $\pi r^2 = 144\pi$

Divide both sides by  $\pi$ :  $r^2 = 144$

Square root:  $r = 12 \text{ cm}$ .

$$12$$

## Step 7 Contextual Problems

### CORE CONCEPT & METHOD

Read the context carefully to decide if you need **Area** (covering a surface like turf or paint) or **Circumference** (going around the edge like a fence), then multiply by the unit cost.

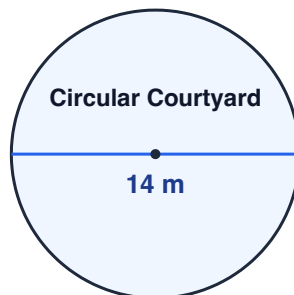
**COMMON MISTAKE:** Fencing/edging goes around the outside (Circumference); grass seed/paint covers the inside (Area).

### WORKED EXAMPLE

#### EXAMPLE 7.1

[CALCULATOR](#)

A circular courtyard has a diameter of 14 m. Paving sealant costs £2 per square metre. How much will it cost to seal the whole courtyard?



#### STEP-BY-STEP SOLUTION:

We need the **Area** to cover the courtyard with sealant.

Radius  $r = 7$  m.

$$\text{Area} = \pi \times 7^2 = 49\pi \approx 153.94 \text{ m}^2$$

$$\text{Cost} = 153.94 \times 2 = \text{£}307.88.$$

**£307.88**

## Step 8 Shaded Rings (Annulus)

### CORE CONCEPT & METHOD

To find the area of a shaded ring (annulus), calculate the **Area of the Outer Circle** and subtract the **Area of the Inner Circle**.

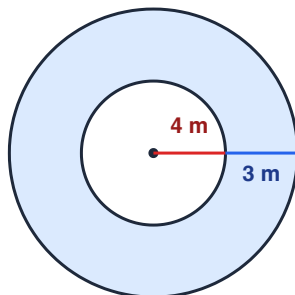
**COMMON MISTAKE:** Square each radius separately ( $\pi R^2 - \pi r^2$ ) — never subtract the radii first!

## WORKED EXAMPLE

## EXAMPLE 8.1

 CALCULATOR

A circular walkway of width 3 m surrounds a circular fountain of radius 4 m. Find the exact area of the walkway in terms of  $\pi$ .



## STEP-BY-STEP SOLUTION:

Inner radius = 4 m.

Outer radius =  $4 + 3 = 7$  m.

Outer area =  $\pi \times 7^2 = 49\pi$

Inner area =  $\pi \times 4^2 = 16\pi$

Shaded walkway =  $49\pi - 16\pi = 33\pi \text{ m}^2$ .

---


$$33\pi$$

## Step 9

## Perimeters of Sectors

## CORE CONCEPT &amp; METHOD

To find the total perimeter of a quarter-circle or three-quarter circle, find the curved arc length first and then **add the two straight radii**.

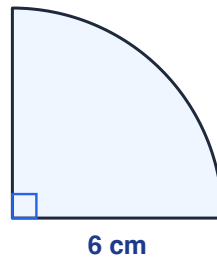
**COMMON MISTAKE:** Don't forget to add the two straight radius sides ( $+2r$ ) to the curved arc length.

## WORKED EXAMPLE

## EXAMPLE 9.1

 CALCULATOR

Find the perimeter of this quarter-circle with radius 6 cm to 1 decimal place.



## STEP-BY-STEP SOLUTION:

$$\text{Curved arc} = \frac{1}{4} \times 2\pi \times 6 = 3\pi \approx 9.42 \text{ cm.}$$

Add the two straight radii ( $6 + 6 = 12$  cm):

$$\text{Perimeter} = 9.42 + 12 = 21.4 \text{ cm.}$$

---

21.4

**Step 10** Comparing Circle Areas

## CORE CONCEPT &amp; METHOD

Keep areas in terms of  $\pi$  when comparing two circular shapes — the  $\pi$  terms cancel out cleanly!

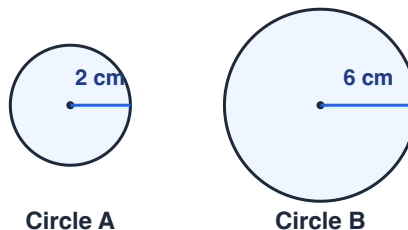
**COMMON MISTAKE:** Keep  $\pi$  in both expressions — the  $\pi$  cancels out cleanly without rounding errors.

## WORKED EXAMPLE

## EXAMPLE 10.1

 NON-CALCULATOR

Circle A has radius 2 cm and Circle B has radius 6 cm. How many times larger is the area of Circle B than Circle A?



