

GCSE MATHEMATICS (9-1) — TUTORIALS & WORKED EXAMPLES

Grade 4 Mathematics — Tutorials & Examples

(Foundation & Higher Crossover)

Foundation & Higher Crossover Tutorials, Common Mistake Warnings & Step-by-Step Worked Examples

- Core Crossover Concept Notes & Precision Geometric/Graph Diagrams
- Exam-Focused "Common Mistake" Warnings in every topic
- Model Worked Examples tagged Calculator or Non-Calculator
- Streamlined Crossover Revision Guide (Tutorials & Worked Examples Only)

Student Name: _____

Class / Teacher: _____

Target Grade: _____



SCAN FOR ONLINE PRACTICE & WORKED SOLUTIONS

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Interactive Step-by-Step GCSE Mathematics Mastery

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Percentage Multipliers

CORE CONCEPT & METHOD

To increase or decrease an amount by a percentage quickly, we use a **Decimal Multiplier**.

The original amount is always 100%.

- **For an Increase:** Add the percentage to 100%, then turn it into a decimal. (e.g. A 15% increase is 115%, so the multiplier is 1.15).
- **For a Decrease:** Subtract the percentage from 100%, then turn it into a decimal. (e.g. A 20% decrease is 80%, so the multiplier is 0.80).

Common Mistake: A 5% increase is $\times 1.05$ (NOT $\times 1.5$!), and a 15% decrease is $100\% - 15\% = 85\%$, so multiply by 0.85 (never 0.15).

MODEL WORKED EXAMPLES

EXAMPLE 1

 **Non-Calculator**

Increase £60 by 15%

STEP-BY-STEP SOLUTION:

1. Start with 100% and add 15%:

$$100\% + 15\% = 115\%$$

2. Turn into a decimal multiplier: $\frac{115}{100} = 1.15$

3. Multiply: $60 \times 1.15 = 69$

£69

EXAMPLE 2

 **Non-Calculator**

Decrease £90 by 20%

STEP-BY-STEP SOLUTION:

1. Start with 100% and subtract 20%:

$$100\% - 20\% = 80\%$$

2. Turn into a decimal multiplier: $\frac{80}{100} = 0.80$

3. Multiply: $90 \times 0.80 = 72$

£72

Compound Interest

1 Compound Interest

📖 CORE CONCEPT & METHOD

There are **two main ways** to calculate compound interest and depreciation.

Method 1: The Official Formula

Edexcel GCSE Exam Aid: For the 2025-2027 GCSE exams, you are given a formula sheet which includes the Compound Interest equation below. You do not need to memorize it!

$$\text{Total accrued} = P \left(1 + \frac{r}{100} \right)^n$$

Calculator Reminder: In algebra, $P \left(1 + \frac{r}{100} \right)^n$ has a hidden multiplication sign between P and the bracket. When typing this into your calculator, **you must include the times (×) symbol before the bracket: Total accrued = P × (1 + $\frac{r}{100}$)ⁿ**

- P is the Principal (starting amount), r is the interest rate, and n is the number of years.

For **depreciation** (decay), simply change the plus to a minus (and remember the × symbol before the bracket): $P \times \left(1 - \frac{r}{100} \right)^n$

Method 2: Using Percentage Multipliers

You can also use a percentage multiplier, which is often faster:

$$\text{Final Amount} = \text{Original} \times (\text{Multiplier})^{\text{years}}$$

- For **5% interest**, the multiplier is 1.05.
- For **15% depreciation**, the multiplier is 0.85 (since $100\% - 15\% = 85\%$).

Common Mistake: Never multiply the percentage by the number of years — raise the decimal multiplier to the power n ($\text{Original} \times \text{Multiplier}^n$), and subtract the original principal if asked for the total INTEREST earned.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 Calculator**Method 1 (Formula):** £600 is invested at 3% compound interest for 4 years. How much is it worth?

STEP-BY-STEP SOLUTION:

1. Identify the values: $P = 600, r = 3, n = 4$
2. Substitute into the Edexcel formula (remembering to include the $\times$ symbol before the bracket):

$$600 \times \left(1 + \frac{3}{100}\right)^4$$

3. Calculate the result: 675.31

£675.31

EXAMPLE 1.2

 Calculator**Method 1 (Formula):** A car is bought for £16,000. It **depreciates** by 15% each year. How much is it worth after 3 years?

STEP-BY-STEP SOLUTION:

1. Identify the values: $P = 16000, r = 15, n = 3$
2. For depreciation, use a minus sign (and include the $\times$ symbol before the bracket):

$$16000 \times \left(1 - \frac{15}{100}\right)^3$$

3. Calculate the result: 9826.00

£9,826.00

EXAMPLE 1.3

 Calculator**Method 2 (Multiplier):** £900 is invested at 4% compound interest for 5 years. Calculate the final amount using a percentage multiplier.

STEP-BY-STEP SOLUTION:

1. Find the multiplier for a 4% increase: $100\% + 4\% = 104\%$, so the multiplier is 1.04
2. Set up the calculation: **Original** $\times$ **Multiplier**^{years}
3. Substitute the values: 900×1.04^5
4. Calculate the result: 1094.99

£1,094.99

EXAMPLE 1.4

Method 2 (Multiplier): A computer costs £1,250. Its value **depreciates** by 20% each year. What is its value after 4 years using a percentage multiplier?

STEP-BY-STEP SOLUTION:

1. Find the multiplier for a 20% decrease: $100\% - 20\% = 80\%$, so the multiplier is 0.80
2. Set up the calculation: $\text{Original} \times \text{Multiplier}^{\text{years}}$
3. Substitute the values: 1250×0.80^4
4. Calculate the result: 512.00

£512.00

2 Finding Total Interest**□ CORE CONCEPT & METHOD**

The standard formula gives you the **total amount** at the end. If the question asks for the **interest earned**, you must subtract the original starting amount from your final answer!

MODEL WORKED EXAMPLES**EXAMPLE 2.1**

Maya invests £6,000 in a bank account that pays 3.5% compound interest per year. Calculate the **total interest** she has earned after 3 years. Give your answer to the nearest penny.

STEP-BY-STEP SOLUTION:

1. Find the multiplier: $100\% + 3.5\% = 103.5\% \rightarrow 1.035$.
2. Find the total amount in the bank after 3 years: $£6,000 \times 1.035^3 = £6,652.30725$.
3. To find the **interest earned**, subtract her original money: $£6,652.31 - £6,000 = £652.31$.

3 Variable Rates**□ CORE CONCEPT & METHOD**

If the rate changes, simply multiply the starting amount by the sequence of different multipliers.

e.g. $\text{Original} \times (\text{Yr1 Multiplier}) \times (\text{Yr2 Multiplier})$

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Calculator

An apartment was bought for £180,000.
In the first year, its value increased by 5%.
In the second year, its value decreased by 2%.
Calculate the value of the apartment after the two years.

STEP-BY-STEP SOLUTION:

1. First year multiplier (5% increase): 1.05
2. Second year multiplier (2% decrease): 0.98
3. Multiply them all in a chain: $£180,000 \times 1.05 \times 0.98$
4. Final value: £185,220.

4 Comparing Investments

 CORE CONCEPT & METHOD

Calculate both options entirely separately, then find the difference between their final totals.

Remember: **Simple Interest** is calculated only on the original amount. The amount of interest is the same every year.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

 Calculator

Zainab has £8,000 to invest for 3 years. She has two options:
Option A: 3.5% compound interest per year.
Option B: 4% simple interest per year.
Which option gives her more money at the end of the 3 years, and by how much?

STEP-BY-STEP SOLUTION:

1. **Option A (Compound):** $£8,000 \times 1.035^3 = £8,869.74$.
2. **Option B (Simple):** Interest per year is 4% of £8,000 = £320. Over 3 years: $3 \times 320 = £960$. Total = £8,960.
3. Option B is better by: $£8,960 - £8,869.74 = £90.26$.

5 Finding 'n' (Years)

 CORE CONCEPT & METHOD

If you know the start and end amount, you need to find the power n . The easiest way is to use **trial and error** on your calculator: keep increasing the power until your answer matches the target!

MODEL WORKED EXAMPLES

EXAMPLE 5.1

 Calculator

Rohan invests £3,000 in a bank account that pays 5% compound interest per year.
How many full years will it take for his investment to exceed £3,900?

STEP-BY-STEP SOLUTION:

1. The multiplier is 1.05. We want: $3000 \times 1.05^n > 3900$.
2. Divide by 3000: $1.05^n > 1.30$.
3. Use **Trial & Error** on your calculator:
Try $n = 4$: $1.05^4 = 1.215\dots$
Try $n = 5$: $1.05^5 = 1.276\dots$
Try $n = 6$: $1.05^6 = 1.340\dots$ (which is > 1.30)
4. Therefore, it takes 6 years.

Prime Factor Trees

CORE CONCEPT & METHOD

A **Prime Number** has exactly two factors: 1 and itself (e.g. 2, 3, 5, 7, 11, 13...). Note: 1 is NOT a prime number!

Every number can be broken down into a unique 'product of prime factors' using a factor tree. When you hit a prime number, draw a circle around it because it can't be broken down any further!

Common Mistake: 1 is NOT a prime number — stop branching at prime numbers (2, 3, 5, 7, 11, ...) and write multiplication signs ($\times$) between your prime factors, never addition (+).

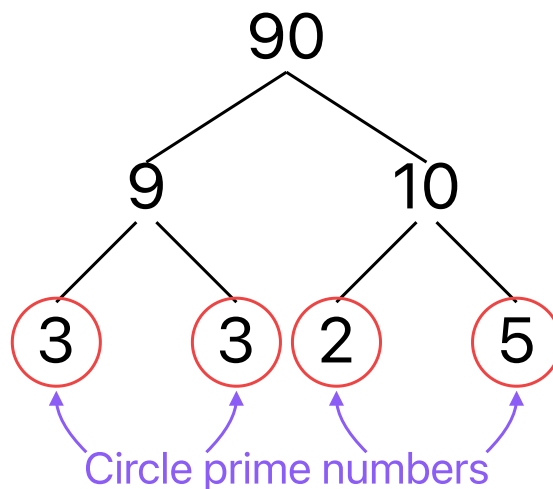
MODEL WORKED EXAMPLES

EXAMPLE 1

 Non-Calculator

Express 90 as a product of its prime factors in index form.

STEP-BY-STEP SOLUTION:



1. The prime factors are 2, 3, 3, 5.
2. Write as $2 \times 3 \times 3 \times 5$
3. In index form: $2 \times 3^2 \times 5$

$$2 \times 3 \times 3 \times 5$$

Index form: $2 \times 3^2 \times 5$

HCF & LCM

1 HCF & LCM

CORE CONCEPT & METHOD

The **Highest Common Factor (HCF)** is the largest number that divides exactly into both numbers.

The **Lowest Common Multiple (LCM)** is the smallest number that is in the times tables of both numbers.

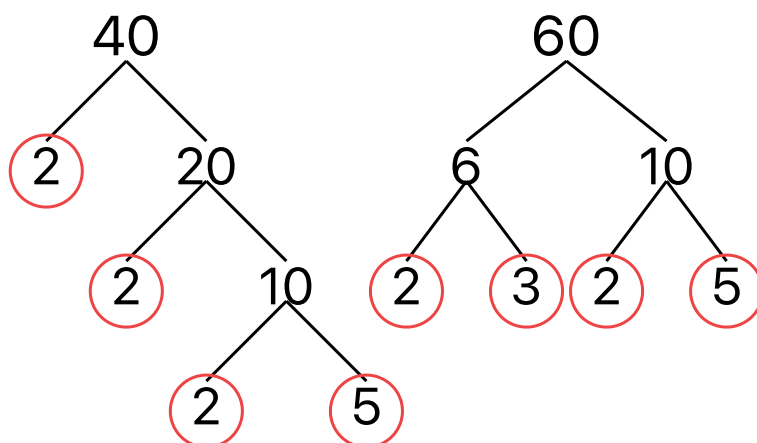
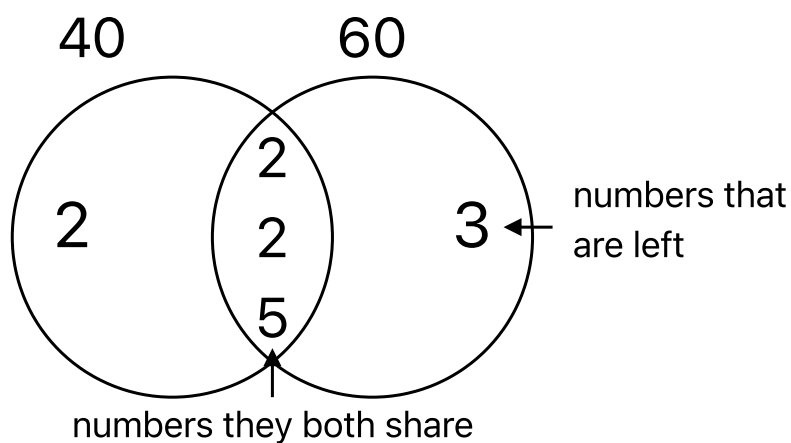
Tip: For word problems, use HCF when splitting items into equal smaller groups. Use LCM when finding when repeating events will happen at the same time again (like buses)!

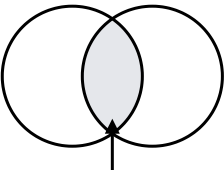
Common Mistake: Do not muddle the names: HCF (Factor) is smaller than or equal to the numbers (overlap only in the Venn diagram), whereas LCM (Multiple) is larger than or equal to the numbers (multiply ALL regions in the Venn diagram).

MODEL WORKED EXAMPLES

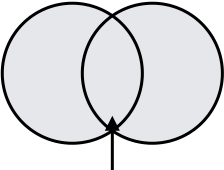
EXAMPLE 1.1 **Non-Calculator**

Find the HCF and LCM of 40 and 60

STEP-BY-STEP SOLUTION:

HCF:  $2 \times 2 \times 5 = 20$

multiply these numbers

LCM:  $2 \times 2 \times 2 \times 3 \times 5 = 120$

multiply all numbers

EXAMPLE 1.2 **Non-Calculator****Alternative Method:** Finding HCF & LCM by listing (useful if you haven't learned Venn diagrams!)**STEP-BY-STEP SOLUTION:****HCF of 16 and 24**

Find factors of both numbers:

16: 1, 2, 4, 8, 16

24: 1, 2, 3, 4, 6, 8, 12, 24

HCF = 8 [Highest factor they both share]

LCM of 5 and 7

Do multiples (times table) of both numbers:

5: 5, 10, 15, 20, 25, 30, 35, 40

7: 7, 14, 21, 28, 35, 42

LCM = 35 [first time they both share a multiple]

2 HCF & LCM with Prime Factors **CORE CONCEPT & METHOD**When numbers are given in **prime factor form** (with powers):**Finding HCF**Take the **lowest power** of each prime number that appears in **BOTH** lists.**Finding LCM**Take the **highest power** of **EVERY** prime number that appears in **EITHER** list.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

☒ Non-Calculator

Given that $A = 2^4 \times 3^2 \times 5$ and $B = 2^3 \times 3^4 \times 7$

Find the Highest Common Factor (HCF) of A and B .

STEP-BY-STEP SOLUTION:

$A =$	2^4	$\times$	3^2	$\times$	5	
$B =$	2^3	$\times$	3^4	$\times$	7	
Both have =	$2^3 \times 3^2$		= 72		(HCF)	

1. Line up the prime factors vertically and find what **both** numbers share:

- For 2: both share 2^3 (lowest power)
- For 3: both share 3^2 (lowest power)
- 5 and 7 are not shared by both.

2. Multiply the shared powers together:

$$\text{HCF} = 2^3 \times 3^2 = 8 \times 9 = 72$$

72

EXAMPLE 2.2 **Non-Calculator**

Given that $A = 2^2 \times 3$ and $B = 2^3 \times 7$

Find the Lowest Common Multiple (LCM) of A and B .

STEP-BY-STEP SOLUTION:

A =	2^2	×	3	×	7	
B =	2^3		3	×	7	

Take highest = $2^3 \times 3 \times 7 = 168$ (LCM)

1. Line up the prime factors and take the **highest power** of every prime factor in either list:

- For 2: highest power is 2^3
- For 3: highest power is 3
- For 7: highest power is 7

2. Multiply them together:

$$\text{LCM} = 2^3 \times 3 \times 7 = 8 \times 3 \times 7 = 168$$

168

3 HCF & LCM of Three Numbers **CORE CONCEPT & METHOD**

Finding HCF and LCM for **three numbers** works the exact same way as for two numbers:

HCF of 3 Numbers

Must divide into **ALL THREE** numbers.
Find the factors of each, and choose the largest shared one.

LCM of 3 Numbers

Must be in the times table of **ALL THREE** numbers. Using prime factors, take the highest power of every prime.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Non-Calculator

Find the Highest Common Factor (HCF) of 16, 24 and 40.

STEP-BY-STEP SOLUTION:

List the factors of each number:

16: 1, 2, 4, **8**, 16

24: 1, 2, 3, 4, 6, **8**, 12, 24

40: 1, 2, 4, 5, **8**, 10, 20, 40

HCF = 8 [Highest factor all three share]

1. Factors that appear in **all three** lists: 1, 2, 4, 8.
2. The **highest** common factor is 8.

8

EXAMPLE 3.2

 Non-Calculator

Find the Lowest Common Multiple (LCM) of 4, 6 and 15.

STEP-BY-STEP SOLUTION:

1. Write each number as a product of prime factors:

• $4 = 2^2$

• $6 = 2 \times 3$

• $15 = 3 \times 5$

2. Take the **highest power** of every prime that appears (2^2 , 3, and 5):

$$\text{LCM} = 2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60$$

60

4 Reverse HCF & LCM

□ CORE CONCEPT & METHOD

To find two mystery numbers from their **HCF** and **LCM**:

The 3-Step Method:

- Both numbers are multiples of the HCF: $\text{First} = \text{HCF} \times a$ and $\text{Second} = \text{HCF} \times b$.
- Find the product $a \times b$ by dividing LCM by HCF:

$$a \times b = \frac{\text{LCM}}{\text{HCF}}$$

- Find pairs of factors for (a, b) that share no common factors and satisfy any condition (e.g. "greater than 10").

MODEL WORKED EXAMPLES

EXAMPLE 4.1

✎ Non-Calculator

Tariq chooses two whole numbers, both greater than 12.

- The highest common factor (HCF) of his numbers is 8.
- The lowest common multiple (LCM) of his numbers is 120.

Find the two numbers.

STEP-BY-STEP SOLUTION:

- The two numbers are $8 \times a$ and $8 \times b$.

- Find $a \times b$ using $\frac{\text{LCM}}{\text{HCF}}$:

$$a \times b = 120 \div 8 = 15$$

- Find factor pairs of 15 that share no common factors:

- $1 \times 15 \rightarrow 8 \times 1 = 8$ and $8 \times 15 = 120$ (*rejected: 8 is not greater than 12*)
- $3 \times 5 \rightarrow 8 \times 3 = 24$ and $8 \times 5 = 40$ (*both are > 12!*)

- The two numbers are 24 and 40.

24, 40

Laws of Indices

📖 CORE CONCEPT & METHOD

Indices (Powers) tell you how many times to multiply a base number by itself.

There are 4 golden rules:

- **Multiply:** Add the powers ($x^a \times x^b = x^{a+b}$)
- **Divide:** Subtract the powers ($x^a \div x^b = x^{a-b}$ or $\frac{x^a}{x^b} = x^{a-b}$)
- **Brackets:** Multiply the powers ($(x^a)^b = x^{ab}$)
- **Power of Zero:** Anything to the power of 0 is exactly 1 ($x^0 = 1$)

Common Mistake: Only add/subtract powers when multiplying/dividing the SAME base letter, and remember any non-zero number to the power of 0 is 1 (e.g. $63^0 = 1$, never 0).

MODEL WORKED EXAMPLES

EXAMPLE 1

🚫 Non-Calculator

Simplify $y^5 \times y^4$

STEP-BY-STEP SOLUTION:

When multiplying, we add the powers:

$$5 + 4 = 9$$

$$y^9$$

EXAMPLE 2

🚫 Non-Calculator

Simplify $\frac{a^9}{a^3}$

STEP-BY-STEP SOLUTION:

A fraction means divide. When dividing, we subtract the powers:

$$9 - 3 = 6$$

$$a^6$$

EXAMPLE 3

🚫 Non-Calculator

Simplify $(p^4)^6$

STEP-BY-STEP SOLUTION:

When raising a power to a power, we multiply them:

$$4 \times 6 = 24$$

$$p^{24}$$

Indices with Numbers & Letters

📖 CORE CONCEPT & METHOD

When working with terms that have both regular numbers (coefficients) and letters, just remember: **handle the numbers normally, and apply the index laws to the letters!**

Common Mistake: Treat front coefficients as normal numbers and exponents with index laws: in $(2y^3)^3$, cube the number $2^3 = 8$ AND multiply the power $3 \times 3 = 9$ to get $8y^9$ (never $6y^9$).

MODEL WORKED EXAMPLES

EXAMPLE 1

🚫 Non-Calculator

Simplify $4a^5 \times 6a^3$

STEP-BY-STEP SOLUTION:

1. Multiply the numbers normally: $4 \times 6 = 24$
2. Add the powers (Index Law): $5 + 3 = 8$

$$24a^8$$

EXAMPLE 2

🚫 Non-Calculator

Simplify $\frac{28y^9}{7y^2}$

STEP-BY-STEP SOLUTION:

1. Divide the numbers normally: $28 \div 7 = 4$
2. Subtract the powers (Index Law): $9 - 2 = 7$

$$4y^7$$

EXAMPLE 3

🚫 Non-Calculator

Simplify $(3x^4)^3$

STEP-BY-STEP SOLUTION:

When a term with a number is in brackets, the power applies to BOTH the number and the letter.

1. Number: $3^3 = 27$
2. Letter: $(x^4)^3 = x^{12}$

$$27x^{12}$$

Brackets (Single)

1 Expanding Single Brackets

📖 CORE CONCEPT & METHOD

To expand a bracket, you must multiply the term on the **outside** by **everything on the inside**!

Be very careful with negative signs!

Common Mistake: A minus sign in front of a bracket flips the sign of EVERY term inside when expanding: $-2(3x - 4) = -6x + 8$, never $-6x - 8$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

🚫 Non-Calculator

Expand $4(x + 3)$

STEP-BY-STEP SOLUTION:

1. Multiply $4 \times x = 4x$
 2. Multiply $4 \times 3 = +12$
- Put them together:

$$4x + 12$$

EXAMPLE 1.2

🚫 Non-Calculator

Expand $5(2y - 3)$

STEP-BY-STEP SOLUTION:

1. Multiply $5 \times 2y = 10y$
 2. Multiply $5 \times -3 = -15$
- Put them together:

$$10y - 15$$

EXAMPLE 1.3

🚫 Non-Calculator

Expand $-4(x - 3)$

STEP-BY-STEP SOLUTION:

Tricky! When a negative is outside, BOTH signs inside flip!

1. Multiply $-4 \times x = -4x$
 2. Multiply $-4 \times -3 = +12$ (negative $\times$ negative = positive!)
- Put them together:

$$-4x + 12$$

EXAMPLE 1.4 **Non-Calculator**Expand $-6(2x + 5)$ **STEP-BY-STEP SOLUTION:****The negative outside flips both signs!**

1. Multiply $-6 \times 2x = -12x$

2. Multiply $-6 \times +5 = -30$ (negative $\times$ positive = negative!)

Put them together:

$$-12x - 30$$

2 Expanding & Collecting **CORE CONCEPT & METHOD**

Sometimes you'll have multiple brackets. Just expand them one at a time, write it all out in a long line, and then **collect the like terms** like we learned earlier!

MODEL WORKED EXAMPLES**EXAMPLE 2.1** **Non-Calculator**Expand and simplify $3(x + 4) + 2(x + 5)$ **STEP-BY-STEP SOLUTION:**

1. Expand the first bracket: $3(x + 4) = 3x + 12$

2. Expand the second bracket: $+2(x + 5) = +2x + 10$

Write it out: $3x + 12 + 2x + 10$

Collect terms:

$3x + 2x = 5x$

$12 + 10 = 22$

$$5x + 22$$

EXAMPLE 2.2 **Non-Calculator**Expand and simplify $6(x + 3) - 2(x - 4)$ **STEP-BY-STEP SOLUTION:****The minus in front of the second bracket flips BOTH signs inside!**

1. Expand the first bracket: $6(x + 3) = 6x + 18$

2. Expand the second bracket: $-2(x - 4) = -2x + 8$

$(-2 \times -4 = +8!)$

Write it out: $6x + 18 - 2x + 8$

Collect terms:

$6x - 2x = 4x$

$18 + 8 = 26$

$$4x + 26$$

EXAMPLE 2.3 **Non-Calculator**Expand and simplify $4(2x - 3) - 2(4x + 5)$ **STEP-BY-STEP SOLUTION:****Watch the negative bracket carefully!**

1. Expand the first bracket: $4(2x - 3) = 8x - 12$
2. Expand the second bracket: $-2(4x + 5) = -8x - 10$
 $(-2 \times +5 = -10!)$

Write it out: $8x - 12 - 8x - 10$

Collect terms:

$$8x - 8x = 0$$

$$-12 - 10 = -22$$

$$-22$$

3 Factorising **CORE CONCEPT & METHOD**

Factorising is the exact **opposite** of expanding! You are trying to find the highest common factor to pull *outside* the bracket.

MODEL WORKED EXAMPLES**EXAMPLE 3.1** **Non-Calculator**Factorise completely: $10x + 15$ **STEP-BY-STEP SOLUTION:**

1. What is the biggest number that divides into both 10 and 15? It's 5!
 2. Put 5 outside the bracket: $5(\quad)$
 3. What do you multiply 5 by to get $10x$? $2x$
 4. What do you multiply 5 by to get 15? $+3$
- Put them inside!

$$5(2x + 3)$$

EXAMPLE 3.2 **Non-Calculator**Factorise completely: $18x - 12$ **STEP-BY-STEP SOLUTION:**

1. What is the biggest number that divides into both 18 and 12? It's 6!
 2. Put 6 outside the bracket: $6(\quad)$
 3. What do you multiply 6 by to get $18x$? $3x$
 4. What do you multiply 6 by to get -12 ? -2 **(keep the negative!)**
- Put them inside!

$$6(3x - 2)$$

4 Expanding with Letters

📖 CORE CONCEPT & METHOD

If there is a letter outside the bracket, remember your index laws: $x \times x = x^2$,
 $2x \times 3x = 6x^2$, $a \times b = ab$.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

 **Non-Calculator**

Expand $3x(2x - 5)$

STEP-BY-STEP SOLUTION:

1. Multiply $3x \times 2x = 6x^2$.
2. Multiply $3x \times (-5) = -15x$.
3. Put them together: $6x^2 - 15x$.

5 Factorising Fully

📖 CORE CONCEPT & METHOD

To factorise **fully**, you must pull out the highest common number AND any common letters!

If both terms have an x , pull x outside the bracket.

MODEL WORKED EXAMPLES

EXAMPLE 5.1

 **Non-Calculator**

Factorise fully $12x^2 + 18xy$

STEP-BY-STEP SOLUTION:

1. Highest common factor of the numbers 12 and 18 is 6.
2. Both terms contain the letter x , so the common factor is $6x$.
3. $12x^2 \div 6x = 2x$ and $18xy \div 6x = 3y$.
4. Answer: $6x(2x + 3y)$.

6 Solving with Brackets

📖 CORE CONCEPT & METHOD

When an equation has a bracket, the easiest method is usually to **expand the bracket first**.

Once expanded, just solve it like a normal linear equation!

MODEL WORKED EXAMPLES**EXAMPLE 6.1** **Non-Calculator****Solve** $3(2x - 5) = 27$ **STEP-BY-STEP SOLUTION:**

1. Expand the bracket: $6x - 15 = 27$.
2. Add 15 to both sides: $6x = 42$.
3. Divide both sides by 6: $x = 7$.

Forming & Solving Equations

1 Unknowns on Both Sides

📖 CORE CONCEPT & METHOD

If there are letters on both sides of the equals sign, you need to collect them on one side! Move the **smallest** amount of letters so you don't end up with annoying negative numbers.

Common Mistake: When unknowns appear on both sides, eliminate the SMALLEST x -term first (adding if it is negative, e.g. $+2x$ to both sides) to keep your x -coefficient positive.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 Non-Calculator

Solve $7x = 4x + 15$

STEP-BY-STEP SOLUTION:

Move the $4x$ to the other side by doing the opposite ($-4x$).

$$\begin{array}{rcl} 7x & = & 4x + 15 \\ -4x & & -4x \\ \hline 3x & = & 15 \end{array}$$

Now divide by 3:

$$\begin{array}{rcl} 3x & = & 15 \\ \div 3 & & \div 3 \\ \hline x & = & 5 \end{array}$$

$$x = 5$$

EXAMPLE 1.2 **Non-Calculator**

Solve $6y + 3 = 2y + 19$

STEP-BY-STEP SOLUTION:1. Subtract $2y$ from both sides:

$$\begin{array}{rcl}
 6y + 3 & = & 2y + 19 \\
 -2y & & -2y \\
 \hline
 4y + 3 & = & 19
 \end{array}$$

2. Subtract 3 from both sides:

$$\begin{array}{rcl}
 4y + 3 & = & 19 \\
 -3 & & -3 \\
 \hline
 4y & = & 16
 \end{array}$$

3. Divide by 4:

$$\begin{array}{rcl}
 4y & = & 16 \\
 \div 4 & & \div 4 \\
 \hline
 y & = & 4
 \end{array}$$

$$y = 4$$

EXAMPLE 1.3 **Non-Calculator**

Solve $3x - 4 = 17 - 4x$

STEP-BY-STEP SOLUTION:1. We have a negative $-4x$. Move it to the other side by adding $4x$ to both sides! This avoids dealing with negatives.

$$\begin{array}{rcl}
 3x - 4 & = & 17 - 4x \\
 +4x & & +4x \\
 \hline
 7x - 4 & = & 17
 \end{array}$$

2. Add 4 to both sides:

$$\begin{array}{rcl}
 7x - 4 & = & 17 \\
 +4 & & +4 \\
 \hline
 7x & = & 21
 \end{array}$$

3. Divide by 7:

$$\begin{array}{rcl}
 7x & = & 21 \\
 \div 7 & & \div 7 \\
 \hline
 x & = & 3
 \end{array}$$

$$x = 3$$

2 Forming & Solving

CORE CONCEPT & METHOD

Sometimes we have to build the equation ourselves from a word problem! Think about the steps carefully, write down the equation, and then solve it using the balancing method.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

I think of a number. I multiply it by 3 and add 5. The answer is 26. What was my number?

STEP-BY-STEP SOLUTION:

1. Form the equation. Let the number be x .

Multiply it by 3 ($3x$) and add 5 ($+5$):

$$3x + 5 = 26$$

2. Solve it! Subtract 5:

$$\begin{array}{rcl} 3x + 5 & = & 26 \\ -5 & & -5 \\ \hline 3x & = & 21 \end{array}$$

3. Divide by 3:

$$\begin{array}{rcl} 3x & = & 21 \\ \div 3 & & \div 3 \\ \hline x & = & 7 \end{array}$$

$$x = 7$$

EXAMPLE 2.2 **Non-Calculator**

Tariq has x stickers.

Meera has 8 more stickers than Tariq.

Kieran has 2 fewer stickers than Tariq.

Together they have 66 stickers in total.

Form an equation and solve it to find how many stickers Tariq has.

STEP-BY-STEP SOLUTION:

1. Write an expression for each person:

$$\text{Tariq} = x$$

$$\text{Meera} = x + 8$$

$$\text{Kieran} = x - 2$$

2. Add them all together to equal 66:

$$x + (x + 8) + (x - 2) = 66$$

Collect the terms ($x + x + x = 3x$ and $8 - 2 = 6$):

$$3x + 6 = 66$$

3. Solve it!

$$\begin{array}{rcl} 3x + 6 & = & 66 \\ -6 & & -6 \\ \hline 3x & = & 60 \end{array}$$

Now divide by 3:

$$\begin{array}{rcl} 3x & = & 60 \\ \div 3 & & \div 3 \\ \hline x & = & 20 \end{array}$$

$$x = 20$$

EXAMPLE 2.3 **Non-Calculator**

A student museum pass costs £ x .

An adult museum pass costs £11.

Nadia bought 4 student passes and 3 adult passes.

The total cost was £65.

Form an equation to find the price of a student museum pass.

STEP-BY-STEP SOLUTION:

1. Find the cost of the adult passes first:

$$3 \times £11 = £33$$

2. Set up the total cost equation:

4 student passes plus the adult passes equals £65.

$$4x + 33 = 65$$

3. Solve it!

$$\begin{array}{rcl} 4x + 33 & = & 65 \\ -33 & & -33 \\ \hline 4x & = & 32 \end{array}$$

Now divide by 4:

$$\begin{array}{rcl} 4x & = & 32 \\ \div 4 & & \div 4 \\ \hline x & = & 8 \end{array}$$

$$x = 8$$

3 Forming Equations from Shapes **CORE CONCEPT & METHOD**

Geometry rules can help us form equations!

- **Perimeter:** Add up all the outside edges and set equal to the total.
- **Angles in a triangle:** Add them up and set equal to 180.
- **Angles around a point:** Add them up and set equal to 360.
- **Rectangles/Parallelograms:** Opposite sides are equal in length (so set them equal to each other!).

MODEL WORKED EXAMPLES

EXAMPLE 3.1 **Non-Calculator**

A rectangle has a length of $4x + 1$ cm and a width of $x + 3$ cm.
 Given the rectangle has a perimeter of 58 cm, find the value of x .

STEP-BY-STEP SOLUTION:

1. A rectangle has two lengths and two widths. Add all four sides to form an expression for the perimeter:

$$2(4x + 1) + 2(x + 3) = 8x + 2 + 2x + 6 = 10x + 8$$

2. Set the perimeter equal to 58 and solve:

$$\begin{array}{rcl} 10x + 8 & = & 58 \\ -8 & & -8 \\ \hline 10x & = & 50 \end{array}$$

Divide by 10:

$$x = 5$$

$$x = 5$$

EXAMPLE 3.2 **Non-Calculator**

The sizes of the angles, in degrees, of a triangle are $2x + 15$, $x + 25$ and $2x$.
 Work out the value of x .

STEP-BY-STEP SOLUTION:

1. Angles in a triangle always add up to 180° .

2. Add the three angle expressions together and set equal to 180:

$$\begin{aligned} (2x + 15) + (x + 25) + 2x &= 180 \\ 5x + 40 &= 180 \end{aligned}$$

3. Solve for x :

$$\begin{array}{rcl} 5x + 40 & = & 180 \\ -40 & & -40 \\ \hline 5x & = & 140 \end{array}$$

Divide by 5:

$$x = 28$$

$$x = 28$$

Linear Inequalities

1 Number Lines

📖 CORE CONCEPT & METHOD

Inequalities show a range of values instead of just one exact answer!

- $<$ means **Less Than**
- $>$ means **Greater Than**
- $\leq$ means **Less Than OR Equal To**
- $\geq$ means **Greater Than OR Equal To**

On a number line, we use circles:

- An **empty circle** is used for $<$ and $>$ (the number is NOT included).
- A **filled circle** is used for $\leq$ and $\geq$ (the number IS included).

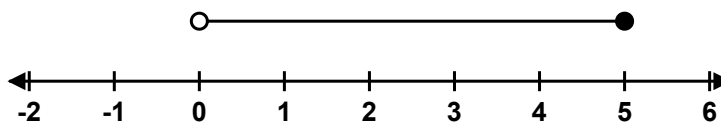
Common Mistake: If you multiply or divide both sides of an inequality by a **NEGATIVE** number, you **MUST** flip the inequality sign (e.g. $-3x \geq -18$ becomes $x \leq 6$), and check strict ($<$) vs inclusive ($\leq$) when listing integers.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

🚫 Non-Calculator

Write down the inequality shown on the number line below for x :



STEP-BY-STEP SOLUTION:

1. The line starts at 0 with an **empty circle**, meaning $x > 0$.
2. The line ends at 5 with a **filled circle**, meaning $x \leq 5$.
3. Combine them into one inequality with x in the middle:
 $0 < x \leq 5$

$$0 < x \leq 5$$

EXAMPLE 1.2 **Non-Calculator**

n is an integer such that $-3 \leq n < 2$.

Write down all the possible values of n .

STEP-BY-STEP SOLUTION:

1. The symbol $\leq$ next to -3 means n **can** be exactly -3 .
2. The symbol $<$ next to 2 means n **cannot** be 2 . It must stop at 1 .
3. The possible integers are $-3, -2, -1, 0, 1$.

$$-3, -2, -1, 0, 1$$

2 Linear Inequalities **CORE CONCEPT & METHOD**

Solving a linear inequality is exactly the same as solving a normal linear equation!
 You use the exact same balancing method.

The Golden Rule:

If you ever multiply or divide by a NEGATIVE number, you MUST flip the inequality sign!

MODEL WORKED EXAMPLES**EXAMPLE 2.1** **Non-Calculator**

Solve $5t + 4 \leq 24$

STEP-BY-STEP SOLUTION:

Solve this exactly like $5t + 4 = 24$:

$$\begin{array}{rcl} 5t + 4 & \leq & 24 \\ -4 & & -4 \\ \hline 5t & \leq & 20 \end{array}$$

Divide by 5:

$$\begin{array}{rcl} 5t & \leq & 20 \\ \div 5 & & \div 5 \\ \hline t & \leq & 4 \end{array}$$

$$t \leq 4$$

EXAMPLE 2.2 **Non-Calculator****Solve** $5x + 4 \leq x + 28$ **STEP-BY-STEP SOLUTION:**Move the smallest x term first. The smallest is x , so we subtract x from both sides:

$$\begin{array}{r} 5x + 4 \leq x + 28 \quad -x \quad -x \\ \hline 4x + 4 \leq 28 \end{array}$$

Subtract 4:

$$\begin{array}{r} 4x + 4 \leq 28 \\ -4 \quad -4 \\ \hline 4x \leq 24 \end{array}$$

Divide by 4:

$$\begin{array}{r} 4x \leq 24 \\ \div 4 \quad \div 4 \\ \hline x \leq 6 \end{array}$$

$$x \leq 6$$

EXAMPLE 2.3 **Non-Calculator****Solve** $15 - 4x < 31$ **STEP-BY-STEP SOLUTION:**

1. Subtract 15 from both sides:

$$\begin{array}{r} 15 - 4x < 31 \quad -15 \quad -15 \\ \hline -4x < 16 \end{array}$$

2. Divide by -4. Because we are dividing by a **negative number**, we **MUST** flip the sign!

$$\begin{array}{r} -4x < 16 \\ \div -4 \quad \div -4 \\ \hline x > -4 \end{array}$$

$$x > -4$$

3 Compound Integer Inequalities **CORE CONCEPT & METHOD**For inequalities like $-5 < 2y < 8$, you want to find y by itself in the middle.Whatever you do to the middle, you must do to **both the left AND the right!**1. Divide all three parts by 2: $-2.5 < y < 4$ 2. Then list the integers that fit! (e.g. $-2, -1, 0, 1, 2, 3$)

MODEL WORKED EXAMPLES

EXAMPLE 3.1 **Non-Calculator** $-12 < 4y \leq 8$ where y is an integer.Write down all the possible values of y .**STEP-BY-STEP SOLUTION:**

1. Divide all three parts by 4:

$$-3 < y \leq 2$$

2. List the integers greater than -3 and up to (including) 2 :

$$-2, -1, 0, 1, 2$$

$$-2, -1, 0, 1, 2$$

EXAMPLE 3.2 **Non-Calculator** $-7 \leq 2y + 1 < 5$ where y is an integer.Write down all the possible values of y .**STEP-BY-STEP SOLUTION:**

1. Subtract 1 from all three parts:

$$-8 \leq 2y < 4$$

2. Divide all three parts by 2:

$$-4 \leq y < 2$$

3. List the integers from -4 (included) up to 1 (2 is not included):

$$-4, -3, -2, -1, 0, 1$$

$$-4, -3, -2, -1, 0, 1$$

4 Inequalities with Brackets **CORE CONCEPT & METHOD**When an inequality has brackets, **expand them first!**

Once expanded, solve it exactly like a normal linear inequality, making sure to flip the sign if you divide by a negative!

MODEL WORKED EXAMPLES

EXAMPLE 4.1 **Non-Calculator****Solve** $4(2x - 3) \geq 20$ **STEP-BY-STEP SOLUTION:**

1. Expand the bracket first:

$$8x - 12 \geq 20$$

2. Add 12 to both sides:

$$8x \geq 32$$

3. Divide both sides by 8:

$$x \geq 4$$

$$x \geq 4$$

EXAMPLE 4.2 **Non-Calculator****Solve** $6(x + 3) < 2x + 38$ **STEP-BY-STEP SOLUTION:**

1. Expand the bracket first:

$$6x + 18 < 2x + 38$$

2. Subtract $2x$ from both sides:

$$4x + 18 < 38$$

3. Subtract 18 from both sides and divide by 4:

$$4x < 20 \implies x < 5$$

$$x < 5$$

Linear & Special Sequences

1 Linear Sequences

📖 CORE CONCEPT & METHOD

To find the n th term of a linear sequence, first find what it goes up (or down) by each time. That number goes in front of the n !

Then, work backwards to find out what the 'zeroth' term would have been. That number goes on the end!

Common Mistake: The number in front of n is the step difference (not the first term!), and when testing if a value is in a sequence, n MUST work out to a positive whole number (integer).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

🚫 Non-Calculator

Find the n th term of the sequence: 7, 11, 15, 19, ...

STEP-BY-STEP SOLUTION:

Zero term	3	7	11	15	19	
	- -		\\	\\	\\	
			4	4	4	Difference

Difference = +4, so $+4n$

Zero term = +3

$$4n + 3$$

EXAMPLE 1.2 **Non-Calculator**Find the n th term of the sequence: 13, 8, 3, -2 , ...**STEP-BY-STEP SOLUTION:**

Zero term	18	13	8	3	-2	
	--		\\	\\	\\	
		-5	-5	-5		Difference

Difference = -5 , so $-5n$ Zero term = $+18$

$$-5n + 18$$

2 Special Sequences **CORE CONCEPT & METHOD**

Not all sequences go up by adding a fixed number! In GCSE Maths you need to recognise two special types:

- **Fibonacci Sequences:** You add the previous TWO terms to get the next term.
Example: 1, 1, 2, 3, 5, 8, 13
- **Geometric Sequences:** You MULTIPLY by a fixed number (the common ratio) to get the next term.
Example: 2, 6, 18, 54 (multiply by 3)

MODEL WORKED EXAMPLES**EXAMPLE 2.1** **Non-Calculator**

Here are the first five terms of a Fibonacci sequence.

$$3 \quad 5 \quad 8 \quad 13 \quad 21$$

Find the 7th term of this sequence.**STEP-BY-STEP SOLUTION:**

In a Fibonacci sequence, add the previous two terms to get the next term.

$$6^{\text{th}} \text{ term} = 13 + 21 = 34$$

$$7^{\text{th}} \text{ term} = 21 + 34 = 55$$

$$55$$

EXAMPLE 2.2 **Non-Calculator**

Here are the first four terms of a sequence.

4 12 36 108

What is the next term in the sequence?

STEP-BY-STEP SOLUTION:

Check the differences: $+8, +24 \dots$

They are not constant, so it's not arithmetic.

Check the ratios: $12 \div 4 = 3, 36 \div 12 = 3$.

It is a **Geometric** sequence where we multiply by 3 each time.