## STEP-BY-STEP SOLUTION:

$$\text{Area of Circle A} = \pi \times 2^2 = 4\pi$$

$$\text{Area of Circle B} = \pi \times 6^2 = 36\pi$$

$$36\pi \div 4\pi = 9 \text{ times larger.}$$

---

9

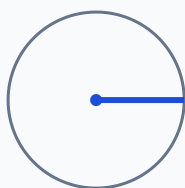
## GEOMETRY AND MEASURES • CIRCLES

Target Grade 3

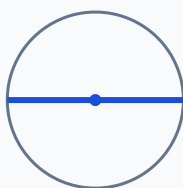
## TOPIC 17: Parts of a Circle

## CORE CONCEPT &amp; METHOD

## Parts of a Circle



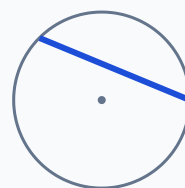
Radius



Diameter



Circumference



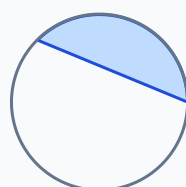
Chord



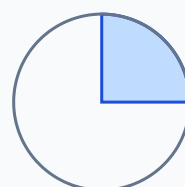
Arc



Tangent



Segment



Sector

**COMMON MISTAKE:** Do not mix up Sector (pizza slice touching the centre) and Segment (crust cut off by a chord).

## WORKED EXAMPLE

## EXAMPLE 1

NON-CALCULATOR

What is the geometric name for a straight line that passes right through the centre of a circle from one side of the circumference to the other?

## STEP-BY-STEP SOLUTION:

A straight line passing through the centre from one edge of the circle to the opposite edge is called the **diameter** (which is exactly twice the length of the radius).

## GEOMETRY AND MEASURES • 2D SHAPES

## TOPIC 18: Area of Compound Shapes

Target Grade 3

## Step 1 Additive Compound Shapes

## CORE CONCEPT &amp; METHOD

To find the area of a complex shape, split it into simpler shapes you know (rectangles, triangles, trapeziums).

Find the area of each part and **add them together**.

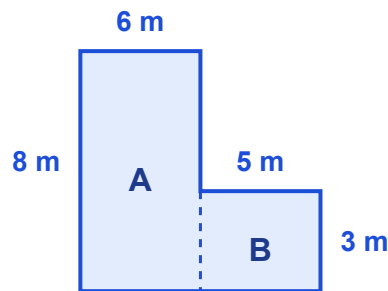
**COMMON MISTAKE:** Work out any missing side lengths using opposite parallel sides before multiplying.

## WORKED EXAMPLE

## EXAMPLE 1.1

NON-CALCULATOR

An L-shape is formed by joining a  $6\text{ m} \times 8\text{ m}$  rectangle to a  $5\text{ m} \times 3\text{ m}$  rectangle. Find the total area.



## STEP-BY-STEP SOLUTION:

Split the shape into two rectangles.

$$\text{Area 1 (Rectangle A)} = 6 \times 8 = 48\text{ m}^2$$

$$\text{Area 2 (Rectangle B)} = 5 \times 3 = 15\text{ m}^2$$

$$\text{Total Area} = 48 + 15 = 63\text{ m}^2$$

## Step 2 Subtractive Compound Shapes

## CORE CONCEPT &amp; METHOD

Sometimes it's easier to find the area of the whole shape (like a large rectangle) and **subtract** the missing pieces (the holes or cutouts).

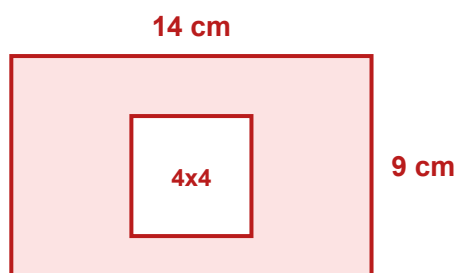
**COMMON MISTAKE:** Check units match on every side before subtracting the cut-out rectangle.

## WORKED EXAMPLE

## EXAMPLE 2.1

NON-CALCULATOR

A rectangle measures 14 cm by 9 cm. A square of side 4 cm is cut out from its center. Find the area of the remaining shape.



## STEP-BY-STEP SOLUTION:

$$\text{Area of large rectangle} = 14 \times 9 = 126 \text{ cm}^2$$

$$\text{Area of square hole} = 4 \times 4 = 16 \text{ cm}^2$$

$$\text{Shaded Area} = 126 - 16 = 110 \text{ cm}^2$$

## Step 3 Problem Solving in Context

## CORE CONCEPT &amp; METHOD

Combine your area skills with real-world problems. You might need to multiply the total area by a cost per square metre to find a final price!

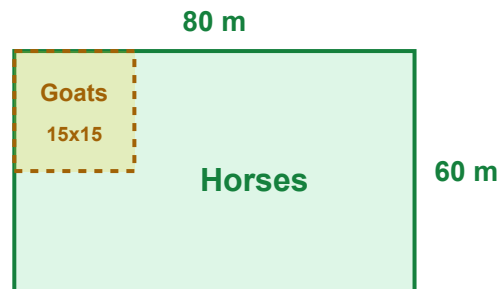
**COMMON MISTAKE:** Remember the area of a triangle is  $\frac{1}{2} \times \text{base} \times \text{height}$  — don't forget to halve it!

## WORKED EXAMPLE

## EXAMPLE 3.1

NON-CALCULATOR

A landowner has a rectangular meadow 80 m by 60 m. She fences off a 15 m by 15 m square enclosure for goats. How much area is left for horses? If wildflower seed costs £3 per square metre, how much will it cost to seed the horse paddock?