Next term $= 108 \times 3 = 324$

324

3 Testing Terms **CORE CONCEPT & METHOD**

Sometimes you will be asked if a specific number belongs to a sequence.

- **Step 1:** Set the n th term rule equal to the number you are testing.
- **Step 2:** Solve the equation to find n .
- **Step 3:** If n is a **positive whole number** (an integer), then YES, the number is in the sequence! If n is a decimal, fraction, or negative, then NO, it is not.

MODEL WORKED EXAMPLES**EXAMPLE 3.1** **Non-Calculator**

The n th term of a sequence is $5n + 2$.

Is 47 a term in this sequence?

STEP-BY-STEP SOLUTION:

Set the rule equal to 47:

$$5n + 2 = 47$$

Solve for n :

$$5n = 45$$

$$n = 9$$

Because 9 is a positive whole number, **YES**, 47 is the 9th term in the sequence!

Yes

EXAMPLE 3.2 **Non-Calculator**

The n th term of a sequence is $n^2 + 3$.
Is 50 a term in this sequence?

STEP-BY-STEP SOLUTION:**Method 1: Solve Algebraically**

Set the rule equal to 50:

$$n^2 + 3 = 50$$

Solve for n :

$$n^2 = 47$$

$$n = \sqrt{47} \approx 6.86$$

Because n is not a whole number, **NO**, 50 is not in the sequence.

Method 2: Trial and Error

Try substituting whole numbers for n :

$$\bullet n = 5 \implies 5^2 + 3 = 28$$

$$\bullet n = 6 \implies 6^2 + 3 = 39$$

← 50 lies in between!

$$\bullet n = 7 \implies 7^2 + 3 = 52$$

50 is between $n = 6$ and $n = 7$, so n is not a whole number.

No

EXAMPLE 3.3 **Non-Calculator**

The first term in a sequence is 4. The term-to-term rule is add 5.
Is 86 a term in the sequence?

STEP-BY-STEP SOLUTION:

First, find the n th term rule:

It goes up by 5, so it starts with $5n$.

Working backwards, the 0th term is $4 - 5 = -1$.

So the rule is $5n - 1$.

Set it equal to 86:

$$5n - 1 = 86$$

$$5n = 87$$

$$n = 17.4$$

Because n is not a whole number, **NO**, 86 is not in the sequence.

No

4 Generating Sequence Terms **CORE CONCEPT & METHOD**

If you are given the rule for the **n th term**, you can find any term in the sequence!

- To find the 1st term, substitute $n = 1$.
- To find the 2nd term, substitute $n = 2$.
- To find the 10th term, substitute $n = 10$.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

 Non-Calculator

Linear Sequence: The n th term of a sequence is $5n - 2$.
Find the first three terms and the 10th term of this sequence.

STEP-BY-STEP SOLUTION:

Substitute $n = 1, 2, 3$ and 10 into $5n - 2$:

- 1st term ($n = 1$): $5(1) - 2 = 5 - 2 = 3$
- 2nd term ($n = 2$): $5(2) - 2 = 10 - 2 = 8$
- 3rd term ($n = 3$): $5(3) - 2 = 15 - 2 = 13$
- 10th term ($n = 10$): $5(10) - 2 = 50 - 2 = 48$

3, 8, 13 and 48

EXAMPLE 4.2

 Non-Calculator

Quadratic Sequence: The n th term of a sequence is $2n^2 + 3$.
Find the first three terms and the 5th term of this sequence.

STEP-BY-STEP SOLUTION:

Remember **BIDMAS**: square n first before multiplying by 2!

- 1st term ($n = 1$): $2(1)^2 + 3 = 2 \times 1 + 3 = 5$
- 2nd term ($n = 2$): $2(2)^2 + 3 = 2 \times 4 + 3 = 11$
- 3rd term ($n = 3$): $2(3)^2 + 3 = 2 \times 9 + 3 = 21$
- 5th term ($n = 5$): $2(5)^2 + 3 = 2 \times 25 + 3 = 53$

5, 11, 21 and 53

Real Life Graphs

1 Distance-Time Graphs

📖 CORE CONCEPT & METHOD

On a distance-time graph:

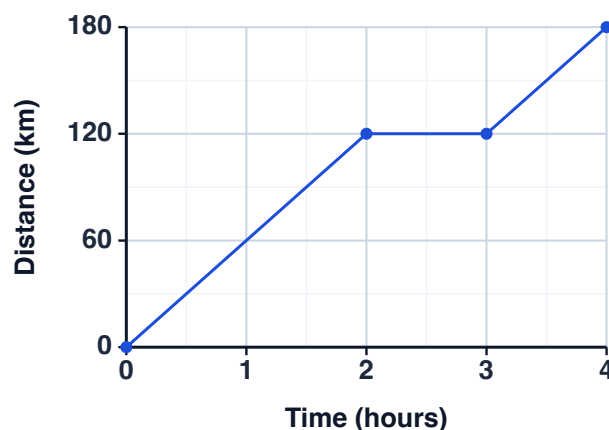
- The **vertical (y) axis** shows distance from the start.
- The **horizontal (x) axis** shows time.
- A **horizontal (flat) line** means the object is stationary (resting).
- The **gradient (steepness)** of the line represents the **speed** ($\text{speed} = \frac{\text{change in distance}}{\text{change in time}}$).

Common Mistake: On a cost graph, subtract the fixed starting charge (the y -intercept at $x = 0$) BEFORE dividing by the time or distance to find the rate per unit.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

🚫 Non-Calculator



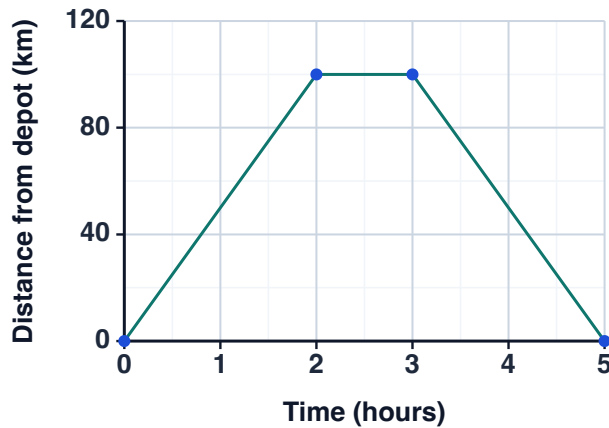
The distance-time graph shows a 4-hour delivery van journey.

- (a) What was the van's speed in the first 2 hours?
 (b) What was happening between 2 and 3 hours?

STEP-BY-STEP SOLUTION:

(a) $\text{Speed} = \text{gradient} = \frac{\text{change in distance}}{\text{change in time}} = \frac{120 - 0}{2 - 0} = \frac{120}{2} = 60 \text{ km/h.}$

(b) The line is **horizontal** at 120 km, so the distance is not changing — the delivery van is **stationary (at rest)** for 1 hour.

EXAMPLE 1.2 **Non-Calculator**

The distance-time graph shows a courier's journey away from the depot and back again.

- (a) How far from the depot did the courier stop for a break?
 (b) Calculate the courier's speed on the return journey to the depot.

STEP-BY-STEP SOLUTION:

- (a) The horizontal section is halfway between 80 and 120 at $y = 100$, so the courier stopped **100 km** from the depot.
 (b) The return journey is from $t = 3$ hours to $t = 5$ hours (a time of $5 - 3 = 2$ hours) covering 100 km:

$$\text{Return speed} = \frac{100}{2} = 50 \text{ km/h}$$

2 Cost & Fixed Charges **CORE CONCEPT & METHOD**

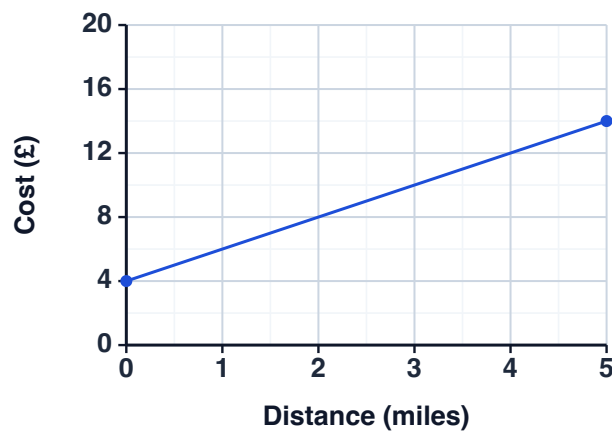
For real-life pricing graphs (like taxis, plumbers, or tool hire), the line starts above 0 on the vertical axis.

- The **y-intercept** (where the line crosses the cost axis at $x = 0$) is the **fixed starting charge**.
- The **gradient** ($\frac{\text{rise}}{\text{run}}$) tells you the **cost per mile / hour / day**.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator



The parcel delivery graph shows the cost (£) against distance (miles). The line crosses the vertical axis at (0, 4) and passes through (5, 14).

- (a) What is the fixed starting charge?
 (b) What is the cost per mile?

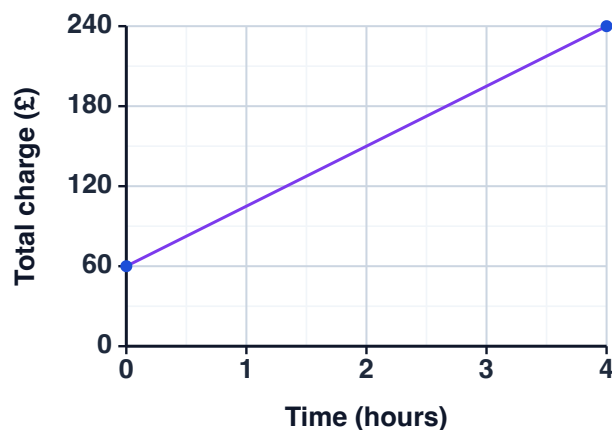
STEP-BY-STEP SOLUTION:

(a) The **y-intercept** at distance = 0 is **£4**. This is the fixed starting charge.

(b) Gradient = $\frac{\text{change in cost}}{\text{change in miles}} = \frac{14 - 4}{5 - 0} = \frac{10}{5} = \text{£}2$ per mile.

EXAMPLE 2.2

Non-Calculator



The graph shows the total charge (£) of a plumber for jobs up to 4 hours. Work out the total charge for a **6-hour** job.

STEP-BY-STEP SOLUTION:

- Fixed call-out charge (**y-intercept**) = **£60**.
- Hourly rate (gradient) = $\frac{240 - 60}{4 - 0} = \frac{180}{4} = \text{£}45$ per hour.
- For a 6-hour job: Total = $60 + 45 \times 6 = 60 + 270 = \text{£}330$.

3 Rates of Flow & Consumption

□ CORE CONCEPT & METHOD

To find a rate of flow or consumption from a straight-line graph, calculate the

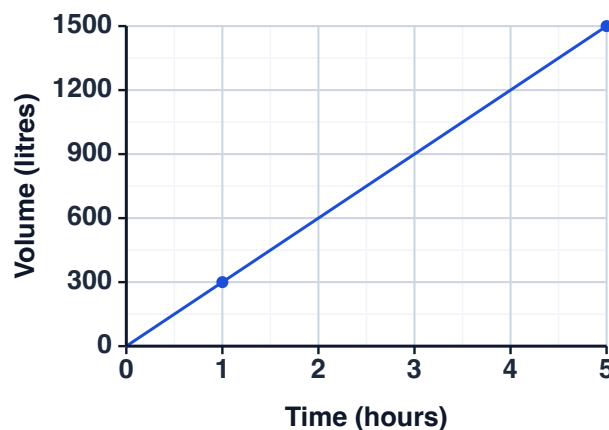
gradient $\left(\frac{\text{change in } y}{\text{change in } x}\right)$.

- An **upward slope** shows a tank filling (positive rate).
- A **downward slope** shows a tank draining or fuel being used up.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 **Non-Calculator**



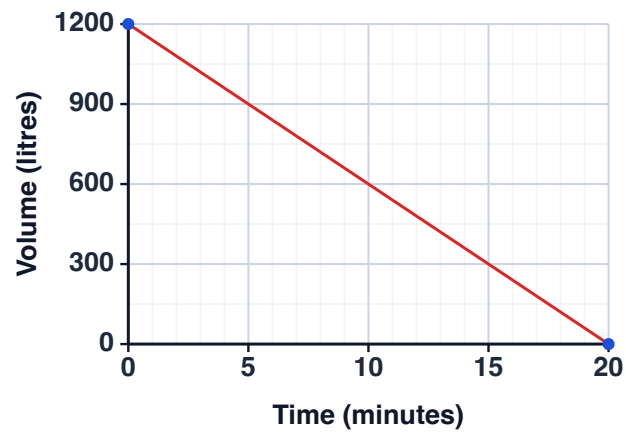
The graph shows the volume of water in a storage reservoir (litres) against time (hours). The volume increases from 300 litres at 1 hour to 1500 litres at 5 hours.

Calculate the rate of flow of water into the reservoir.

STEP-BY-STEP SOLUTION:

The rate of flow is the **gradient** of the graph:

$$\text{Rate} = \frac{\text{change in volume}}{\text{change in time}} = \frac{1500 - 300}{5 - 1} = \frac{1200}{4} = 300 \text{ litres per hour}$$

EXAMPLE 3.2**Non-Calculator**

The graph shows the volume of liquid in an industrial vat as it drains completely over 20 minutes. Calculate the rate at which the liquid is draining in litres per minute.

STEP-BY-STEP SOLUTION:

1. At $t = 0$ minutes, volume = 1200 litres. At $t = 20$ minutes, volume = 0 litres.

2. Drain rate = $\frac{1200 - 0}{20 - 0} = 60$ litres per minute.

Angles in Parallel Lines

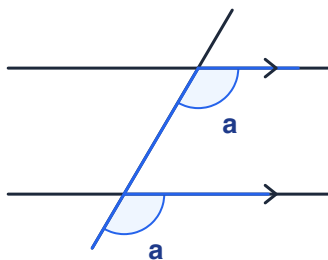
1 Alternate, Corresponding & Co-Interior Angles

CORE CONCEPT & METHOD

Angles in Parallel Lines Reference

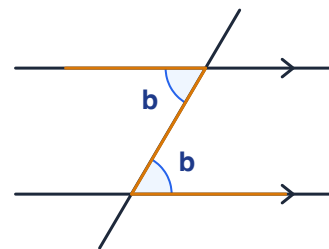
When a straight line (called a **transversal**) crosses two parallel lines (marked with matching arrows), three special angle pairs are formed.

1. Corresponding Angles ("F-shape")



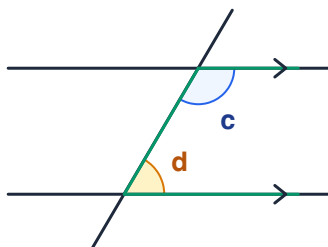
Corresponding angles sit in the same relative position at each intersection. **Corresponding angles are equal ($a = a$).**

2. Alternate Angles ("Z-shape")



Alternate angles lie on opposite sides of the transversal between the parallel lines. **Alternate angles are equal ($b = b$).**

3. Co-Interior / Allied Angles ("C-shape")



Co-interior (or allied) angles lie on the same side of the transversal between the parallel lines.

Co-interior angles add up to 180° ($c + d = 180^\circ$).

GCSE Exam Tip: Always state the full geometrical term ("*Corresponding angles are equal*", "*Alternate angles are equal*", or "*Co-interior angles add to 180°* "). Never write "*F-angles*" or "*Z-angles*" as a reason in an exam!

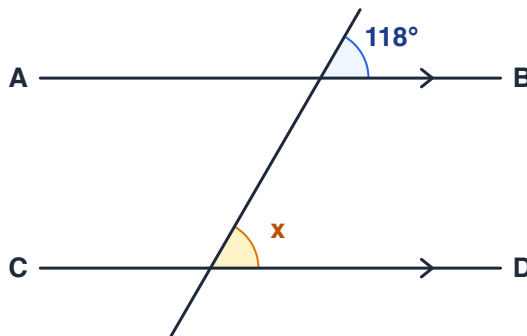
Common Mistake: Always use official geometric terminology in reasons ("*Alternate angles are equal*", "*Corresponding angles are equal*", "*Co-interior angles sum to 180°* ") — Edexcel awards zero marks for "*Z-angles*" or "*F-angles*".

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

Lines AB and CD are parallel.



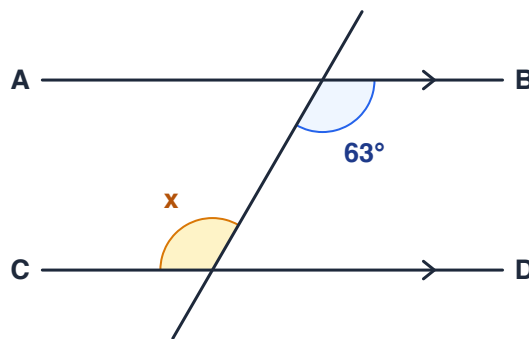
Work out the size of angle x .

STEP-BY-STEP SOLUTION:

1. Angle x and the 118° angle are in the same relative position at each intersection (an "F-shape").
2. Because **corresponding angles are equal**:

$$x = 118^\circ$$

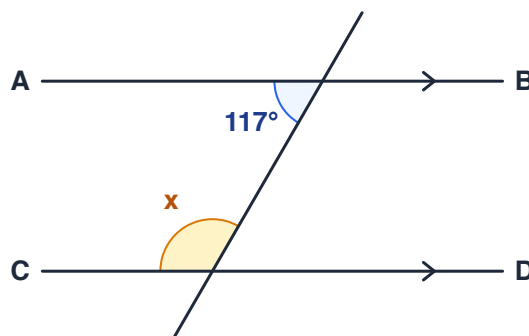
$$118^\circ$$

EXAMPLE 1.2 **Non-Calculator**Lines AB and CD are parallel.Work out the size of angle x .**STEP-BY-STEP SOLUTION:**

- Angle x and the 63° angle lie on opposite sides of the transversal between the parallel lines (a "Z-shape").
- Because **alternate angles are equal**:

$$x = 63^\circ$$

$$63^\circ$$

EXAMPLE 1.3 **Non-Calculator**Lines AB and CD are parallel.Work out the size of angle x .**STEP-BY-STEP SOLUTION:**

- Angle x and the 117° angle lie between the parallel lines on the same side of the transversal (a "C-shape").
- Because **co-interior angles add up to 180°** :

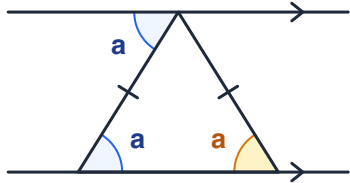
$$x = 180^\circ - 117^\circ = 63^\circ$$

$$63^\circ$$

2 Triangles & Parallelograms in Parallel Lines **CORE CONCEPT & METHOD****Triangles & Parallelograms in Parallel Lines**

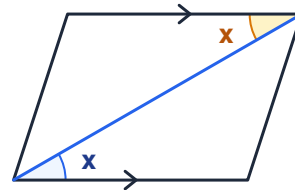
Multi-step GCSE questions embed isosceles triangles, intersecting transversals, and parallelograms inside parallel lines.

1. Isosceles Triangles Between Parallel Lines



First use **alternate** or **corresponding** angles to move an outside angle into the triangle. Then use the equal sides (tick marks) to identify the two equal base angles!

2. Parallelograms with a Diagonal



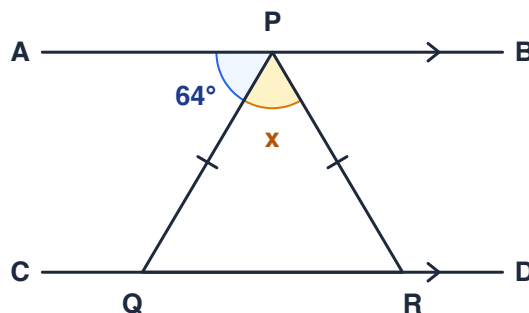
A diagonal acts as a transversal across both pairs of parallel sides! Therefore, alternate angles on opposite sides of the diagonal are equal ($\angle CAB = \angle ACD$).

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Line APB is parallel to line $CQRD$, and $PQ = PR$.
Angle $APQ = 64^\circ$.

Non-Calculator



Work out the size of angle QPR (x).

STEP-BY-STEP SOLUTION:

1. By alternate angles between parallel lines AB and CD :

$$\angle PQR = \angle APQ = 64^\circ$$

2. Since $PQ = PR$, $\triangle PQR$ is isosceles so $\angle PRQ = \angle PQR = 64^\circ$.

3. Angles in $\triangle PQR$ add up to 180° :

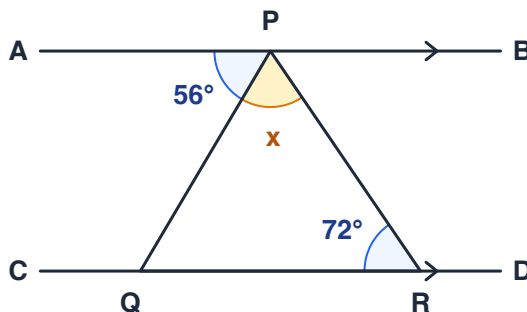
$$x = 180^\circ - (64^\circ + 64^\circ) = 52^\circ$$

52°

EXAMPLE 2.2 **Non-Calculator**

Line APB is parallel to line $CQRD$.

Angle $APQ = 56^\circ$ and angle $PRQ = 72^\circ$.



Work out the size of angle QPR (x).

STEP-BY-STEP SOLUTION:

1. By alternate angles, $\angle PQR = \angle APQ = 56^\circ$.

2. Angles in $\triangle PQR$ add up to 180° :

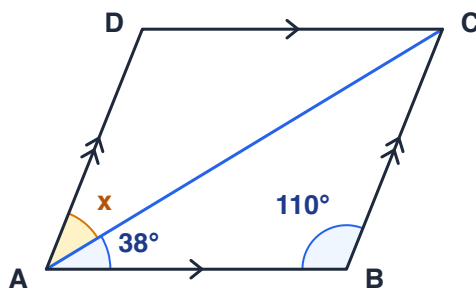
$$x = 180^\circ - (56^\circ + 72^\circ) = 52^\circ$$

52°

EXAMPLE 2.3 **Non-Calculator**

$ABCD$ is a parallelogram with diagonal AC .

Angle $CAB = 38^\circ$ and angle $ABC = 110^\circ$.



Work out the size of angle DAC (x).

STEP-BY-STEP SOLUTION:

1. Co-interior angles $\angle DAB$ and $\angle ABC$ add up to 180° :

$$\angle DAB = 180^\circ - 110^\circ = 70^\circ$$

2. Subtract $\angle CAB = 38^\circ$:

$$x = 70^\circ - 38^\circ = 32^\circ$$

32°

3 Algebraic & Ratio Angles in Parallel Lines

□ CORE CONCEPT & METHOD

Algebraic & Ratio Angles in Parallel Lines

To solve algebraic parallel-line problems, first decide whether the two algebraic expressions are **equal** or **add up to 180°** :

- **Corresponding or Alternate Angles:** The angles are equal, so set up the equation:

$$ax + b = cx + d$$

- **Co-Interior Angles or Angles on a Straight Line:** The angles add to 180° , so set up the equation:

$$(ax + b) + (cx + d) = 180$$

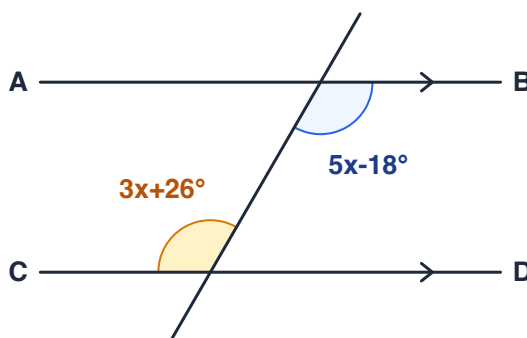
- **Ratio Problems on Parallel Lines:** Find the total angle (180° for co-interior/straight line) and divide by the total number of ratio parts.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Lines AB and CD are parallel.

✎ Non-Calculator



Work out the value of x .

STEP-BY-STEP SOLUTION:

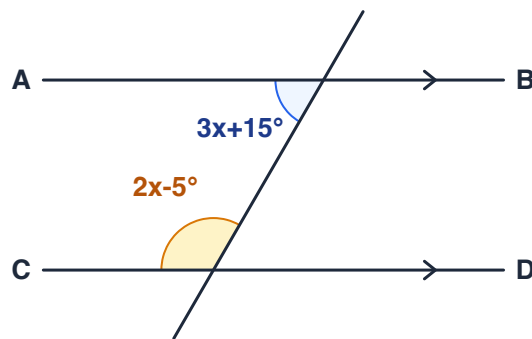
1. Alternate angles between parallel lines are equal:

$$5x - 18 = 3x + 26$$

2. Subtract $3x$ from both sides and add 18:

$$2x = 44 \implies x = 22$$

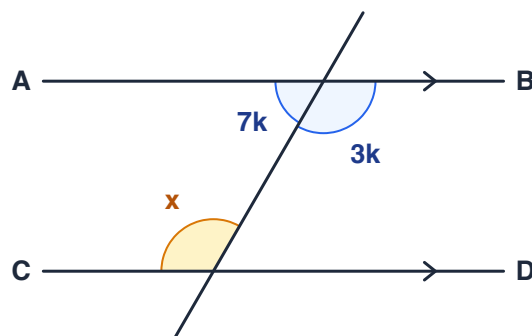
22

EXAMPLE 3.2 **Non-Calculator**Lines AB and CD are parallel.Work out the value of x .**STEP-BY-STEP SOLUTION:**1. Co-interior angles between parallel lines add up to 180° :

$$(3x + 15) + (2x - 5) = 180$$

2. Collect like terms and solve:

$$5x + 10 = 180 \implies 5x = 170 \implies x = 34$$

34
EXAMPLE 3.3 **Non-Calculator**Lines AB and CD are parallel. The two interior angles on line AB are in the ratio 7 : 3.Work out the size of angle x .**STEP-BY-STEP SOLUTION:**1. Angles on straight line AB add up to 180° , split in the ratio 7 : 3 (10 parts total):

$$1 \text{ part} = 180^\circ \div 10 = 18^\circ$$

2. The acute angle is 3 parts:

$$3 \times 18^\circ = 54^\circ$$

3. By alternate angles, $x = 54^\circ$.

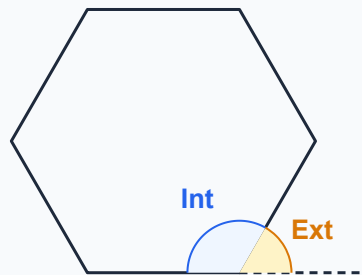
 54°

Angles in Polygons

1 Regular Polygons: Interior & Exterior Angles

▣ CORE CONCEPT & METHOD

In a **regular polygon**, all sides are equal in length and all interior angles are equal in size.



- **Straight Line Rule:** At any vertex, the interior and exterior angles lie on a straight line:

$$\text{Interior Angle} + \text{Exterior Angle} = 180^\circ$$

- **Exterior Angle Sum Rule:** The exterior angles of *any* polygon always add up to 360° . For a regular n -sided polygon:

$$\text{Exterior Angle} = \frac{360^\circ}{n} \quad \text{and} \quad n = \frac{360^\circ}{\text{Exterior Angle}}$$

- **Centre Angles & Diagonals:** Lines from the centre O to adjacent vertices form n congruent isosceles triangles with centre angle $\frac{360^\circ}{n}$.

Polygon	Sides (n)	Regular Exterior	Regular Interior
Pentagon	5	72°	108°
Hexagon	6	60°	120°
Octagon	8	45°	135°
Nonagon	9	40°	140°
Decagon	10	36°	144°
Dodecagon	12	30°	150°

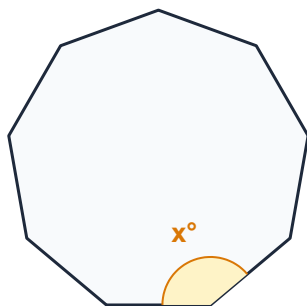
Common Mistake: For any regular polygon, find the EXTERIOR angle first using $360^\circ \div n$ (since exterior angles always sum to 360°), then use $\text{Interior} = 180^\circ - \text{Exterior}$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

Work out the size of each exterior angle and each interior angle of a regular nonagon (9 sides).



STEP-BY-STEP SOLUTION:

Step 1: Find the exterior angle

A nonagon has $n = 9$ equal exterior angles that sum to 360° :

$$\text{Exterior Angle} = \frac{360^\circ}{9} = 40^\circ$$

Step 2: Find the interior angle

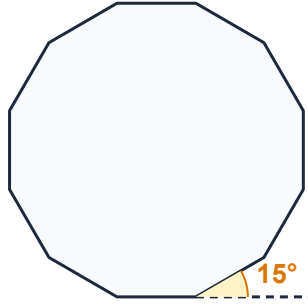
Interior and exterior angles add up to 180° on a straight line:

$$x = 180^\circ - 40^\circ = 140^\circ$$

$$140^\circ$$

EXAMPLE 1.2 **Non-Calculator**

The exterior angle of a regular polygon is 15° .
Work out the number of sides the regular polygon has.

**STEP-BY-STEP SOLUTION:**

Step 1: Use the Exterior Angle Sum formula

The sum of exterior angles of any polygon is 360° :

$$n = \frac{360^\circ}{\text{Exterior Angle}}$$

Step 2: Substitute 15°

$$n = \frac{360^\circ}{15^\circ} = 24 \text{ sides}$$

24

EXAMPLE 1.3 **Non-Calculator**

Each interior angle of a regular polygon is 168° .
Work out how many sides the regular polygon has.

STEP-BY-STEP SOLUTION:

Step 1: Find the exterior angle first

$$\text{Exterior Angle} = 180^\circ - 168^\circ = 12^\circ$$

Step 2: Divide 360° by the exterior angle

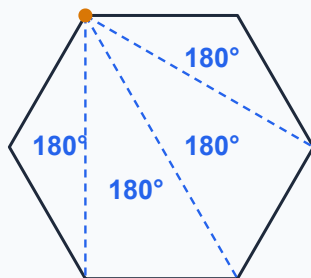
$$n = \frac{360^\circ}{12^\circ} = 30 \text{ sides}$$

30

2 Irregular Polygons & Sum of Interior Angles

□ CORE CONCEPT & METHOD

Any n -sided polygon can be split into $(n - 2)$ **triangles** by drawing diagonals from a single vertex.



Since the angles in each triangle add up to 180° :

$$\text{Sum of Interior Angles} = (n - 2) \times 180^\circ$$

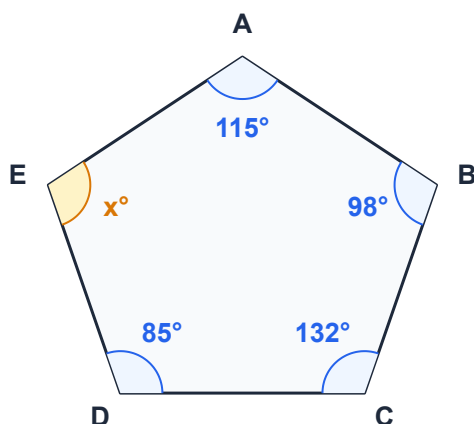
- **Pentagon (5 sides):** $(5 - 2) \times 180^\circ = 540^\circ$
- **Hexagon (6 sides):** $(6 - 2) \times 180^\circ = 720^\circ$
- **Heptagon (7 sides):** $(7 - 2) \times 180^\circ = 900^\circ$
- **Octagon (8 sides):** $(8 - 2) \times 180^\circ = 1080^\circ$

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

Work out the size of angle x in the irregular pentagon $ABCDE$.



STEP-BY-STEP SOLUTION:

Step 1: Find the sum of interior angles of a pentagon ($n = 5$)

$$\text{Sum} = (5 - 2) \times 180^\circ = 3 \times 180^\circ = 540^\circ$$

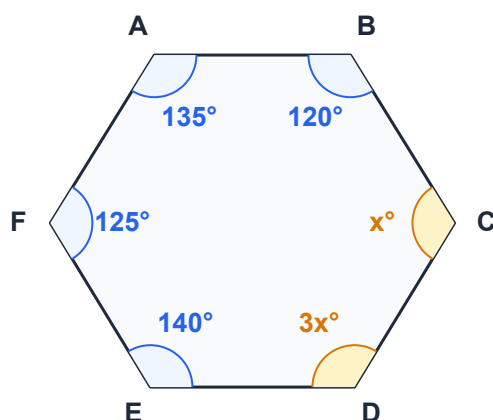
Step 2: Subtract the known angles from 540°

$$115^\circ + 98^\circ + 132^\circ + 85^\circ = 430^\circ$$
$$x = 540^\circ - 430^\circ = 110^\circ$$

$$x = 110^\circ$$

EXAMPLE 2.2**Non-Calculator**

In the irregular hexagon $ABCDEF$, $\angle CDE$ is 3 times the size of $\angle BCD$.
Work out the size of $\angle CDE$.

**STEP-BY-STEP SOLUTION:**

Step 1: Find the sum of interior angles of a hexagon ($n = 6$)

$$\text{Sum} = (6 - 2) \times 180^\circ = 720^\circ$$

Step 2: Form an equation in x

$$\begin{aligned} 135^\circ + 120^\circ + 140^\circ + 125^\circ &= 520^\circ \\ x + 3x + 520^\circ &= 720^\circ \implies 4x = 200^\circ \implies x = 50^\circ \end{aligned}$$

Step 3: Calculate $\angle CDE = 3x$

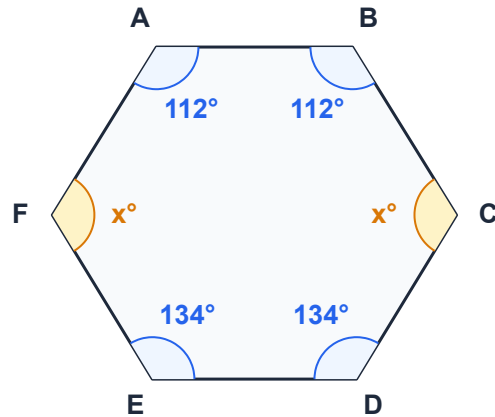
$$\angle CDE = 3 \times 50^\circ = 150^\circ$$

$$150^\circ$$

EXAMPLE 2.3**Non-Calculator**

Hexagon $ABCDEF$ is symmetrical with $\angle A = \angle B = 112^\circ$, $\angle D = \angle E = 134^\circ$, and $\angle C = \angle F = x^\circ$.

Work out the value of x .

**STEP-BY-STEP SOLUTION:**

Step 1: Sum of interior angles of a hexagon ($n = 6$)

$$(6 - 2) \times 180^\circ = 720^\circ$$

Step 2: Add the four known angles

$$112^\circ + 112^\circ + 134^\circ + 134^\circ = 492^\circ$$

Step 3: Divide the remaining angle sum equally between the two equal angles x

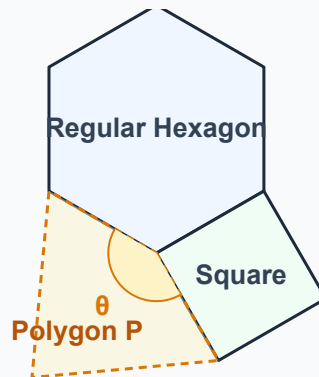
$$2x = 720^\circ - 492^\circ = 228^\circ \implies x = 114^\circ$$

$$x = 114^\circ$$

3 Compound & Tessellating Regular Polygons

📖 CORE CONCEPT & METHOD

In GCSE compound polygon problems, regular polygons share sides and meet at a common vertex.



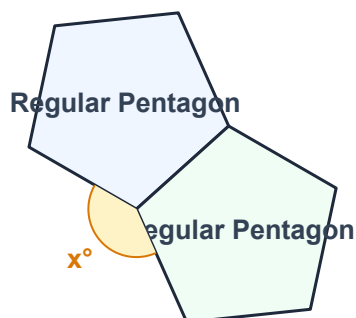
1. Find the **interior angle** of each known regular polygon meeting at the vertex.
2. Use **angles around a point** $= 360^\circ$ to find the missing angle x or the interior angle θ of the mystery regular polygon P .
3. To find how many sides regular polygon P has, find its exterior angle $(180^\circ - \theta)$ and divide 360° by that exterior angle:

$$n = \frac{360^\circ}{180^\circ - \theta}$$

MODEL WORKED EXAMPLES

EXAMPLE 3.1 **Non-Calculator**

Two regular pentagons share a side and meet at a point.
Work out the size of the angle marked x° .

**STEP-BY-STEP SOLUTION:**

Step 1: Find the interior angle of a regular pentagon ($n = 5$)

$$\text{Exterior Angle} = \frac{360^\circ}{5} = 72^\circ \implies \text{Interior Angle} = 180^\circ - 72^\circ = 108^\circ$$

Step 2: Use angles around a point = 360°

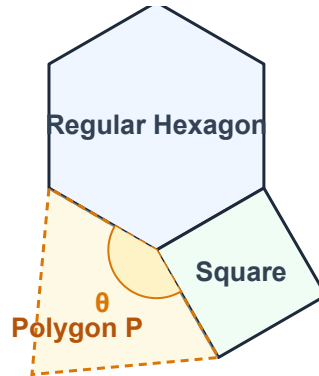
$$x = 360^\circ - (108^\circ + 108^\circ) = 360^\circ - 216^\circ = 144^\circ$$

$$x = 144^\circ$$

EXAMPLE 3.2 **Non-Calculator**

A regular hexagon and a square share sides with a third regular polygon P , meeting at a point with no gaps.

Calculate how many sides regular polygon P has.

**STEP-BY-STEP SOLUTION:**

Step 1: Find the interior angles of the regular hexagon and square

Regular hexagon interior angle = 120° ; Square interior angle = 90° .

Step 2: Find the interior angle θ of Polygon P

$$\theta = 360^\circ - (120^\circ + 90^\circ) = 150^\circ$$

Step 3: Find the exterior angle and number of sides of Polygon P

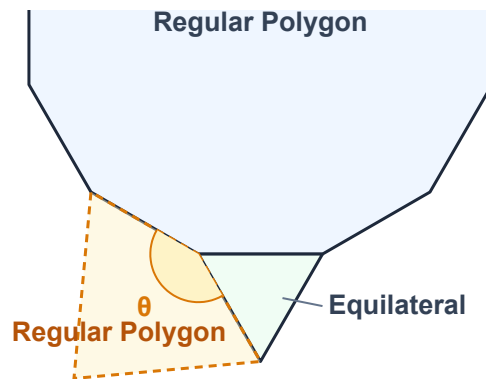
$$\text{Exterior Angle} = 180^\circ - 150^\circ = 30^\circ$$

$$n = \frac{360^\circ}{30^\circ} = 12 \text{ sides}$$

12

EXAMPLE 3.3 **Non-Calculator**

Two identical regular polygons and an equilateral triangle meet at a point with no gaps.
Work out the number of sides of each identical regular polygon.

**STEP-BY-STEP SOLUTION:**

Step 1: Use angles around a point = 360°

The interior angle of an equilateral triangle is 60° . The remaining angle is split equally between the two identical regular polygons:

$$2\theta = 360^\circ - 60^\circ = 300^\circ \implies \theta = 150^\circ$$

Step 2: Find the exterior angle and number of sides n

$$\text{Exterior Angle} = 180^\circ - 150^\circ = 30^\circ$$

$$n = \frac{360^\circ}{30^\circ} = 12 \text{ sides}$$

12

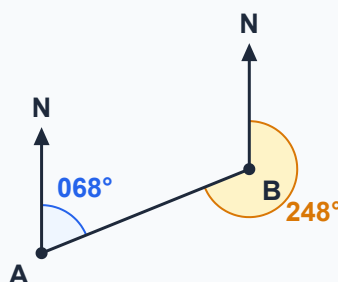
Bearings

1 Three-Figure Bearings & Back Bearings

□ CORE CONCEPT & METHOD

A **three-figure bearing** describes a direction using three strict rules:

1. **Measure from North:** Always start at the vertical North line at the point you are measuring **from**.
2. **Measure Clockwise:** Always turn clockwise from North to the direction line.
3. **Write 3 Digits:** Pad angles under 100° with leading zeros (e.g. 045° , 008° , 248°).



Back Bearings (The bearing of A from B):

Because the North lines at A and B are parallel, co-interior angles add up to 180° , and a full turn is 360° . This gives the shortcut rule:

- If the forward bearing $\theta < 180^\circ$, **add** 180° :
Back Bearing = $\theta + 180^\circ$.
- If the forward bearing $\theta \geq 180^\circ$, **subtract** 180° :
Back Bearing = $\theta - 180^\circ$.

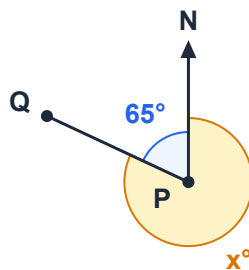
Common Mistake: Every bearing must be measured **CLOCKWISE** from the **NORTH** line at the "FROM" point and written as 3 digits (e.g. 037° , never 37°).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Work out the three-figure bearing of Q from P .



STEP-BY-STEP SOLUTION:

Step 1: Recall the rules of bearings

Bearings are measured **clockwise** from North at P .

Step 2: Subtract the anticlockwise angle from 360°

$$x = 360^\circ - 65^\circ = 295^\circ$$

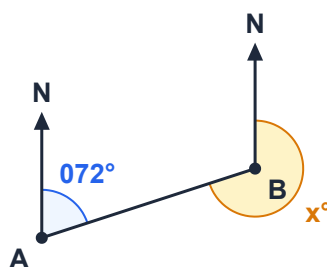
295°

EXAMPLE 1.2

Non-Calculator

The bearing of B from A is 072° .

Work out the three-figure bearing of A from B .



STEP-BY-STEP SOLUTION:

Step 1: Use co-interior angles between parallel North lines

The anticlockwise angle from North at B to the line BA is:

$$180^\circ - 72^\circ = 108^\circ$$

Step 2: Subtract from 360° (or add 180° to 072°)

$$\text{Bearing of } A \text{ from } B = 360^\circ - 108^\circ = 252^\circ$$

252°

EXAMPLE 1.3 **Non-Calculator**

Port B is on a bearing of 218° from Port A .

Find the three-figure bearing of Port A from Port B .

STEP-BY-STEP SOLUTION:

Step 1: Apply the Back Bearing rule

Since $218^\circ \geq 180^\circ$, subtract 180° :

$$218^\circ - 180^\circ = 38^\circ$$

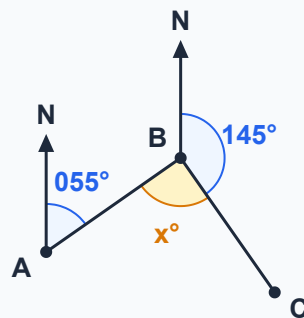
Step 2: Write as a three-figure bearing

Pad with a leading zero to give 038° .

038°

2 Bearings with Scale & Interior Angles **CORE CONCEPT & METHOD**

When a ship or aircraft changes course at a turning point B , draw a **North line at B** parallel to the North line at A .

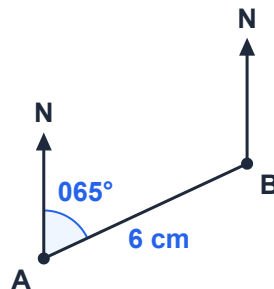


- **Step 1 (Co-interior Angles):** The angle between North at B and the line BA is $180^\circ - \text{Bearing}(B \text{ from } A)$.
- **Step 2 (Angles around B):** Subtract the clockwise bearing at B and the co-interior angle from 360° to find the interior angle $\angle ABC$.
- **Map Scales:** Multiply the map length (in cm) by the scale factor to find the real distance, or divide to work backwards.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 **Non-Calculator**

On a map with scale 1 cm represents 3.5 km, Island B is 6 cm from Port A on a bearing of 065° . Work out the actual distance between Port A and Island B in km.