## STEP-BY-STEP SOLUTION:

## 1. Area for Horses:

$$\text{Total Area} = 80 \times 60 = 4800 \text{ m}^2$$

$$\text{Goat Area} = 15 \times 15 = 225 \text{ m}^2$$

$$\text{Horse Area} = 4800 - 225 = 4575 \text{ m}^2$$

## 2. Cost:

$$4575 \times £3 = £13,725$$

## Step 4 Multiple Holes

## CORE CONCEPT &amp; METHOD

If there are multiple holes, find the area of ONE hole, multiply it by the number of holes, and then subtract that total from the large area.

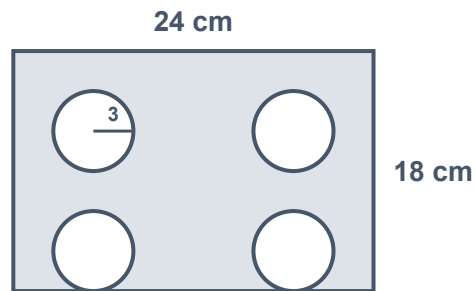
**COMMON MISTAKE:** Multiply the area of one identical hole by the number of holes before subtracting.

## WORKED EXAMPLE

## EXAMPLE 4.1

 CALCULATOR

A rectangular aluminium plate is 24 cm by 18 cm. Four identical circles of radius 3 cm are cut out. Find the remaining area. Give your answer to 2 decimal places.



## STEP-BY-STEP SOLUTION:

$$\text{Area of rectangle} = 24 \times 18 = 432 \text{ cm}^2$$

$$\text{Area of ONE circle} = \pi \times 3^2 = 9\pi$$

$$\text{Area of FOUR circles} = 4 \times 9\pi = 36\pi \approx 113.10 \text{ cm}^2$$

$$\text{Remaining area} = 432 - 36\pi \approx 318.90 \text{ cm}^2$$

## Step 5 Triangles with Holes

## CORE CONCEPT &amp; METHOD

Find the area of the large triangle (Half Base x Height), then subtract the area of the rectangular hole inside it.

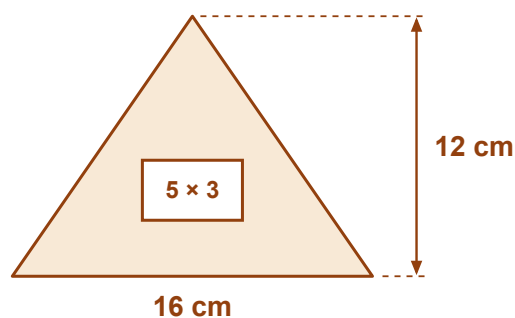
**COMMON MISTAKE:** Halve base  $\times$  height for the outer triangle, not for the rectangular hole inside.

## WORKED EXAMPLE

## EXAMPLE 5.1

 NON-CALCULATOR

A wooden triangle has a base of 16 cm and a height of 12 cm. A rectangular hole measuring 5 cm by 3 cm is cut out. What is the area of the wood left?



## STEP-BY-STEP SOLUTION:

$$\text{Area of the triangle} = \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$$

$$\text{Area of the rectangular hole} = 5 \times 3 = 15 \text{ cm}^2$$

$$\text{Remaining area} = 96 - 15 = 81 \text{ cm}^2.$$

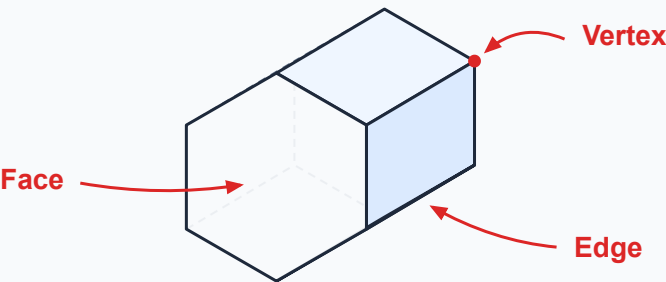
TOPIC 19: 3D Solids, Plans & Elevations

Target Grade 3

Step 1 3D Solids: Faces, Edges, Vertices & Nets

CORE CONCEPT & METHOD

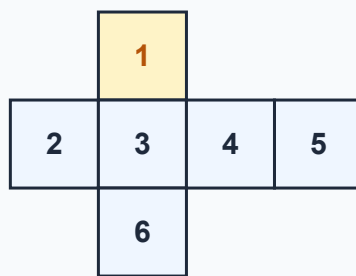
Three-dimensional (3D) solids are described by their **Faces ( $F$ )** (flat or curved surfaces), **Edges ( $E$ )** (lines where two faces meet), and **Vertices ( $V$ )** (corners where edges meet).