**STEP-BY-STEP SOLUTION:**

Step 1: Multiply the map distance by the scale

Each 1 cm represents 3.5 km:

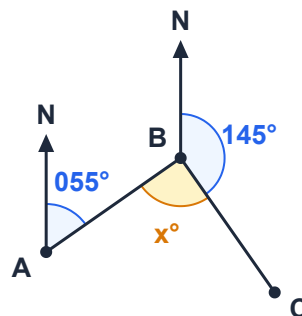
$$\text{Actual Distance} = 6 \times 3.5 = 21 \text{ km}$$

21 km

EXAMPLE 2.2 **Non-Calculator**

A ship sails from A to B on a bearing of 055° , then changes course at B and sails to C on a bearing of 145° .

Work out the size of angle ABC (marked x°).

**STEP-BY-STEP SOLUTION:**

Step 1: Find the co-interior angle at B

Since the North lines at A and B are parallel:

$$\text{Angle between North at } B \text{ and } BA = 180^\circ - 55^\circ = 125^\circ$$

Step 2: Use angles around point B (360°)

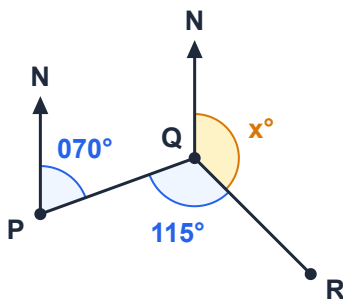
$$x = 360^\circ - (125^\circ + 145^\circ) = 360^\circ - 270^\circ = 90^\circ$$

$x = 90^\circ$

EXAMPLE 2.3 **Non-Calculator**

A hiker walks from P to Q on a bearing of 070° , then turns clockwise so that the interior angle $\angle PQR = 115^\circ$.

Work out the three-figure bearing of R from Q .

**STEP-BY-STEP SOLUTION:**

Step 1: Find the co-interior angle at Q

$$180^\circ - 70^\circ = 110^\circ$$

Step 2: Subtract from 360° around point Q

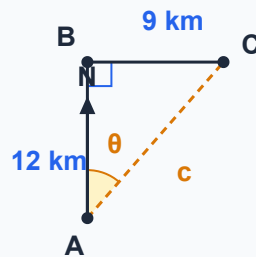
$$\text{Bearing of } R \text{ from } Q = 360^\circ - (110^\circ + 115^\circ) = 135^\circ$$

$$135^\circ$$

3 Bearings with Pythagoras & Trigonometry

□ CORE CONCEPT & METHOD

In higher-mark GCSE bearing questions, a journey along compass directions (North/South and East/West) or perpendicular bearings forms a **right-angled triangle**.



- **Direct Distance (Pythagoras' Theorem):**

$$c = \sqrt{a^2 + b^2}$$

- **Finding the Bearing (Trigonometry / SOHCAHTOA):**

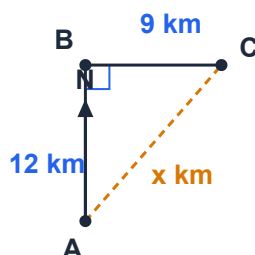
Use $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$ to find the acute angle θ inside the triangle, then combine θ with 000° , 090° , 180° , 270° , or 360° to write the three-figure bearing.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Non-Calculator

A ship sails 12 km due North from A to B , and then 9 km due East from B to C .
Work out the direct distance AC .



STEP-BY-STEP SOLUTION:

Step 1: Use Pythagoras' Theorem in right-angled triangle ABC

$$AC^2 = 12^2 + 9^2 = 144 + 81 = 225$$

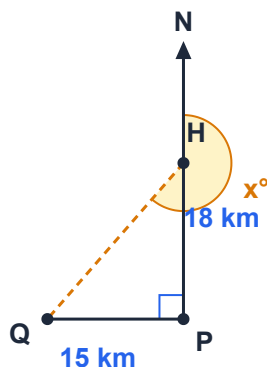
Step 2: Square root to find AC

$$AC = \sqrt{225} = 15 \text{ km}$$

15 km

 **Calculator**
EXAMPLE 3.2

A helicopter flies 18 km due South from base H to point P , then 15 km due West from P to Q . Work out the three-figure bearing of Q from H , correct to the nearest degree.

**STEP-BY-STEP SOLUTION:**

Step 1: Find acute angle $\angle PHQ$ using $\tan$

$$\tan(\angle PHQ) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{15}{18}$$

$$\angle PHQ = \tan^{-1}\left(\frac{15}{18}\right) = 39.8^\circ \approx 40^\circ$$

Step 2: Add to 180° (since P is due South and Q is clockwise past South)

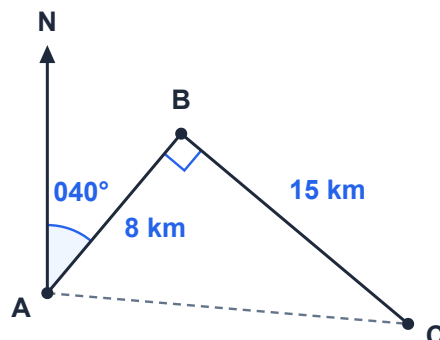
$$\text{Bearing} = 180^\circ + 40^\circ = 220^\circ$$

220°

EXAMPLE 3.3 **Calculator**

A yacht sails 8 km from A to B on a bearing of 040° , then turns 90° clockwise and sails 15 km from B to C on a bearing of 130° .

Work out the three-figure bearing of C from A , correct to the nearest degree.

**STEP-BY-STEP SOLUTION:**

Step 1: Find $\angle BAC$ in right-angled triangle ABC

$$\tan(\angle BAC) = \frac{15}{8} = 1.875$$

$$\angle BAC = \tan^{-1}(1.875) = 61.9^\circ \approx 62^\circ$$

Step 2: Add $\angle BAC$ to the bearing of B from A (040°)

$$\text{Bearing of } C \text{ from } A = 040^\circ + 62^\circ = 102^\circ$$

102°

Translations, Reflections & Rotations

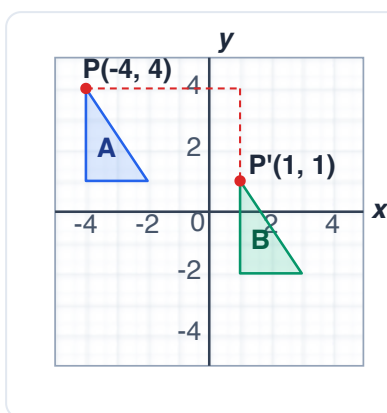
1 Translations & Column Vectors

CORE CONCEPT & METHOD

Translations & Column Vectors

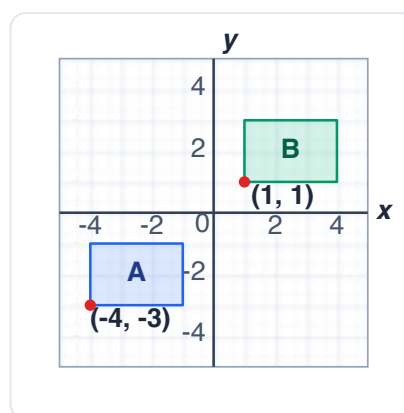
A **translation** slides every vertex of a shape by the exact same horizontal and vertical distance without rotating or flipping it. We write a translation using a **column vector** $\begin{pmatrix} x \\ y \end{pmatrix}$.

1. Reading a Column Vector $\begin{pmatrix} x \\ y \end{pmatrix}$



- **Top number (x):** horizontal shift (+ right, − left).
- **Bottom number (y):** vertical shift (+ up, − down).
- From $P(-4, 4)$ on A to $P'(1, 1)$ on B is 5 right and 3 down: $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

2. Match Corresponding Corners (Not the Gap!)



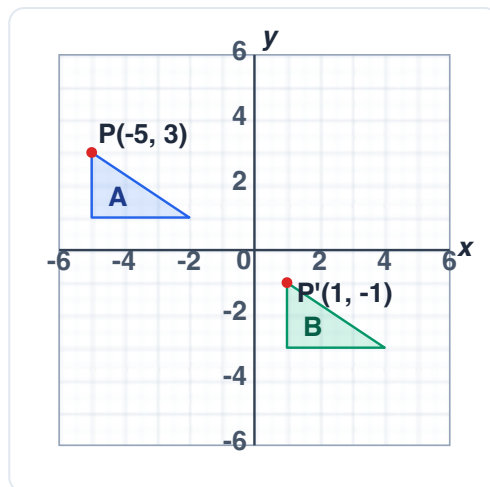
- **Never count the empty space** between two shapes!
- Pick one corner on A — e.g. bottom-left $(-4, -3)$ — and count to the **same corner** on B $(1, 1)$: $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$.
- Reversing ($B \rightarrow A$) negates both signs: $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$.

Common Mistake: Count translation vectors from one corner on Shape A to the MATCHING corner on Shape B (never count the gap between shapes!), and remember $x = k$ is a VERTICAL mirror line while $y = k$ is HORIZONTAL.

MODEL WORKED EXAMPLES

EXAMPLE 1.1 **Non-Calculator**

Triangle A and Triangle B are drawn on the coordinate grid below.



Triangle A is translated onto Triangle B by the column vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find the column vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

STEP-BY-STEP SOLUTION:

Step 1: Choose a pair of corresponding vertices on A and B

The top-left vertex of Triangle A is $P(-5, 3)$, and the corresponding top-left vertex of Triangle B is $P'(1, -1)$.

Step 2: Find the horizontal and vertical changes

$$p = 1 - (-5) = 6 \text{ (6 units right)}$$

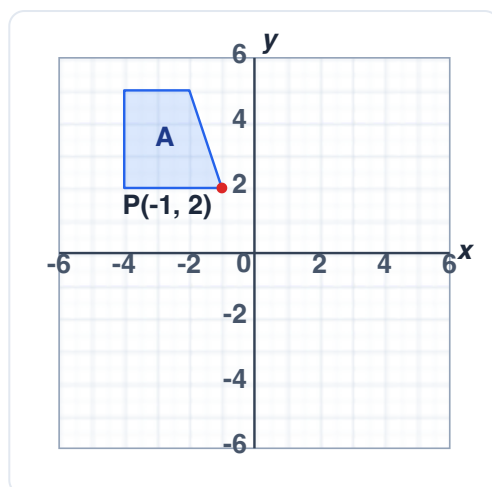
$$q = -1 - 3 = -4 \text{ (4 units down)}$$

So the translation vector is $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$.

$$\begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

EXAMPLE 1.2 **Non-Calculator**

Trapezium A is shown on the coordinate grid below with vertex $P(-1, 2)$.



Trapezium A is translated by the column vector $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ to give Trapezium B .

Find the coordinates of the image of vertex P .

STEP-BY-STEP SOLUTION:

Step 1: Interpret the column vector $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

Add 5 to the x -coordinate (move 5 units right) and subtract 4 from the y -coordinate (move 4 units down).

Step 2: Apply the shift to $P(-1, 2)$

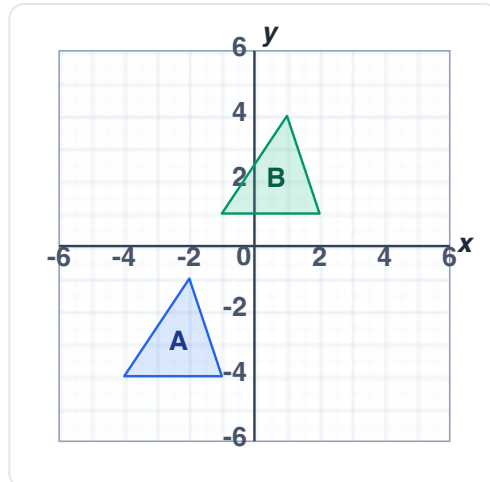
$$\begin{aligned} x' &= -1 + 5 = 4 \\ y' &= 2 + (-4) = -2 \end{aligned}$$

The image of vertex P is $(4, -2)$.

$$(4, -2)$$

EXAMPLE 1.3 **Non-Calculator**

Triangle A and Triangle B are shown on the coordinate grid below.



Write down the column vector that translates Triangle B back onto Triangle A .

STEP-BY-STEP SOLUTION:

Step 1: Pick a corner on Triangle B and the matching corner on Triangle A

The top vertex of B is $(1, 4)$ and the top vertex of A is $(-2, -1)$.

Step 2: Count from B to A

Horizontal change: $-2 - 1 = -3$ (3 units left).

Vertical change: $-1 - 4 = -5$ (5 units down).

Therefore, the vector mapping B onto A is $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

$$\begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

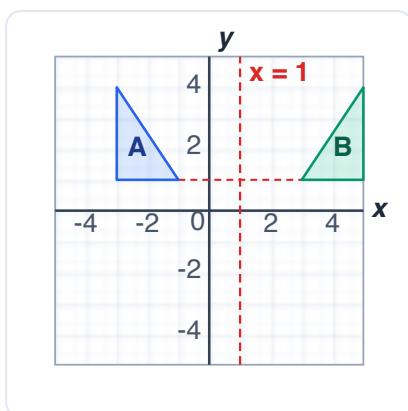
2 Reflections on a Coordinate Grid

CORE CONCEPT & METHOD

Reflections on a Coordinate Grid

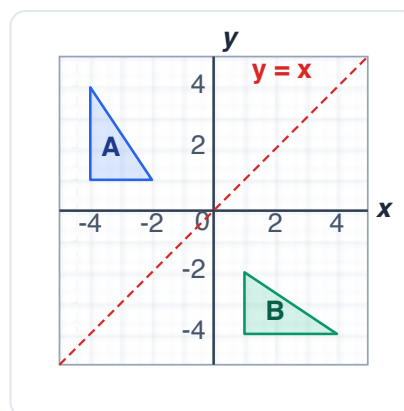
A **reflection** flips a shape across a **mirror line** (line of reflection). Every vertex and its image are the exact same perpendicular distance from the mirror line.

1. Vertical ($x = k$) & Horizontal ($y = k$) Mirror Lines



- **y-axis** has equation $x = 0$; **x-axis** has equation $y = 0$.
- Count squares **perpendicularly** from each corner to the mirror line $x = k$, then count the same number of squares to the other side.
- The mirror line is always the **midpoint** between matching corners!

2. Diagonal Mirror Lines ($y = x$ and $y = -x$)

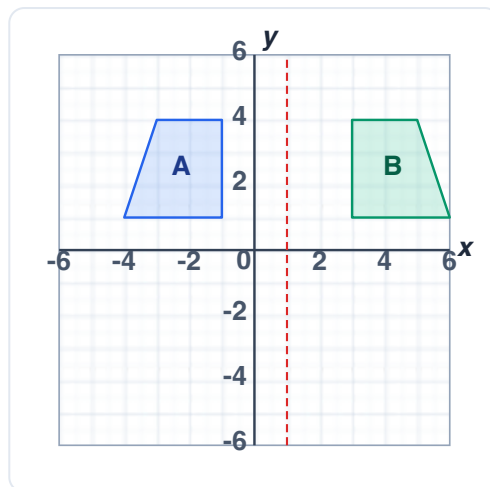


- **Reflect in $y = x$** : swap coordinates $\rightarrow (x, y) \mapsto (y, x)$.
Example: $(-4, 1) \mapsto (1, -4)$.
- **Reflect in $y = -x$** : swap and negate both $\rightarrow (x, y) \mapsto (-y, -x)$.
Example: $(-4, 1) \mapsto (-1, 4)$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 **Non-Calculator**

Shape *A* and Shape *B* are drawn on the coordinate grid below.



Write down the equation of the mirror line that reflects Shape *A* onto Shape *B*.

STEP-BY-STEP SOLUTION:**Step 1: Find the halfway point between a pair of matching vertices**

The right-hand vertical edge of Shape *A* lies on $x = -1$, and the matching left-hand vertical edge of Shape *B* lies on $x = 3$.

Step 2: Average the x -coordinates

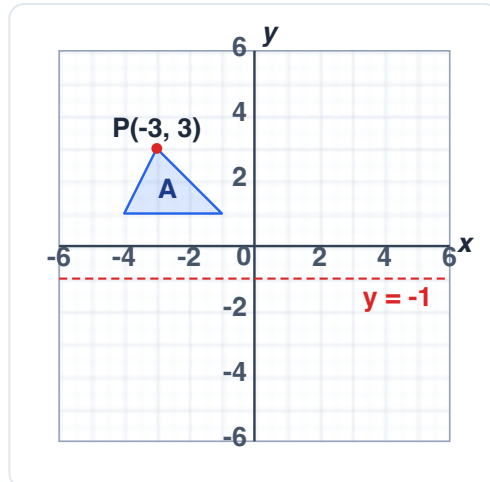
$$x = \frac{-1 + 3}{2} = 1$$

Since the mirror line is vertical and crosses the x -axis at 1, its equation is $x = 1$.

$$x = 1$$

EXAMPLE 2.2**Non-Calculator**

Triangle A is plotted on the coordinate grid below with vertex $P(-3, 3)$.



Triangle A is reflected in the horizontal line $y = -1$.
Find the coordinates of the image of vertex P .

STEP-BY-STEP SOLUTION:

Step 1: Find the perpendicular distance from $P(-3, 3)$ to the line $y = -1$

From $y = 3$ down to $y = -1$ is a vertical distance of $3 - (-1) = 4$ units.

Step 2: Move the same distance below $y = -1$

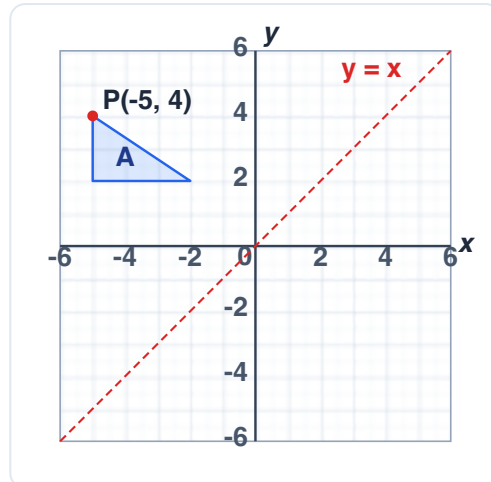
The x -coordinate stays -3 , and the new y -coordinate is $-1 - 4 = -5$.

So the image of P is $(-3, -5)$.

$(-3, -5)$

EXAMPLE 2.3 **Non-Calculator**

Triangle A is drawn on the coordinate grid below with vertex $P(-5, 4)$.



Triangle A is reflected in the diagonal line $y = x$.
Write down the coordinates of the image of vertex P .

STEP-BY-STEP SOLUTION:

Step 1: Recall the coordinate rule for reflecting in $y = x$

Reflecting a point (x, y) in the line $y = x$ swaps its x - and y -coordinates: $(x, y) \mapsto (y, x)$.

Step 2: Apply the rule to $P(-5, 4)$

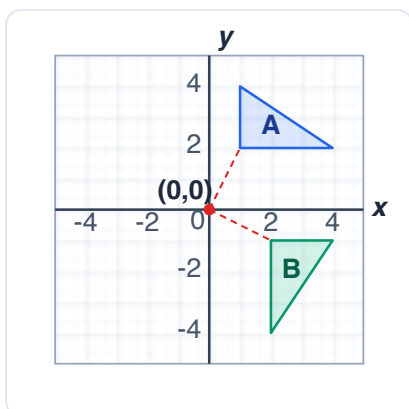
$$(-5, 4) \mapsto (4, -5)$$

$$(4, -5)$$

3 Rotations on a Coordinate Grid **CORE CONCEPT & METHOD****Rotations on a Coordinate Grid**

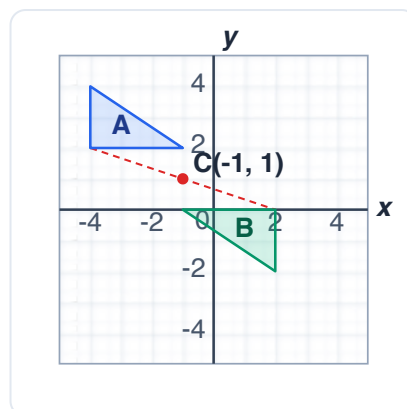
To describe a **rotation** fully in a GCSE exam, state: (1) **Rotation**, (2) **Angle** (90° or 180°), (3) **Direction** (clockwise or anticlockwise — not needed for 180°), and (4) **Centre of rotation** (a, b) .

1. Rotating Using L-Shaped Steps



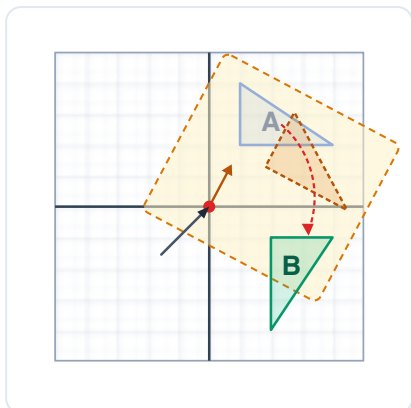
- Count the L-step from the centre C to a vertex P , then rotate that L-step:
- About $(0, 0)$:
 - 90° clockwise: $(x, y) \mapsto (y, -x)$
 - 90° anticlockwise: $(x, y) \mapsto (-y, x)$
 - 180° : $(x, y) \mapsto (-x, -y)$

2. Finding the Centre of Rotation



- **For a 180° rotation:** the centre C is simply the **midpoint** of the line joining any two corresponding vertices!
- **For a 90° rotation:** find the point C where the L-step to a vertex on A turns by 90° to reach the matching vertex on B .