Hexagonal Prism:  $F = 8, E = 18, V = 12$

| 3D Solid                         | Faces ( $F$ )        | Edges ( $E$ ) | Vertices ( $V$ ) |
|----------------------------------|----------------------|---------------|------------------|
| Cube / Cuboid                    | 6                    | 12            | 8                |
| Triangular Prism                 | 5                    | 9             | 6                |
| Pentagonal Prism                 | 7                    | 15            | 10               |
| Hexagonal Prism                  | 8                    | 18            | 12               |
| Tetrahedron (Triangular Pyramid) | 4                    | 6             | 4                |
| Square-Based Pyramid             | 5                    | 8             | 5                |
| Cylinder                         | 3 (2 flat, 1 curved) | 2 curved      | 0                |
| Cone                             | 2 (1 flat, 1 curved) | 1 curved      | 1 (apex)         |
| Sphere                           | 1 curved             | 0             | 0                |

- **2D Nets of a Cube:** A net is a flat 2D pattern that folds along its edges to make a 3D solid. On a straight row of squares in a cube net, faces separated by **one square** fold to be **opposite** each other.



**COMMON MISTAKE:** Vertices are corners, Edges are lines, and Faces are flat/curved surfaces — do not mix up the three words.

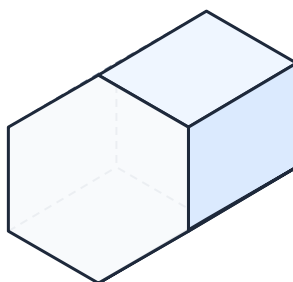
### WORKED EXAMPLES

#### EXAMPLE 1.1

NON-CALCULATOR

The diagram shows a hexagonal prism.

Work out the number of faces, edges, and vertices of a hexagonal prism.



Hexagonal Prism

#### STEP-BY-STEP SOLUTION:

##### Exam Tip: Use a highlighter!

Use a highlighter (or coloured pen) to mark off each face, edge, and corner on the diagram as you count so you do not miss any or count one twice.

**Faces ( $F$ ):** 2 hexagonal end faces (front and back) + 6 rectangular side faces going around the middle = 8.

**Edges ( $E$ ):** Highlight the 6 edges on the front hexagon, 6 edges on the back hexagon, and 6 straight edges joining them:  $6 + 6 + 6 = 18$ .

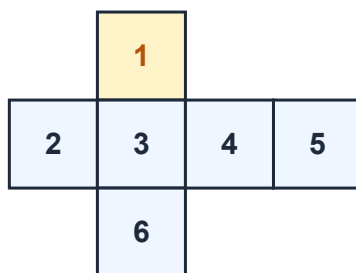
**Vertices ( $V$ ):** Highlight and dot the 6 corners on the front hexagon and 6 corners on the back hexagon:  $6 + 6 = 12$ .

$$F = 8, E = 18, V = 12$$

**EXAMPLE 1.2**

NON-CALCULATOR

The diagram shows the net of a cube with faces numbered 1 to 6.  
When the net is folded to make a cube, which face is opposite face 1?

**STEP-BY-STEP SOLUTION:****Step 1: Choose face 3 as the bottom base of the cube**

Fold the attached squares up around base 3: faces 2 and 4 fold up as the left and right sides (with face 5 folding over to form the top), while face 1 folds up as the back face and face 6 folds up as the front face.

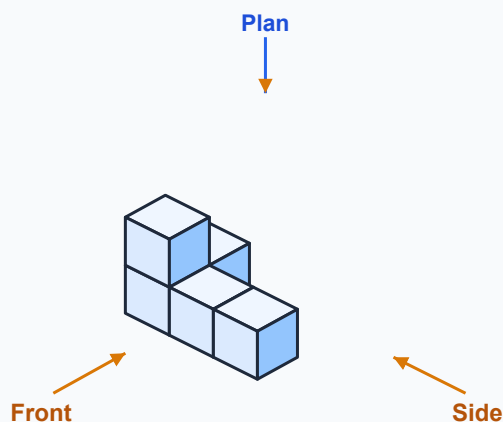
**Step 2: Identify the opposite face**

Face 1 (back) and face 6 (front) are separated by one square (3) in a straight line, so they face each other directly across the cube.

6

**Step 2 Plan View, Front Elevation & Side Elevation****CORE CONCEPT & METHOD**

**Plans and elevations** are accurate 2D orthographic drawings of a 3D object from three perpendicular directions:



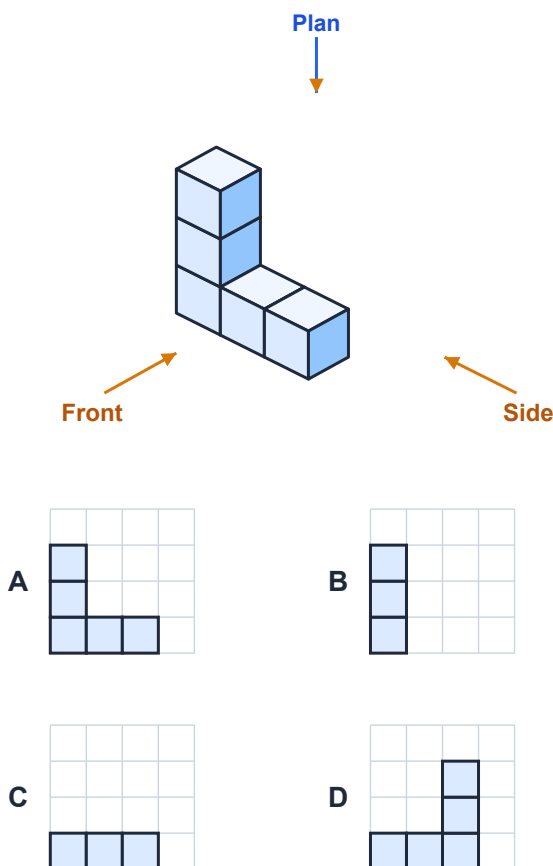
- **Plan View (Top View):** What you see looking straight down from directly above (bird's-eye view).
- **Front Elevation:** What you see standing directly in front and looking horizontally along the *Front* arrow.
- **Side Elevation:** What you see looking horizontally from the side along the *Side* arrow.
- **Solid vs Dashed Lines:** Visible edges where the height or depth steps change are drawn with **solid lines**. Hidden internal steps behind a taller wall are drawn with **dashed lines**.

**COMMON MISTAKE:** Draw flat 2D grid outlines only — never try to sketch 3D depth on a plan or elevation.

## WORKED EXAMPLES

**EXAMPLE 2.1****NON-CALCULATOR**

The diagram shows a 3D solid made from 5 centimetre cubes (each  $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ ). Which grid (**A**, **B**, **C**, or **D**) shows the **Front Elevation** from the direction of the *Front* arrow?

**STEP-BY-STEP SOLUTION:****Step 1: Look straight along the Front arrow**

Reading from left to right across the front face:

- Column 1 (left) is 3 cubes high.
- Column 2 (middle) is 1 cube high.
- Column 3 (right) is 1 cube high.

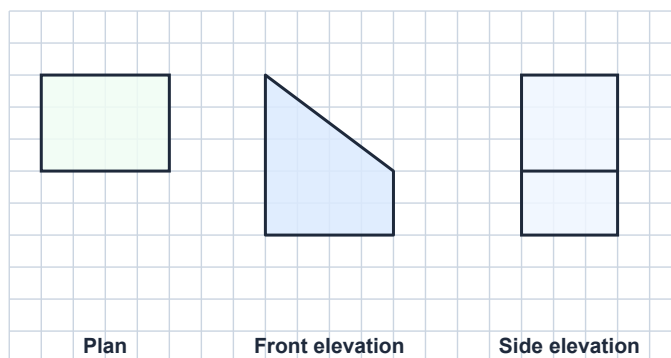
**Step 2: Match to the 2D grid**

Grid **A** has 3 squares in the left column and 1 square in the middle and right columns.

**A**

**EXAMPLE 2.2**
 **NON-CALCULATOR**

The diagram shows the plan, front elevation, and side elevation of a solid prism on a 1 cm square grid. Describe the 3D solid and state the area of its cross-section.

**STEP-BY-STEP SOLUTION:****Step 1: Identify the cross-section from the Front Elevation**

The front elevation is a right-angled trapezium of width 4 cm and parallel vertical sides 5 cm and 2 cm, so the solid is a **trapezoidal prism**.

**Step 2: Calculate the area of the trapezium cross-section**

$$\text{Area} = \frac{5 + 2}{2} \times 4 = 14 \text{ cm}^2$$

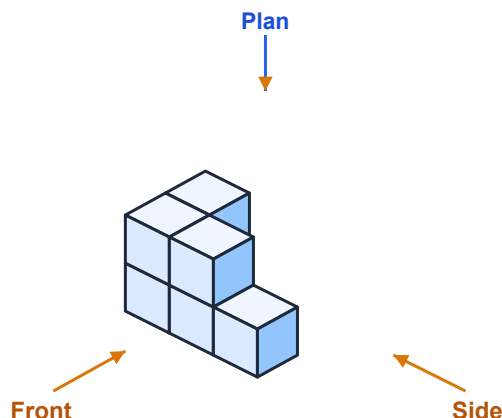
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$$14 \text{ cm}^2$$

**EXAMPLE 2.3****NON-CALCULATOR**

The diagram shows a 3D structure built from centimetre cubes (each  $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ ), with no hidden gaps behind the visible columns.

Work out the total number of centimetre cubes used to build the structure.

**STEP-BY-STEP SOLUTION:****Step 1: Count the cubes column by column**

- Front row ( $y = 0$ ): left column has 2 cubes, middle column has 2 cubes, right column has 1 cube ( $2 + 2 + 1 = 5$ ).
- Back row ( $y = 1$ ): back-left column has 2 cubes.