3. Using Tracing Paper in the Exam

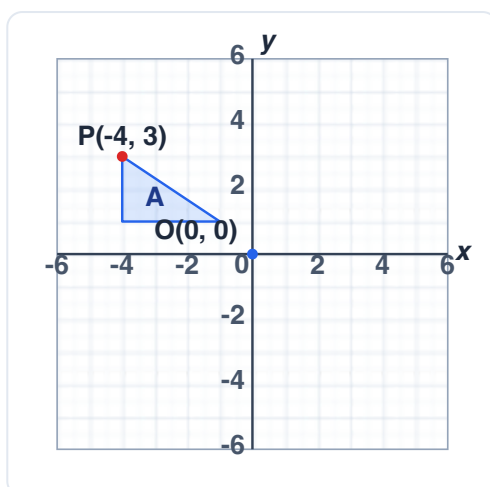


- **Step 1:** Place tracing paper over the grid, trace shape A , and draw a small upward arrow ($\uparrow$) on the centre of rotation $C(a, b)$.
- **Step 2:** Hold your pencil point firmly on $C(a, b)$ and turn the paper (90° when the arrow points right/left, 180° when it points down) to read off the new vertices.
- **Trial & Improvement:** To find an unknown centre C , trace A and test grid points with your pencil tip until turning the paper lands A directly onto B .

MODEL WORKED EXAMPLES

EXAMPLE 3.1 **Non-Calculator**

Triangle A is drawn on the coordinate grid below with vertex $P(-4, 3)$.



Triangle A is rotated 90° clockwise about the origin $(0, 0)$.
Find the coordinates of the image of vertex P .

STEP-BY-STEP SOLUTION:

Step 1: Count the L-step from $(0, 0)$ to $P(-4, 3)$

From $(0, 0)$ to $P(-4, 3)$ is 4 units left and 3 units up.

Step 2: Rotate the L-step 90° clockwise

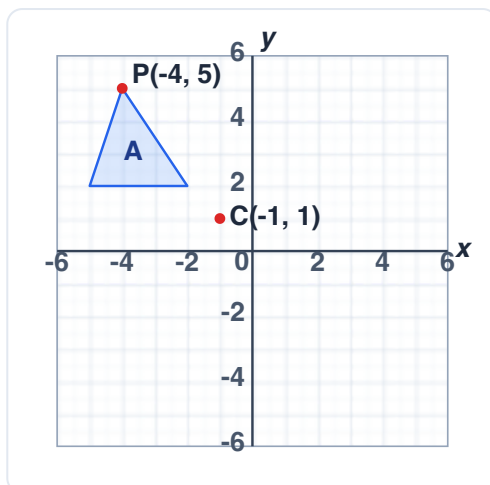
"Left 4, up 3" rotated 90° clockwise becomes "up 4, right 3", or use $(x, y) \mapsto (y, -x)$:

$$(-4, 3) \mapsto (3, 4)$$

$$(3, 4)$$

EXAMPLE 3.2 **Non-Calculator**

Triangle A is drawn on the coordinate grid below with vertex $P(-4, 5)$.



Triangle A is rotated 180° about the point $C(-1, 1)$.
Find the coordinates of the image of vertex P .

STEP-BY-STEP SOLUTION:

Step 1: Find the vector from $C(-1, 1)$ to $P(-4, 5)$

From $(-1, 1)$ to $(-4, 5)$ is 3 units left (-3) and 4 units up $(+4)$.

Step 2: Reverse the step from $C(-1, 1)$ for a 180° rotation

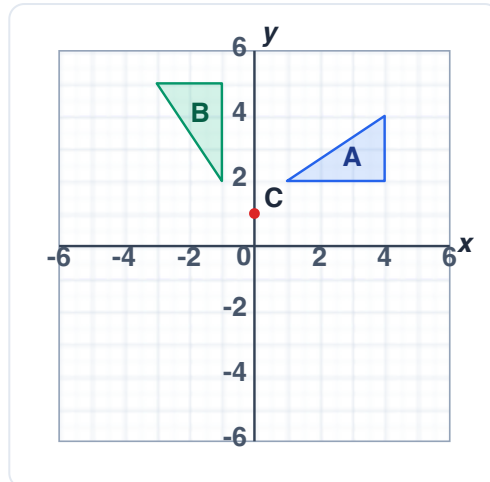
Go 3 units right $(+3)$ and 4 units down (-4) from $(-1, 1)$:

$$P' = (-1 + 3, 1 - 4) = (2, -3)$$

$$(2, -3)$$

EXAMPLE 3.3**Non-Calculator**

Triangle A and Triangle B are drawn on the coordinate grid below.



Triangle A is mapped onto Triangle B by a rotation of 90° anticlockwise.
Write down the coordinates of the centre of rotation C .

STEP-BY-STEP SOLUTION:**Step 1: Compare matching vertices of A and B**

The bottom-left vertex of A is $(1, 2)$ and the matching vertex of B is $(-1, 2)$.

Step 2: Check the L-steps from $C(0, 1)$

From $(0, 1)$ to $(1, 2)$ is 1 right and 1 up. Rotating this L-step 90° anticlockwise gives 1 up and 1 left, reaching $(-1, 2)$.

Checking $(4, 2)$: 4 right, 1 up from $(0, 1)$ rotates 90° anticlockwise to 4 up, 1 left $\rightarrow (-1, 5)$.

So the centre of rotation is $(0, 1)$.

$(0, 1)$

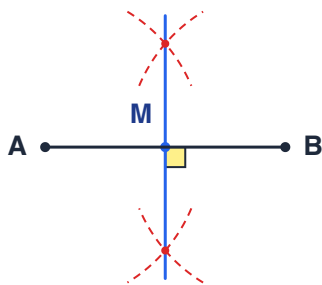
Constructions & Loci

1 Perpendicular Bisectors, Angle Bisectors & Perpendiculars

□ CORE CONCEPT & METHOD

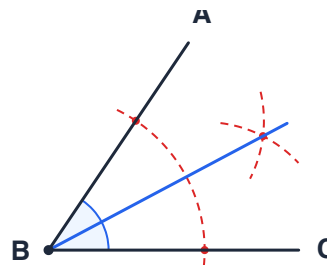
Ruler & Compass Constructions: Bisectors & Perpendiculars

1. Perpendicular Bisector of AB



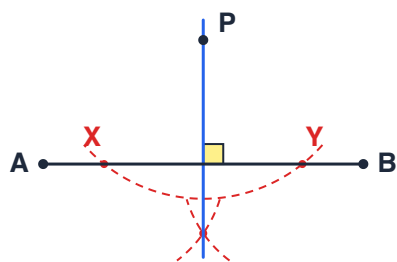
1. Open compasses to **more than half** the length of AB .
2. Place point at A and draw arcs above and below AB .
3. **Keep the exact same compass width**, place at B , and cross both arcs.
4. Join the two intersections with a ruler: this cuts AB at 90° at its exact midpoint M .

2. Angle Bisector of Angle ABC



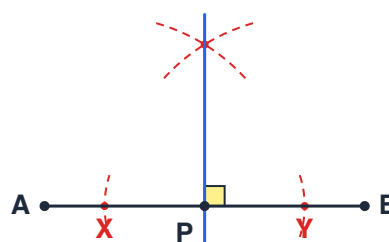
1. Place compass point on vertex B and draw an arc crossing BA and BC .
2. Place compass point on each crossing point in turn and draw two equal-radius arcs inside the angle so they intersect.
3. Draw a straight line from vertex B through the intersection to split $\angle ABC$ into two equal halves.

3. Perpendicular from Point P to AB



1. Place compass point at external point P and draw an arc crossing line AB twice at X and Y .
2. From X and Y , draw two equal-radius arcs on the opposite side of AB to intersect.
3. Join P to this intersection with a ruler to meet AB at 90° (giving the shortest distance from P to AB).

4. Perpendicular at Point P on AB



1. Place compass point on P and draw two small arcs on AB at equal distances either side (X and Y).
2. **Widen your compasses**, place at X and Y , and draw two intersecting arcs above P .
3. Join P to the intersection. **Exam Rule: Never erase your construction arcs!**

Common Mistake: NEVER rub out your compass construction arcs — examiners award method marks for visible intersecting arcs, and remember equidistant from two points A and B is the perpendicular bisector of AB .

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

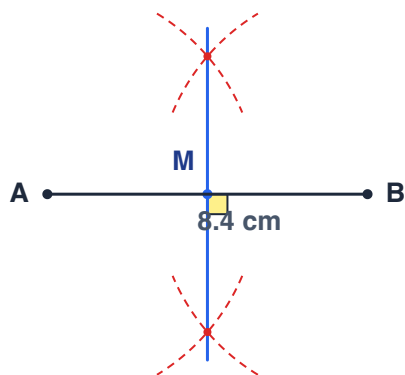
Using a ruler and a pair of compasses only, construct the perpendicular bisector of the line segment AB of length 8.4 cm.



State the distance from A to the midpoint M where the bisector crosses AB .

STEP-BY-STEP SOLUTION:

1. Open your compasses to a radius greater than half of 8.4 cm (i.e. greater than 4.2 cm, for example 5.5 cm).
2. With the compass point at A , draw arcs above and below AB . Keeping the exact same radius, place the compass point at B and draw arcs to intersect the first pair.
3. Draw a straight line through both arc intersections:



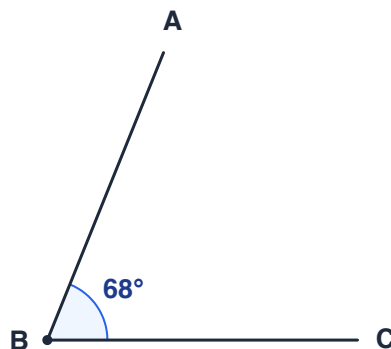
4. The perpendicular bisector crosses AB at 90° at its midpoint M , so:

$$AM = 8.4 \div 2 = 4.2 \text{ cm}$$

4.2 cm

EXAMPLE 1.2**Non-Calculator**

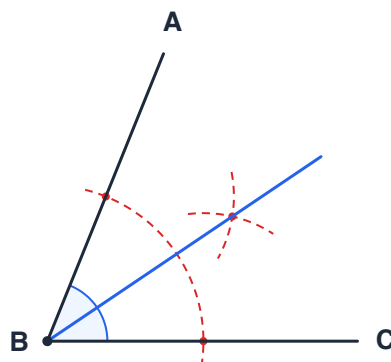
Angle $ABC = 68^\circ$. Using a ruler and a pair of compasses only, construct the angle bisector of $\angle ABC$.



Work out the size of each of the two angles formed by the bisector.

STEP-BY-STEP SOLUTION:

1. Place the compass point on vertex B and draw an arc across both arms BA and BC .
2. From the two points where the arc crosses BA and BC , draw two equal-radius arcs inside the angle so that they cross.
3. Join vertex B to the crossing point with a straight line:



4. The angle bisector divides $\angle ABC = 68^\circ$ into two equal angles:

$$68^\circ \div 2 = 34^\circ$$

$$34^\circ$$

EXAMPLE 1.3 **Non-Calculator**

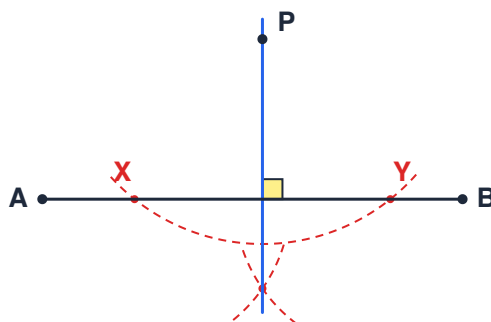
Point P lies above the straight line AB . Construct the perpendicular from P to the line AB .



What angle does this constructed line make with AB , and why must you keep the compass width unchanged when drawing the arcs from X and Y ?

STEP-BY-STEP SOLUTION:

1. Place the compass point at P and draw an arc that crosses line AB at two points X and Y (so $PX = PY$).
2. Using an unchanged compass radius from X and Y , draw two intersecting arcs below AB at Q (so $XQ = YQ$).
3. Join P to Q with a ruler:



4. Keeping the compass width fixed ensures Q is equidistant from X and Y , so line PQ is the perpendicular bisector of segment XY and meets AB at exactly 90° .

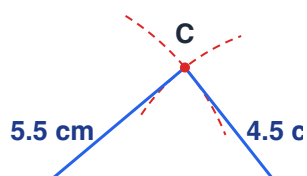
90°

2 Constructing Triangles, 60° / 30° Angles & Polygons

CORE CONCEPT & METHOD

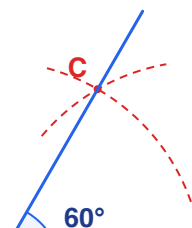
Constructing Triangles, Exact Angles & Polygons

1. SSS Triangle Construction



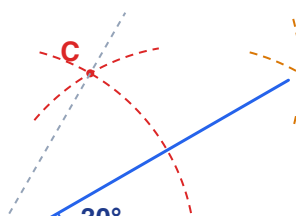
1. Draw the base AB accurately using a ruler.
2. Set your compass radius to side AC , place the point at A , and draw an arc above AB .
3. Set your compass radius to side BC , place at B , and draw an arc intersecting the first arc at C . Join AC and BC .

2. Constructing a 60° Angle



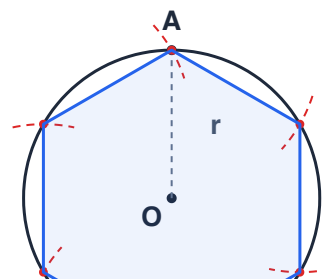
1. Place compass point at A and draw a wide arc crossing AB at X .
2. **Without changing the compass width**, place at X and draw an arc to cross the first arc at C .
3. Since $AX = AC = XC$, $\triangle AXC$ is equilateral, so $\angle CAB = 60^\circ$.

3. Constructing a 30° or 45° Angle



- **For 30° :** First construct a 60° angle (X and C), then bisect it using equal arcs from X and C ($60^\circ \div 2 = 30^\circ$).
- **For 45° :** First construct a 90° perpendicular, then bisect the 90° angle ($90^\circ \div 2 = 45^\circ$).

4. Regular Hexagon in a Circle



1. Set your compass width equal to the circle's radius $OA = r$.
2. Starting at A , step the compasses around the circumference 6 times to mark all 6 vertices.
3. Joining adjacent marks forms a regular hexagon of side length r (6 equilateral triangles).

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

The base AB of triangle ABC has been drawn of length 8 cm.

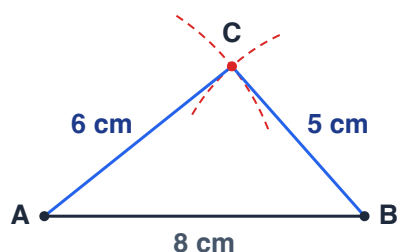
Using a ruler and compasses only, complete the construction of triangle ABC given that $AC = 6$ cm and $BC = 5$ cm.



Work out the perimeter of the completed triangle ABC .

STEP-BY-STEP SOLUTION:

1. Open your compasses to 6 cm, place the point at A , and draw an arc above AB .
2. Open your compasses to 5 cm, place the point at B , and draw an arc intersecting the first arc at C .
3. Join A to C and B to C with a straight ruler:



4. The perimeter of $\triangle ABC$ is:

$$8 + 6 + 5 = 19 \text{ cm}$$

19 cm

EXAMPLE 2.2**Non-Calculator**

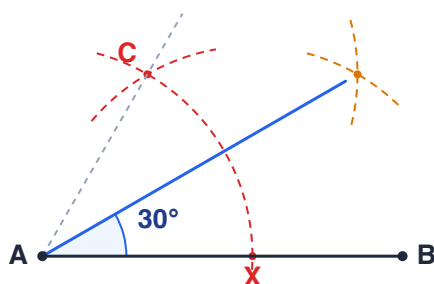
Using a ruler and a pair of compasses only, construct an angle of 30° at point A on the line segment AB .



What angle must you construct first before bisecting it to obtain 30° ?

STEP-BY-STEP SOLUTION:

1. First construct an equilateral-triangle arc of 60° at A : draw an arc from A crossing AB at X , then with the same radius draw an arc from X crossing the first arc at C ($\angle CAB = 60^\circ$).
2. Next, bisect the 60° angle by drawing two equal-radius arcs from X and C to cross at D , and join AD :

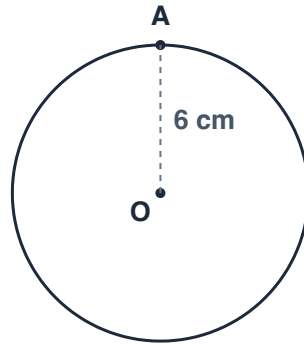


3. Because $60^\circ \div 2 = 30^\circ$, you must first construct a 60° angle.

60°

EXAMPLE 2.3**Non-Calculator**

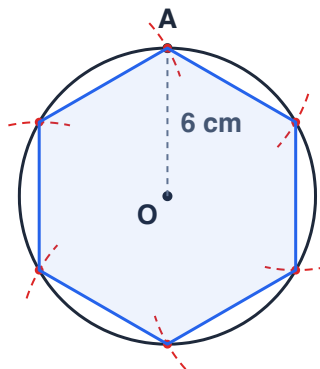
A circle with centre O has radius $OA = 6$ cm. Using compasses only, a regular hexagon $ABCDEF$ is inscribed inside the circle by stepping the radius around the circumference.



Find the total perimeter of the inscribed regular hexagon.

STEP-BY-STEP SOLUTION:

1. Keep the compass radius fixed at $r = 6$ cm and step around the circumference from A to mark B, C, D, E, F :



2. Because the hexagon is made of 6 equilateral triangles meeting at O , each side of the regular hexagon is equal to the radius (6 cm).

3. Perimeter of the regular hexagon:

$$6 \times 6 \text{ cm} = 36 \text{ cm}$$

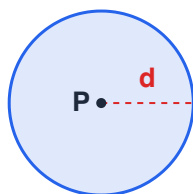
36 cm

3 Loci & Shading Regions on Scale Drawings

CORE CONCEPT & METHOD

The 4 Fundamental Loci Rules & Scale Region Shading

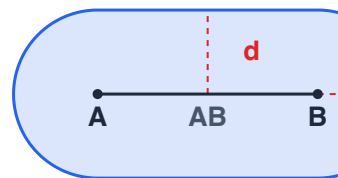
1. Fixed Distance from a Point P



The locus of points a fixed distance d from a single point P is a **circle of radius d centred at P** .

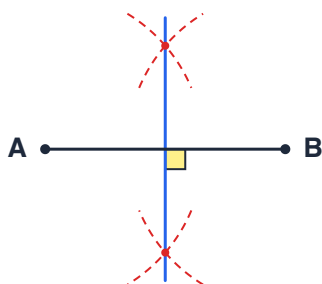
- *Within d of P* means inside the circle.

2. Fixed Distance from a Line Segment AB



The locus of points a distance d from a finite line segment AB consists of **two parallel lines** of distance d joined by **two semicircular ends** of radius d (a stadium shape).

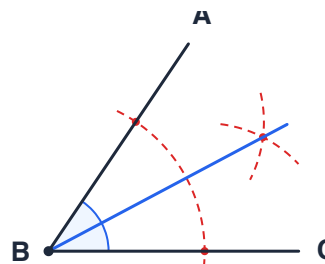
3. Equidistant from Two Points A & B



The locus of points equidistant from two points A and B is the **perpendicular bisector of AB** .

- Points *closer to A than B* lie on A 's side of the bisector.

4. Equidistant from Two Intersecting Lines



The locus of points equidistant from two intersecting lines BA and BC is the **angle bisector of $\angle ABC$** .

- Points *closer to BA than BC* lie on BA 's side of the bisector.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Non-Calculator

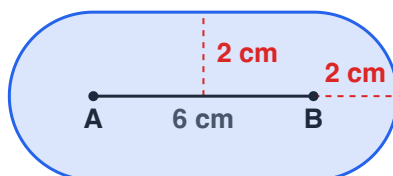
Line segment AB has length 6 cm. Construct and shade the locus of all points that are within 2 cm of the line segment AB .



Calculate the total area of this region in terms of π .

STEP-BY-STEP SOLUTION:

1. Draw two straight lines parallel to AB at a perpendicular distance of 2 cm above and below AB (forming a $6 \text{ cm} \times 4 \text{ cm}$ rectangle).
2. With compass point at A and B and radius 2 cm, draw two semicircular ends:



3. Total area = rectangle area + two semicircles (one full circle of radius 2 cm):

$$\text{Area} = (6 \times 4) + \pi \times 2^2 = 24 + 4\pi \text{ cm}^2$$

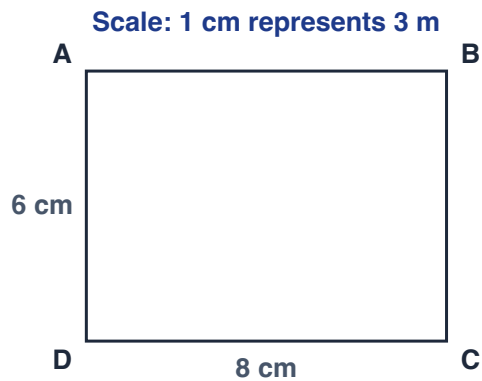
$$24 + 4\pi \text{ cm}^2$$

EXAMPLE 3.2**Non-Calculator**

The scale drawing shows a rectangular courtyard $ABCD$ measuring 8 cm by 6 cm, where 1 cm represents 3 m.

A fountain is to be placed inside the courtyard so that it is:

- closer to side AB than side DC , and
- within 12 m of corner A .



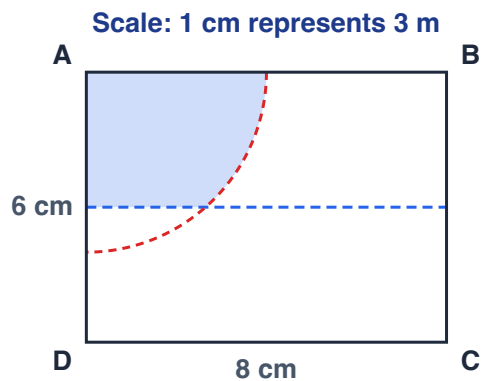
Find the compass radius (in cm) needed on the scale drawing for the arc centred at A .