**Step 2: Find the total**

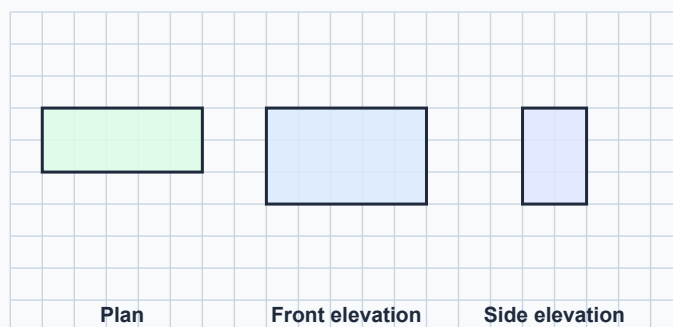
$$5 + 2 = 7 \text{ cubes}$$

---

**7**

**Step 3** Dimensions, Area & Volume from Plans and Elevations**CORE CONCEPT & METHOD**

When the **Plan**, **Front Elevation**, and **Side Elevation** of a 3D solid are drawn side-by-side on a 1 cm square grid, you can read off its exact dimensions to calculate its **Cross-Sectional Area** or **Volume**:



Cuboid: width = 5 cm, height = 3 cm, depth = 2 cm

- **Front Elevation** gives the **width** and **height** (and the cross-sectional area  $A$  of a prism).
- **Side Elevation** and **Plan** give the **depth / length ( $L$ )** of the solid.
- **Scale Drawings (e.g. 2 cm to 1 m)**: Multiply each real-world dimension in metres by the scale factor to find the grid length in cm (notice that for a pyramid or cone, the elevation height uses the **perpendicular height**, not the slant height!).
- **Volume of any Prism**:  $\text{Volume} = \text{Area of Cross-Section} \times \text{Length}$

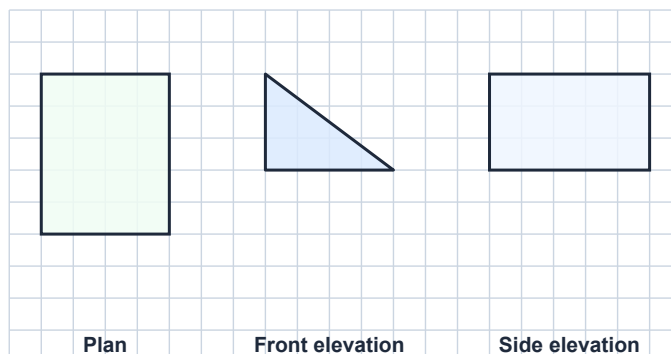
**COMMON MISTAKE:** Use the vertical height from the elevation (not the sloped edge) when finding cross-section area.

## WORKED EXAMPLES

**EXAMPLE 3.1****NON-CALCULATOR**

The plan, front elevation, and side elevation of a right-angled triangular prism are drawn side-by-side on a 1 cm square grid.

Work out the volume of the prism in  $\text{cm}^3$ .

**STEP-BY-STEP SOLUTION:**

**Step 1: Find the area of the triangular cross-section (Front elevation)**

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

**Step 2: Multiply by the depth from the Plan / Side elevation (5 cm)**

$$\text{Volume} = 6 \times 5 = 30 \text{ cm}^3$$

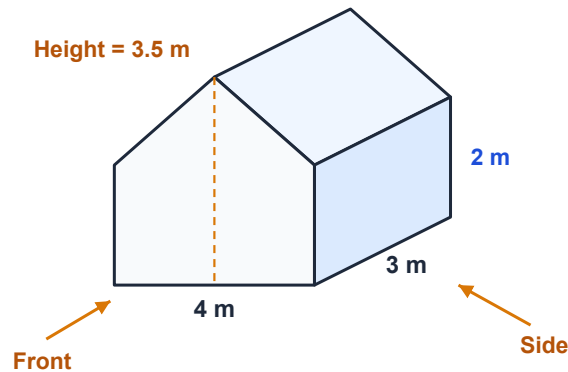
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**30**

**EXAMPLE 3.2****NON-CALCULATOR**

The diagram shows a house-shaped prism with width 4 m, vertical wall height 2 m, total ridge height 3.5 m, and length 3 m.

Work out the area of the pentagonal **Front Elevation** in  $\text{m}^2$ .

**STEP-BY-STEP SOLUTION:**

**Step 1: Split the pentagonal Front Elevation into a rectangle and a triangle**

- Bottom rectangle area =  $4 \times 2 = 8 \text{ m}^2$
- Top roof triangle has base 4 m and height  $3.5 - 2 = 1.5 \text{ m}$ :

$$\text{Triangle Area} = \frac{1}{2} \times 4 \times 1.5 = 3 \text{ m}^2$$

**Step 2: Add the areas together**

$$\text{Total Front Elevation Area} = 8 + 3 = 11 \text{ m}^2$$

---

**11**

## PROBABILITY • FREQUENCY TREES &amp; TABLES

Target Grade 3

## TOPIC 20: Frequency Trees

## Step 1 Completing Frequency Trees

## CORE CONCEPT &amp; METHOD

In a Frequency Tree, the number in a starting bubble ALWAYS splits into the two bubbles connected to it.

If 150 visitors split into 90 Morning visitors and some Afternoon visitors, you use subtraction to find the missing bubble:  $150 - 90 = 60$  Afternoon visitors.

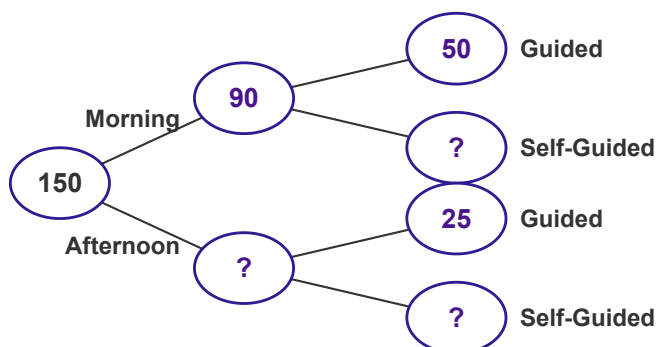
**COMMON MISTAKE:** Each pair of branches leaving a bubble must add up to the number inside that bubble.

## WORKED EXAMPLE

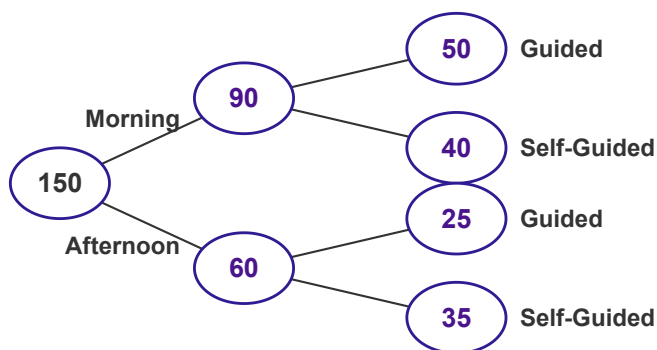
## EXAMPLE 1.1

NON-CALCULATOR

150 museum visitors were asked whether they arrived in the morning or the afternoon, and whether they took a guided tour or a self-guided tour. 90 visitors arrived in the morning, of whom 50 took a guided tour. 25 of the afternoon visitors took a guided tour. Complete the frequency tree and find how many visitors took a self-guided tour.



## STEP-BY-STEP SOLUTION:



**Step 1:** 90 arrived in the morning, so  $150 - 90 = 60$  arrived in the afternoon.

**Step 2:** Of the 90 morning visitors: 50 took a guided tour, so  $90 - 50 = 40$  took a self-guided tour.

**Step 3:** Of the 60 afternoon visitors: 25 took a guided tour, so  $60 - 25 = 35$  took a self-guided tour.

**Step 4:** Total taking a self-guided tour =  $40 + 35 = 75$ .

## Step 2 Probabilities from Trees

### CORE CONCEPT & METHOD

Read the question carefully to find the denominator!

If it says 'One of the **Morning visitors** is chosen at random...', your denominator is the total number of Morning visitors, NOT the total number of visitors!

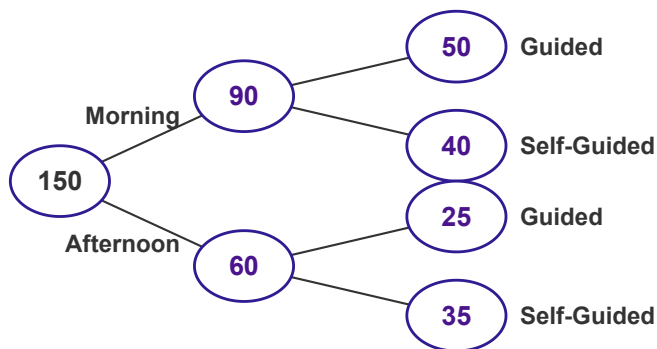
**COMMON MISTAKE:** "Chosen from the Women" means the bottom of your fraction is the total Women, not everyone surveyed.

### WORKED EXAMPLE

#### EXAMPLE 2.1

NON-CALCULATOR

Using the completed museum visitors frequency tree from the previous example:



(a) A visitor is chosen at random. Find  $P(\text{afternoon and self-guided})$ .

(b) A **morning visitor** is chosen at random. Find the probability they took a guided tour.

#### STEP-BY-STEP SOLUTION:

(a) Afternoon visitors taking a self-guided tour = 35. Total visitors = 150.

$$P(\text{afternoon, self-guided}) = \frac{35}{150} = \frac{7}{30}$$

(b) We are told the person is a morning visitor, so the denominator is the total number of morning visitors = 90.  
Morning visitors taking a guided tour = 50.

$$P(\text{guided} \mid \text{morning}) = \frac{50}{90} = \frac{5}{9}$$

TOPIC 21: Two-Way Tables

Target Grade 3

Step 1 Completing Two-Way Tables

CORE CONCEPT & METHOD

In a two-way table, the rows and columns must always add up to their respective **Totals**.

Look for a row or column that only has **ONE** missing number, and use subtraction to find it!

**COMMON MISTAKE:** Start with a row or column that has only ONE gap — never guess when two gaps are empty.

WORKED EXAMPLE

EXAMPLE 1.1

NON-CALCULATOR

90 club members were asked if they prefer Swimming or Cycling.

|         | Swimming | Cycling | Total |
|---------|----------|---------|-------|
| Adults  | 24       | 28      |       |
| Juniors |          | 19      | 38    |
| Total   |          |         | 90    |

Complete the two-way table.