STEP-BY-STEP SOLUTION:

1. Convert the real-life distance 12 m using the scale 1 cm : 3 m:

$$\text{Compass radius} = 12 \div 3 = 4 \text{ cm}$$

2. Construct the perpendicular bisector of AD (horizontal line 3 cm below AB) and draw a quarter-circle arc of radius 4 cm centred at A :



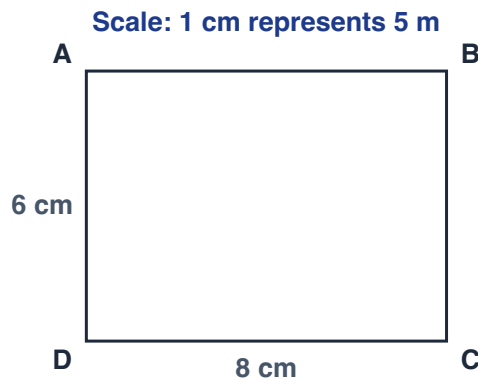
3. Shade the region above the bisector and inside the 4 cm arc from A .

4 cm

EXAMPLE 3.3**Non-Calculator**

On the scale drawing of a park $ABCD$ (8 cm by 6 cm, scale 1 cm represents 5 m), a cafe can be built where it is:

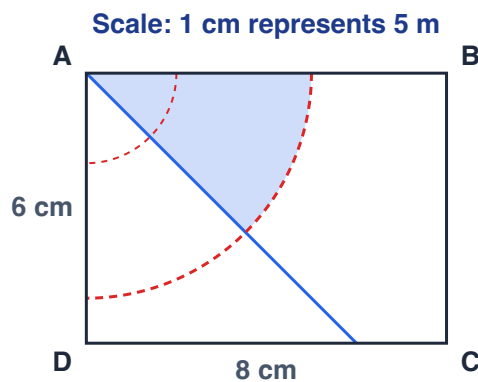
- closer to boundary AB than boundary AD , and
- less than 25 m from corner A .



Construct and shade the region, and state the angle (in degrees) of the sector formed inside corner A .

STEP-BY-STEP SOLUTION:

1. Points equidistant from AB and AD lie on the angle bisector of $\angle BAD$. Since $\angle BAD = 90^\circ$, the angle bisector is at 45° to AB .
2. Using the scale 1 cm : 5 m, 25 m is represented by $25 \div 5 = 5$ cm on the drawing.
3. Draw the 45° angle bisector from A and an arc of radius 5 cm centred at A , then shade the sector between AB and the bisector:



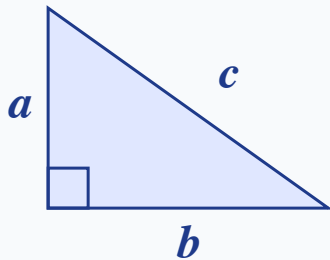
4. The shaded region is a sector of angle $90^\circ \div 2 = 45^\circ$.

45°

Pythagoras' Theorem

1 Finding the Hypotenuse

📖 CORE CONCEPT & METHOD



To find lengths of sides for right-angled triangles, use these two rules.

Note: c is always the longest side, called the **hypotenuse** (even though our memory trick calls it the Diagonal!).

D.S.A. (To find the long side)

Diagonal Side Add.

$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

S.S.S. (To find a short side)

Short Side Subtract.

$$a^2 = c^2 - b^2 \rightarrow a = \sqrt{c^2 - b^2}$$

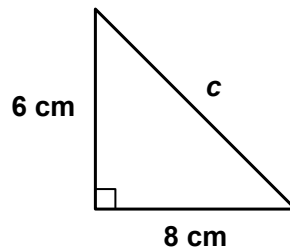
Common Mistake: Check whether you are finding the longest side (Hypotenuse: square and ADD, $a^2 + b^2$) or a shorter leg (square and SUBTRACT, $c^2 - a^2$), and halve the base of an isosceles triangle first.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

A right-angled triangle has legs of length 6 cm and 8 cm. Find the length of the hypotenuse c .



STEP-BY-STEP SOLUTION:

We need to find the hypotenuse, so we **square and add**:

$$c^2 = 6^2 + 8^2 = 36 + 64 = 100$$

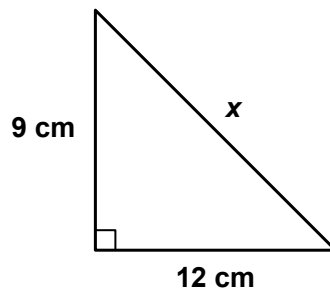
Square root to find c :

$$c = \sqrt{100} = 10 \text{ cm}$$

EXAMPLE 1.2

Non-Calculator

Calculate the length of x in a right-angled triangle with shorter sides 9 cm and 12 cm.



STEP-BY-STEP SOLUTION:

Square and add the two shorter sides:

$$x^2 = 9^2 + 12^2 = 81 + 144 = 225$$

Square root to find x :

$$x = \sqrt{225} = 15 \text{ cm}$$

2 Finding a Shorter Side

CORE CONCEPT & METHOD

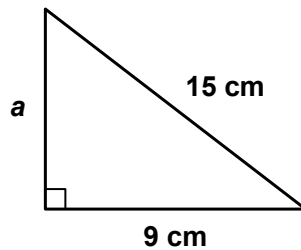
To find a shorter side, you must subtract instead! Square the hypotenuse, square the given shorter side, **subtract** the smaller square from the larger square, and then square root the answer.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

A right-angled triangle has a hypotenuse of 15 cm and one shorter side of 9 cm. Find the length of the other shorter side a .



STEP-BY-STEP SOLUTION:

We need a shorter side, so we **square and subtract**:

$$a^2 = 15^2 - 9^2 = 225 - 81 = 144$$

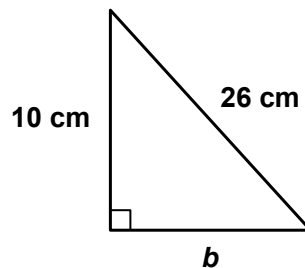
Square root to find a :

$$a = \sqrt{144} = 12 \text{ cm}$$

EXAMPLE 2.2

 Non-Calculator

The hypotenuse of a right-angled triangle is 26 cm and one leg is 10 cm. Find the missing leg b .



STEP-BY-STEP SOLUTION:

Square the hypotenuse, then subtract the square of the known leg:

$$b^2 = 26^2 - 10^2 = 676 - 100 = 576$$

$$b = \sqrt{576} = 24 \text{ cm}$$

3 Problem Solving

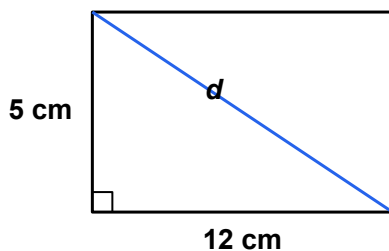
CORE CONCEPT & METHOD

Look for hidden right-angled triangles in shapes like rectangles, isosceles triangles, or real-life problems (like ladders against walls).

MODEL WORKED EXAMPLES

EXAMPLE 3.1 **Non-Calculator**

A rectangle has width 12 cm and height 5 cm. Find the length of the diagonal.

**STEP-BY-STEP SOLUTION:**

The diagonal of a rectangle splits it into two right-angled triangles.

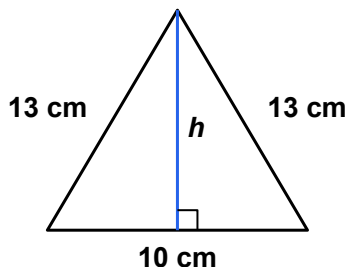
Square and add the width and height:

$$d^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$d = \sqrt{169} = 13 \text{ cm}$$

EXAMPLE 3.2 **Non-Calculator**

An isosceles triangle has equal sides of 13 cm and a base of 10 cm. Find the perpendicular height.

**STEP-BY-STEP SOLUTION:**

The height splits the triangle into two right-angled triangles. The base of each small triangle is $10 \div 2 = 5$ cm.

Use Pythagoras to find the height h :

$$h^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$h = \sqrt{144} = 12 \text{ cm}$$

EXAMPLE 3.3 **Non-Calculator**

ABC and ACD are right-angled triangles sharing the common side AC . Calculate the length of AD .

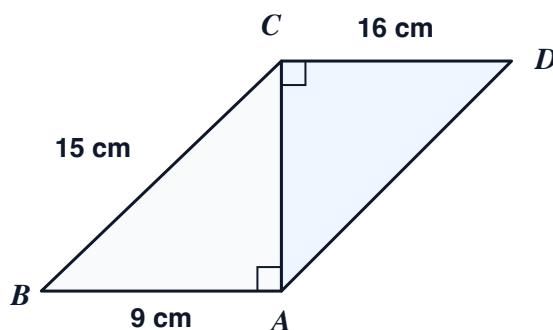


Diagram NOT accurately drawn

STEP-BY-STEP SOLUTION:

Step 1: Use right-angled triangle ABC to find the shared vertical side AC :

$$AC^2 = 15^2 - 9^2 = 225 - 81 = 144 \implies AC = 12 \text{ cm}$$

Step 2: Now use right-angled triangle ACD to find the hypotenuse AD :

$$AD^2 = 12^2 + 16^2 = 144 + 256 = 400$$

$$AD = \sqrt{400} = 20 \text{ cm}$$

EXAMPLE 3.4 **Non-Calculator**

$ABCD$ is a right-angled trapezium. Calculate the length of BC .

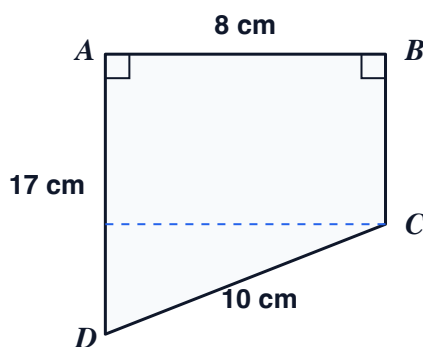


Diagram NOT accurately drawn

STEP-BY-STEP SOLUTION:

Step 1: Draw a horizontal line from C across to AD , splitting the trapezium into a rectangle of width 8 cm and a bottom right-angled triangle with hypotenuse $CD = 10$ cm.

Step 2: Find the vertical leg h of the bottom right-angled triangle:

$$h^2 = 10^2 - 8^2 = 100 - 64 = 36 \implies h = 6 \text{ cm}$$

Step 3: Subtract h from AD to find BC :

$$BC = 17 - 6 = 11 \text{ cm}$$

Surface Area

1 Basic Prisms

📖 CORE CONCEPT & METHOD

To find the **Total Surface Area** of a 3D shape, find the area of *every single face* and add them all together.

A cuboid has 6 rectangular faces (3 identical pairs: Front/Back, Left/Right, Top/Bottom). A triangular prism has 2 triangular faces and 3 rectangular faces.

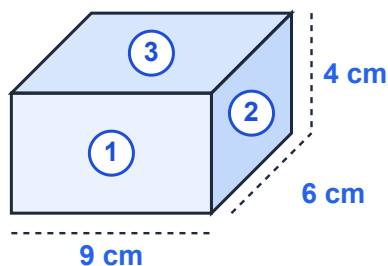
Common Mistake: Do not confuse Volume with Surface Area — for total surface area, calculate the 2D area of EVERY individual face (6 faces on a cuboid, 5 on a triangular prism) and add them together.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

A cuboid has length 9 cm, width 6 cm and height 4 cm. Find its total surface area.



STEP-BY-STEP SOLUTION:

A cuboid has 3 pairs of identical rectangular faces:

$$SA = 2(\text{Area } \textcircled{1} + \text{Area } \textcircled{2} + \text{Area } \textcircled{3})$$

• Area ① (Front): $9 \times 4 = 36 \text{ cm}^2$

• Area ② (Side): $6 \times 4 = 24 \text{ cm}^2$

• Area ③ (Top): $9 \times 6 = 54 \text{ cm}^2$

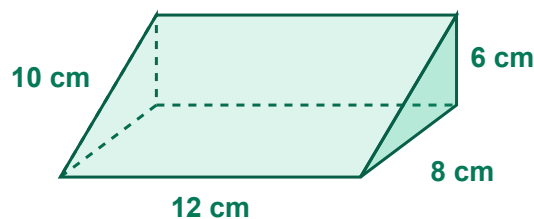
Sum of 3 faces: $36 + 24 + 54 = 114 \text{ cm}^2$

Multiply by 2 (for opposite faces): $114 \times 2 = 228 \text{ cm}^2$

$$SA = 2(36 + 24 + 54) = 2 \times 114 = 228 \text{ cm}^2$$

EXAMPLE 1.2 **Non-Calculator**

A triangular prism has a right-angled cross-section with a base of 8 cm and height of 6 cm (the hypotenuse is 10 cm). The prism is 12 cm long. Find the total surface area.

**STEP-BY-STEP SOLUTION:**

Identify all 5 faces:

2 triangular ends:

$$2 \times \frac{1}{2}(8 \times 6) = 48 \text{ cm}^2$$

3 rectangular faces (base, vertical side, slope):

$$\text{Base: } 8 \times 12 = 96$$

$$\text{Vertical Side: } 6 \times 12 = 72$$

$$\text{Slope: } 10 \times 12 = 120$$

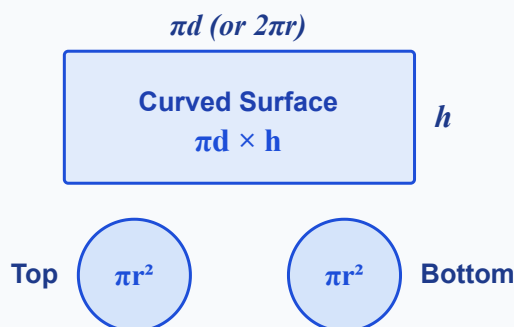
$$\text{Total Rectangles} = 96 + 72 + 120 = 288 \text{ cm}^2$$

$$\text{Total SA} = 48 + 288 = 336 \text{ cm}^2$$

2 Cylinders & Contexts **CORE CONCEPT & METHOD**

Total Surface Area of a Solid Cylinder = $2\pi r^2 + \pi dh$ (or $2\pi r^2 + 2\pi rh$).

When you unroll the net of a cylinder, it is made of **two circular ends** (Top and Bottom, each πr^2) and a **curved surface** that unrolls into a rectangle of width πd (the circumference) and height h :



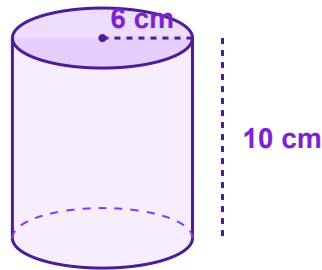
Watch out for questions where you have to paint multiple items or convert between cm^2 and m^2 (remember, $1 \text{ m}^2 = 10,000 \text{ cm}^2$!).

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Calculator

A solid cylinder has radius 6 cm and height 10 cm. Find the total surface area. Give your answer to 1 decimal place.



STEP-BY-STEP SOLUTION:

Using $SA = 2\pi r^2 + 2\pi rh$:

Two circular ends (Top + Bottom): $2 \times \pi \times 6^2 = 72\pi$

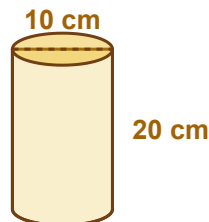
Curved surface ($\pi d \times h$): $2 \times \pi \times 6 \times 10 = 120\pi$

$$SA = 72\pi + 120\pi = 192\pi \approx 603.2 \text{ cm}^2$$

EXAMPLE 2.2

 Calculator

A sealed metal canister has diameter 10 cm and height 20 cm. 4 of these canisters are to be coated with lacquer. Each tin of lacquer covers 1 m^2 . How many tins of lacquer are needed?



STEP-BY-STEP SOLUTION:

Radius $r = 5 \text{ cm}$. Total SA of one canister (top, bottom, curved):

$$SA = 2\pi(5^2) + \pi(10)(20)$$

$$SA = 50\pi + 200\pi = 250\pi \approx 785.4 \text{ cm}^2$$

Convert to m^2 : $785.4 \div 10,000 = 0.07854 \text{ m}^2$

4 canisters: $0.07854 \times 4 = 0.31416 \text{ m}^2$

$$0.31416 \div 1 = 0.31416 \Rightarrow \mathbf{1 \text{ tin needed}}$$

3 Compound Prisms

CORE CONCEPT & METHOD

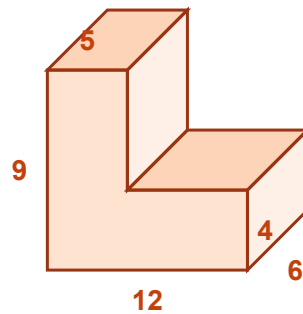
For prisms with complex cross-sections (like L-shapes or trapeziums), you can either add every rectangular side individually, or use the perimeter shortcut:

Total Surface Area = $(2 \times \text{Area of Cross Section}) + (\text{Perimeter of Cross Section} \times \text{Length})$

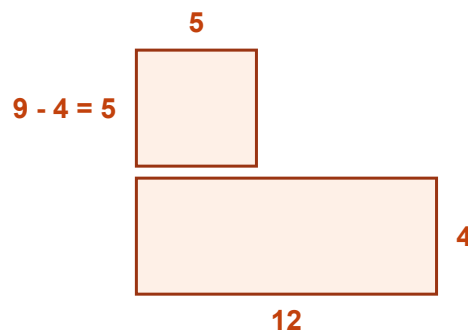
MODEL WORKED EXAMPLES

EXAMPLE 3.1**Non-Calculator**

An L-shaped prism has the dimensions shown below. The depth of the prism is 6 cm. Calculate its total surface area.

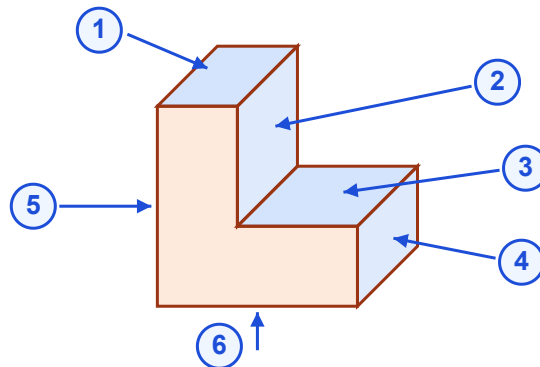
**STEP-BY-STEP SOLUTION:**

1. Find the area of the front L-shaped face:
Split it into two rectangles:



Top Area: $5 \times (9 - 4) = 25 \text{ cm}^2$ | Bottom Area: $12 \times 4 = 48 \text{ cm}^2$
Total Front Area = $25 + 48 = 73 \text{ cm}^2$. Back face is identical = 73 cm^2 .

2. Find the area of the 6 rectangular faces wrapping around it:



- **Face ① (Top):** $5 \times 6 = 30 \text{ cm}^2$
- **Face ② (Inner Vertical):** $(9 - 4) \times 6 = 5 \times 6 = 30 \text{ cm}^2$
- **Face ③ (Inner Horizontal Step):** $(12 - 5) \times 6 = 7 \times 6 = 42 \text{ cm}^2$
- **Face ④ (Right Side):** $4 \times 6 = 24 \text{ cm}^2$
- **Face ⑤ (Left Side):** $9 \times 6 = 54 \text{ cm}^2$
- **Face ⑥ (Bottom):** $12 \times 6 = 72 \text{ cm}^2$

3. Add them all together:

Front & Back ends: $73 + 73 = 146 \text{ cm}^2$

6 wrap-around rectangles: $30 + 30 + 42 + 24 + 54 + 72 = 252 \text{ cm}^2$

Total SA = $146 + 252 = 398 \text{ cm}^2$

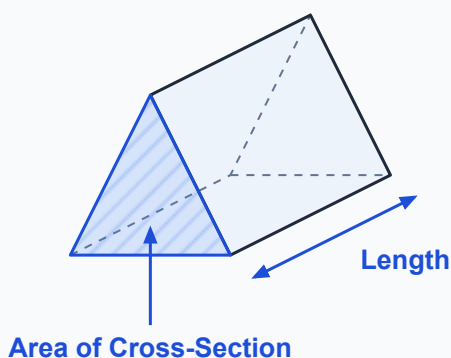
Volume of Prisms

1 Volume of Prisms

□ CORE CONCEPT & METHOD

The volume of **any** prism is exactly: **Volume = Area of Cross-Section × Length.**

First find the area of the shape on the front (the cross-section), then multiply it by how far back the shape goes.



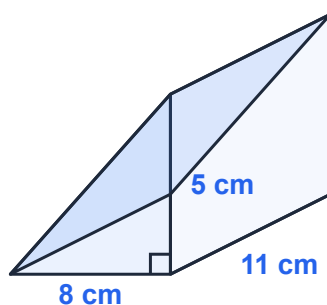
Common Mistake: For a triangular prism, remember to HALVE base × height for the triangular cross-section before multiplying by the prism length, and use 1 litre = 1,000 cm³.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

✎ Non-Calculator

A triangular prism has a right-angled triangular cross-section with base 8 cm and height 5 cm. The prism is 11 cm long. Find the volume.



STEP-BY-STEP SOLUTION:

Step 1: Find the area of the cross-section (the triangle).

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

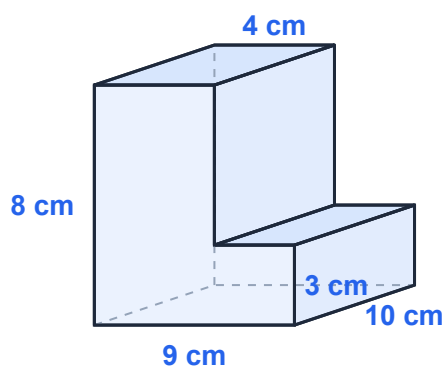
$$\text{Area} = \frac{1}{2} \times 8 \times 5 = 20 \text{ cm}^2$$

Step 2: Multiply by the length (depth) of the prism.

$$\begin{aligned} \text{Volume} &= \text{Area} \times \text{length} \\ \text{Volume} &= 20 \times 11 = 220 \text{ cm}^3 \end{aligned}$$

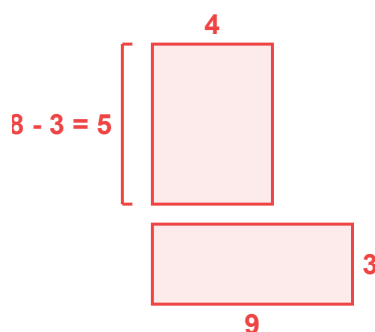
EXAMPLE 1.2 **Non-Calculator**

An L-shaped prism has a cross-section made from two rectangles: a 9 cm × 3 cm piece and a 4 cm × 5 cm piece. The prism is 10 cm long. Find the volume.