STEP-BY-STEP SOLUTION:

**Step 1:** Find the Total Adults.

Swimming + Cycling =  $24 + 28 = 52$ .

**Step 2:** Find the Juniors who prefer Swimming.

Total Juniors — Cycling =  $38 - 19 = 19$ .

**Step 3:** Find the Total Swimming.

Adults Swimming + Juniors Swimming =  $24 + 19 = 43$ .

**Step 4:** Find the Total Cycling.

Adults Cycling + Juniors Cycling =  $28 + 19 = 47$ .

|         | Swimming | Cycling | Total |
|---------|----------|---------|-------|
| Adults  | 24       | 28      | 52    |
| Juniors | 19       | 19      | 38    |
| Total   | 43       | 47      | 90    |

## Step 2 Probabilities from Two-Way Tables

### CORE CONCEPT & METHOD

First, fill out the missing numbers in the table. Then, read the probability question carefully to find your denominator (usually the Grand Total) and your numerator (the specific cell or row total).

**COMMON MISTAKE:** Check whether the person is picked from everyone (Grand Total) or from just one row/column.

### WORKED EXAMPLE

#### EXAMPLE 2.1

 **NON-CALCULATOR**

Here is a completed two-way table for 90 club members:

|         | Swimming | Cycling | Total |
|---------|----------|---------|-------|
| Adults  | 24       | 28      | 52    |
| Juniors | 19       | 19      | 38    |
| Total   | 43       | 47      | 90    |

(a) A member is chosen at random. Find the probability they are an Adult who prefers Swimming.

(b) A **Junior** is chosen at random. Find the probability they prefer Cycling.

#### STEP-BY-STEP SOLUTION:

(a) The denominator is all members (90). The number of Adults who prefer Swimming is 24.

$$P(\text{Adult and Swimming}) = \frac{24}{90} = \frac{4}{15}$$

(b) We are told a **Junior** is chosen. So we ONLY look at the Juniors row! The denominator is Total Juniors (38). The number of Juniors who prefer Cycling is 19.

$$P(\text{Cycling} \mid \text{Junior}) = \frac{19}{38} = \frac{1}{2}$$

## Step 3 Tables from Text

### CORE CONCEPT & METHOD

Draw your own table first! Set up the categories as rows and columns, add Total rows/columns, and place the numbers from the text into the correct cells before solving.

**COMMON MISTAKE:** Tick off each number in the question as you write it into your table, including the Grand Total.

## WORKED EXAMPLE

## EXAMPLE 3.1

NON-CALCULATOR

60 guests at a conference chose Tea or Coffee. There are 34 Women. 16 Men chose Coffee. 21 Women chose Tea. Every guest chose either Tea or Coffee.

Draw a two-way table and find the number of Women who chose Coffee.

## STEP-BY-STEP SOLUTION:

**Step 1:** Set up a table with Rows (Men, Women, Total) and Columns (Tea, Coffee, Total), and fill in the given numbers (60 total, 34 Women, 16 Men with Coffee, 21 Women with Tea).

**Step 2:** Work out the missing values by subtracting:

- Women with Coffee =  $34 - 21 = 13$
- Total Men =  $60 - 34 = 26$ , so Men with Tea =  $26 - 16 = 10$
- Total Tea =  $10 + 21 = 31$  and Total Coffee =  $16 + 13 = 29$

|       | Tea | Coffee | Total |
|-------|-----|--------|-------|
| Men   | 10  | 16     | 26    |
| Women | 21  | 13     | 34    |
| Total | 31  | 29     | 60    |

**Answer:** Number of Women who chose Coffee = 13.

## Step 4 Two-Way Percentages

## CORE CONCEPT &amp; METHOD

Convert all the percentages into actual amounts first by calculating them out of the grand total. Then, fill in your table normally.

**COMMON MISTAKE:** Don't forget to divide by 100 when finding each percentage of the total number of students.

## WORKED EXAMPLE

## EXAMPLE 4.1

NON-CALCULATOR

In a college of 250 students, 60% study Science and the rest study Humanities. 30% of the Science students are in Year 12. 45% of the Humanities students are in Year 12. Every student is in either Year 12 or Year 13.

Find the total number of students in Year 13.

## STEP-BY-STEP SOLUTION:

**Step 1:** Find the amounts using percentages.

- Total Science = 60% of 250 = 150, so Total Humanities =  $250 - 150 = 100$ .
- Science in Year 12 = 30% of 150 = 45.
- Humanities in Year 12 = 45% of 100 = 45.

**Step 2:** Complete the rest of the two-way table by subtracting and adding:

- Science in Year 13 =  $150 - 45 = 105$ .
- Humanities in Year 13 =  $100 - 45 = 55$ .

|            | Year 12 | Year 13 | Total |
|------------|---------|---------|-------|
| Humanities | 45      | 55      | 100   |
| Science    | 45      | 105     | 150   |
| Total      | 90      | 160     | 250   |

**Answer:** Total Year 13 students =  $55 + 105 = 160$ .