**STEP-BY-STEP SOLUTION:**

Step 1: Find the total cross-sectional area.

The L-shape is split into two rectangles:



$$\text{Bottom piece} = 9 \times 3 = 27 \text{ cm}^2$$

$$\text{Top piece} = 4 \times 5 = 20 \text{ cm}^2$$

$$\text{Total Area} = 27 + 20 = 47 \text{ cm}^2$$

Step 2: Multiply by the length.

$$\text{Volume} = \text{Area} \times \text{length}$$

$$\text{Volume} = 47 \times 10 = 470 \text{ cm}^3$$

2 Working Backwards **CORE CONCEPT & METHOD**

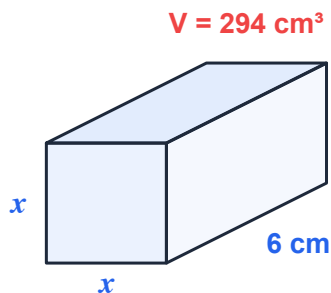
If you are given the Volume, set up your equation: **Area × Length = Volume** and solve backwards to find the missing side x .

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator

A cuboid has a square cross-section of side x cm and a length of 6 cm. Its volume is 294 cm^3 . Find x .



STEP-BY-STEP SOLUTION:

Step 1: Write an expression for the volume.

$$\text{Area of cross-section} = x \times x = x^2$$

$$\text{Volume} = \text{Area} \times \text{length} = x^2 \times 6 = 6x^2$$

Step 2: Set equal to the given volume and solve.

$$6x^2 = 294$$

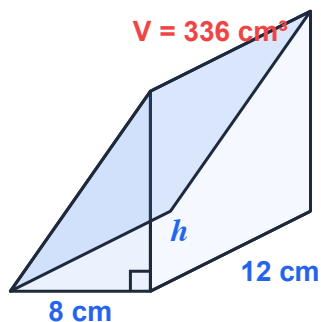
$$x^2 = 294 \div 6 = 49$$

$$x = \sqrt{49} = 7 \text{ cm}$$

EXAMPLE 2.2

Non-Calculator

A triangular prism has a right-angled triangular cross-section with base 8 cm and height h cm. The prism is 12 cm long and has volume 336 cm^3 . Find h .



STEP-BY-STEP SOLUTION:

Step 1: Write an expression for the volume.

$$\text{Area of cross-section} = \frac{1}{2} \times 8 \times h = 4h$$

$$\text{Volume} = 4h \times 12 = 48h$$

Step 2: Set equal to the given volume and solve.

$$48h = 336$$

$$h = 336 \div 48 = 7 \text{ cm}$$

3 Capacity and Contexts

CORE CONCEPT & METHOD

Important conversion: **1 Litre = $1000 \text{ cm}^3 = 1000 \text{ ml}$.**

When packing small boxes into a large container, find out how many boxes fit along the length, width, and height, and multiply those numbers together!

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 **Non-Calculator**

A rectangular glass aquarium measures 50 cm by 30 cm by 40 cm. How many litres of water can it hold?

STEP-BY-STEP SOLUTION:

Step 1: Find the volume in cm^3 .

$$\text{Volume} = 50 \times 30 \times 40 = 60,000 \text{ cm}^3$$

Step 2: Convert to Litres.

Remember that $1000 \text{ cm}^3 = 1 \text{ Litre}$.

$$60,000 \div 1000 = 60 \text{ Litres}$$

Cylinders

1 Cylinder Basics

□ CORE CONCEPT & METHOD

Volume of a Cylinder: $V = \pi r^2 h$

Total Surface Area: $SA = 2\pi r^2 + 2\pi r h$

(Two flat circular ends + the curved rectangular wrapper).

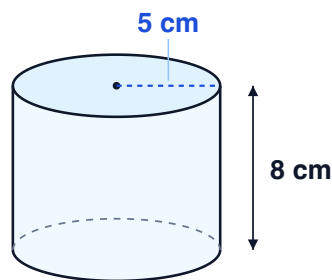
Common Mistake: If given the diameter, HALVE it to get the radius r before using $V = \pi r^2 h$, and for a half-cylinder's surface area remember to include the flat rectangular top face.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

A cylinder has a radius of 5 cm and a height of 8 cm. Find its volume in terms of π .



STEP-BY-STEP SOLUTION:

Step 1: Write down the formula for the volume of a cylinder.

$$\text{Volume} = \pi r^2 h$$

Step 2: Substitute the radius and height into the formula.

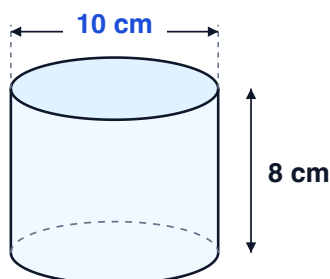
The radius $r = 5$ and height $h = 8$.

$$\text{Volume} = \pi \times 5^2 \times 8$$

$$\text{Volume} = \pi \times 25 \times 8 = 200\pi \text{ cm}^3$$

EXAMPLE 1.2

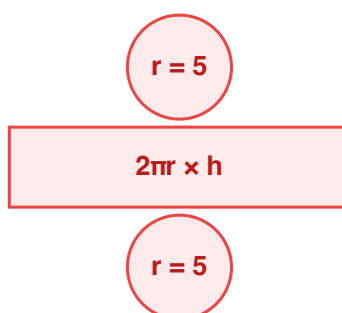
A cylinder has a diameter of 10 cm and a height of 8 cm. Calculate its total surface area to 1 d.p.

**STEP-BY-STEP SOLUTION:****Step 1: Find the radius.**

The diameter is 10, so the radius $r = 10 \div 2 = 5$ cm.

Step 2: Understand the Surface Area components.

The total surface area is made of two flat circular ends and one curved rectangular wrapper.



$$SA = 2\pi r^2 + 2\pi r h$$

Step 3: Substitute and solve.

$$SA = 2\pi(5^2) + 2\pi(5)(8)$$

$$\begin{aligned} SA &= 2\pi(25) + 80\pi \\ SA &= 50\pi + 80\pi = 130\pi \\ 130 \times \pi &\approx 408.4 \text{ cm}^2 \end{aligned}$$

2 Working Backwards & Composites**□ CORE CONCEPT & METHOD**

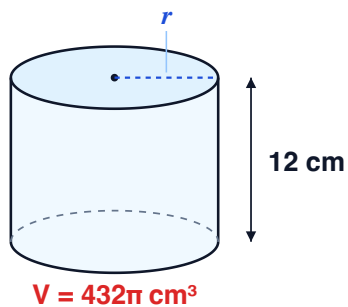
If you are given the Volume or Surface Area, set up an equation with r or h as the unknown and solve backwards.

For a **Semi-Cylinder**, half the circular volume/area, but remember to add the new flat rectangular face!

MODEL WORKED EXAMPLES

EXAMPLE 2.1 **Calculator**

A cylinder has a volume of $432\pi \text{ cm}^3$ and a height of 12 cm. Find its radius.

**STEP-BY-STEP SOLUTION:**

Step 1: Set up the volume equation.

$$\text{Volume} = \pi r^2 h$$

Substitute the given volume and height:

$$432\pi = \pi r^2 (12)$$

Step 2: Solve backwards for r .

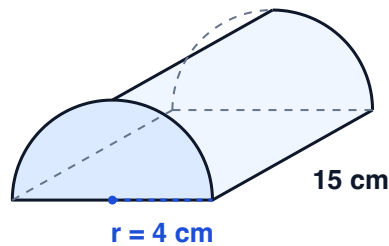
Divide both sides by 12π :

$$r^2 = \frac{432\pi}{12\pi} = 36$$

$$r = \sqrt{36} = 6 \text{ cm}$$

EXAMPLE 2.2 **Non-Calculator**

A solid cylinder is cut in half to form a semi-cylinder with a radius of 4 cm and a length of 15 cm. Find its volume in terms of π .

**STEP-BY-STEP SOLUTION:**

Step 1: Find the volume of a full cylinder.

If it were a full cylinder with radius 4 and height 15:

$$\text{Full Volume} = \pi \times 4^2 \times 15$$

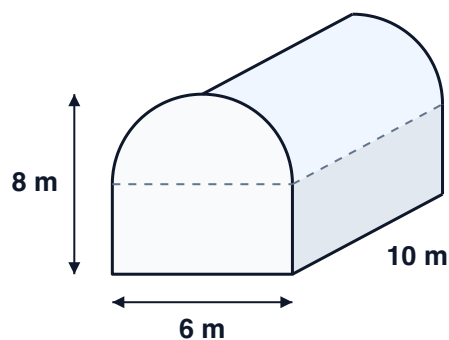
$$\text{Full Volume} = \pi \times 16 \times 15 = 240\pi \text{ cm}^3$$

Step 2: Halve it for a semi-cylinder.

$$\text{Semi-cylinder Volume} = \frac{240\pi}{2} = 120\pi \text{ cm}^3$$

EXAMPLE 2.3 **Calculator**

A solid is formed by placing a half-cylinder on top of a rectangular prism. The solid has a width of 6 m, a total height of 8 m, and a length of 10 m. Work out the total volume to 3 s.f.

**STEP-BY-STEP SOLUTION:**

Step 1: Find the radius and prism height.

Width = 6 m $\implies$ radius of half-cylinder $r = 3$ m.

Height of rectangular prism = $8 - 3 = 5$ m.

Step 2: Add the volumes of the two parts.

$$\text{Prism Volume} = 6 \times 5 \times 10 = 300 \text{ m}^3$$

$$\text{Half-Cylinder Volume} = \frac{1}{2}\pi(3^2)(10) = 45\pi \approx 141.37 \text{ m}^3$$

$$\text{Total Volume} = 300 + 141.37 = 441 \text{ m}^3 \text{ (3 s.f.)}$$

3 Real-world Contexts

□ CORE CONCEPT & METHOD

Remember that $1000 \text{ cm}^3 = 1 \text{ Litre}$.

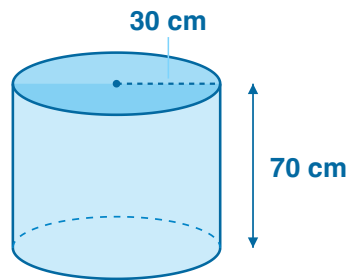
When converting Areas: $1 \text{ m}^2 = 10,000 \text{ cm}^2$ (because $100 \times 100 = 10,000$).

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 **Calculator**

A cylindrical water barrel has a radius of 30 cm and a height of 70 cm. How many Litres of water can it hold? Give your answer to 1 decimal place.



STEP-BY-STEP SOLUTION:

Step 1: Find the volume in cm^3 .

$$\text{Volume} = \pi r^2 h$$

$$\text{Volume} = \pi \times 30^2 \times 70$$

$$\text{Volume} = \pi \times 900 \times 70 = 63,000\pi \text{ cm}^3$$

$$63,000 \times \pi \approx 197,920.337... \text{ cm}^3$$

Step 2: Convert to Litres.

Remember that $1000 \text{ cm}^3 = 1 \text{ Litre}$.

$$197,920.337... \div 1000 \approx 197.9 \text{ Litres (to 1 d.p.)}$$

Averages from Tables

1 Discrete Frequency Tables

📖 CORE CONCEPT & METHOD

To find the mean from a frequency table, you cannot just add up the left column! You must multiply the value by its frequency ($x \times f$) to find the total sum for each row.

$$\text{Mean} = \frac{\text{Total Sum of all rows } (\Sigma fx)}{\text{Total Frequency } (\Sigma f)}$$

Common Mistake: Never divide by the number of rows in the table — always divide the sum of (frequency $\times$ midpoint) by the TOTAL FREQUENCY Σf , and never use midpoints to state the modal class or median class.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Calculator**

The table below shows the number of books read per month by a group of students. Work out the mean.

Books (x)	Frequency (f)
0	4
1	9
2	5
3	2

STEP-BY-STEP SOLUTION:

1. Add an $f \times x$ column to the table:

Books (x)	Frequency (f)	$f \times x$
0	4	$0 \times 4 = 0$
1	9	$1 \times 9 = 9$
2	5	$2 \times 5 = 10$
3	2	$3 \times 2 = 6$
Totals:	$\Sigma f = 20$	$\Sigma fx = 25$

2. Calculate the mean:

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{25}{20} = 1.25$$

EXAMPLE 1.2 **Non-Calculator**

From the same table, write down the **mode**.

Books (x)	Frequency (f)
0	4
1	9
2	5
3	2

STEP-BY-STEP SOLUTION:

Step 1 – The mode is the value with the *highest frequency*.

Step 2 – Looking at the table, frequency 9 is the highest, and it belongs to **1 book**:

Books (x)	Frequency (f)
0	4
1	9 ← Highest frequency
2	5
3	2

Mode = 1 book.

EXAMPLE 1.3 **Non-Calculator**

From the same table, find the **median**.

Books (x)	Frequency (f)
0	4
1	9
2	5
3	2

STEP-BY-STEP SOLUTION:

Step 1 – Find the total frequency: $\Sigma f = 4 + 9 + 5 + 2 = 20$.

Step 2 – Find the median position: $(20 + 1) \div 2 = 10.5\text{th value}$.

Step 3 – Count through the frequencies (running total) to find where the 10.5th value lies:

Books (x)	Freq (f)	Cumulative Positions
0	4	Values 1 to 4
1	9	Values 5 to 13 ← 10.5th value
2	5	Values 14 to 18
3	2	Values 19 to 20

Since 10.5 falls within 5 to 13, the median is **1 book**.

2 Grouped Frequency Tables **CORE CONCEPT & METHOD**

When data is grouped (e.g., $10 < h \leq 20$), we don't know the exact values. To estimate the mean:

1. Find the **midpoint** of each class interval.
2. Multiply the midpoint by the frequency ($M \times f$).
3. Divide the total sum by the total frequency.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Calculator

The table shows the masses (g) of 25 parcels. Estimate the mean.

Mass (m g)	Freq (f)
$140 < m \leq 150$	5
$150 < m \leq 160$	11
$160 < m \leq 170$	6
$170 < m \leq 180$	3

STEP-BY-STEP SOLUTION:

1. Add a **Midpoint (M)** column and an $f \times M$ column to the table:

Mass (m g)	Freq (f)	Midpoint (M)	$f \times M$
$140 < m \leq 150$	5	145	$145 \times 5 = 725$
$150 < m \leq 160$	11	155	$155 \times 11 = 1705$
$160 < m \leq 170$	6	165	$165 \times 6 = 990$
$170 < m \leq 180$	3	175	$175 \times 3 = 525$
Totals:		$\Sigma f = 25$	$\Sigma fM = 3945$

2. Calculate the estimated mean:

$$\text{Estimated Mean} = \frac{\Sigma fM}{\Sigma f} = \frac{3945}{25} = 157.8$$

EXAMPLE 2.2 **Non-Calculator**

From the same table, write down the **modal class**.

Mass (m g)	Freq (f)
$140 < m \leq 150$	5
$150 < m \leq 160$	11
$160 < m \leq 170$	6
$170 < m \leq 180$	3

STEP-BY-STEP SOLUTION:

Step 1 – The modal class is the class interval with the *highest frequency*.

Step 2 – Looking at the frequencies (5, 11, 6, 3), the highest frequency is 11, which belongs to the interval $150 < m \leq 160$:

Mass (m g)	Freq (f)
$140 < m \leq 150$	5
$150 < m \leq 160$	11 ← Highest frequency
$160 < m \leq 170$	6
$170 < m \leq 180$	3

Modal class = $150 < m \leq 160$

EXAMPLE 2.3 **Non-Calculator**

From the same table, find the class interval that contains the **median**.

Mass (m g)	Freq (f)
$140 < m \leq 150$	5
$150 < m \leq 160$	11
$160 < m \leq 170$	6
$170 < m \leq 180$	3

STEP-BY-STEP SOLUTION:

Step 1 – Find the total frequency: $\Sigma f = 5 + 11 + 6 + 3 = 25$.

Step 2 – Find the median position: $(25 + 1) \div 2 = 13\text{th value}$.

Step 3 – Count through the cumulative frequencies to find where the 13th value lies:

Mass (m g)	Freq (f)	Cumulative Positions
$140 < m \leq 150$	5	Values 1 to 5
$150 < m \leq 160$	11	Values 6 to 16 ($5 + 11 = 16$) ← 13th value
$160 < m \leq 170$	6	Values 17 to 22 ($16 + 6 = 22$)
$170 < m \leq 180$	3	Values 23 to 25 ($22 + 3 = 25$)

Since 13 falls within 6 to 16, the median lies in the interval $150 < m \leq 160$.

3 Mean from Bar Charts **CORE CONCEPT & METHOD**

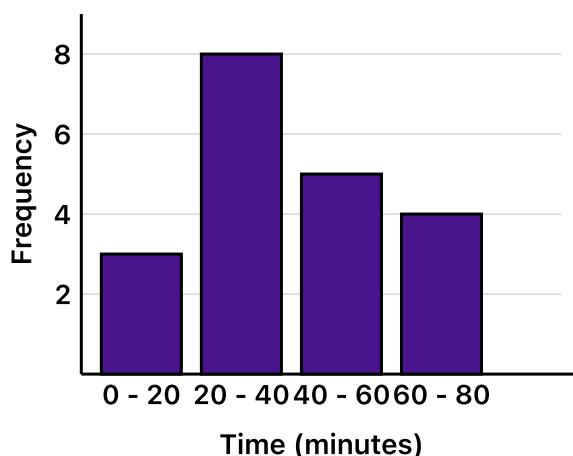
To estimate the mean from a grouped bar chart, you must first read the frequency (height) of each bar. Then, just like a grouped frequency table, find the midpoint of each bar's interval label! Multiply midpoints by frequencies, add them up, and divide by the total number of people.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Calculator

The bar chart below shows the time spent reading by 20 students. Estimate the **mean** time.



STEP-BY-STEP SOLUTION:

1. First, read the frequency (height) for each bar, and find the midpoint (M) of its interval.

Time (mins)	Frequency (f)	Midpoint (M)	$f \times M$
0 - 20	3	10	$10 \times 3 = 30$
20 - 40	8	30	$30 \times 8 = 240$
40 - 60	5	50	$50 \times 5 = 250$
60 - 80	4	70	$70 \times 4 = 280$
Totals:		$\Sigma f = 20$	$\Sigma fM = 800$

2. Calculate the estimated mean by dividing the totals:

$$\text{Estimated Mean} = \frac{\Sigma fM}{\Sigma f} = \frac{800}{20} = 40 \text{ minutes}$$

Relative Frequency

1 Expected Outcomes

CORE CONCEPT & METHOD

To find the expected number of times an event will happen, multiply the **probability** of the event by the **number of trials**.

Example: Probability of winning a raffle prize is 0.15. You buy 300 tickets.

Expected wins = $0.15 \times 300 = 45$.

Common Mistake: Probabilities of exhaustive outcomes in a table must sum to 1, while
Expected Frequency = Probability $\times$ Number of Trials (an integer count, not a probability!).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

The probability of hitting the bullseye in a darts game is 0.35. If a player throws 80 darts, how many times would you expect them to hit the bullseye?

STEP-BY-STEP SOLUTION:

Expected Frequency = Probability $\times$ Number of Trials

$0.35 \times 80 = 28$ times.

2 Advanced Probability Tables

CORE CONCEPT & METHOD

If you are told two probabilities are the same, or one is twice the other, form an equation!

Remember that all probabilities in the table must add up to 1.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

The table shows the probabilities of a spinner landing on Purple, Gold, or Silver. The probabilities of landing on Gold and Silver are equal.

Colour	Purple	Gold	Silver
Probability	0.44	x	x

Find the probability of landing on Gold.

STEP-BY-STEP SOLUTION:

Step 1: The total probability must equal 1. Subtract the known probability.

$$1 - 0.44 = 0.56$$

Step 2: Since Gold and Silver are equal (x), divide the remaining probability by 2.

$$0.56 \div 2 = 0.28$$

Probability of Gold = 0.28.

EXAMPLE 2.2

 Non-Calculator

The table shows the probabilities of a biased four-sided spinner landing on 1, 2, 3, or 4. The probability of landing on 3 is **three times** the probability of landing on 4.

Number	1	2	3	4
Probability	0.15	0.25	$3x$	x

Find the probability of landing on 3.

STEP-BY-STEP SOLUTION:

Step 1: Total probability = 1. Subtract the knowns.

$$1 - (0.15 + 0.25) = 1 - 0.40 = 0.60$$

Step 2: Let the probability of 4 be x . Then the probability of 3 is $3x$. Together they make $4x$.

$$4x = 0.60$$

Step 3: Divide by 4 to find x .

$$x = 0.60 \div 4 = 0.15$$

Step 4: The question asks for the probability of 3, which is $3x$.

$$3 \times 0.15 = 0.45.$$

3 Finding Totals from Probabilities

CORE CONCEPT & METHOD

If you know the probability of an item and how many of them there are, you can find the total number of items.

$$\text{Total Items} = \text{Amount} \div \text{Probability}$$

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 **Non-Calculator**

A biased coin is flipped 250 times. It lands on tails 65 times. What is the relative frequency of getting tails?

STEP-BY-STEP SOLUTION:

Relative Frequency = Successful Trials $\div$ Total Trials

$$65 \div 250 = 0.26 \text{ (or } \frac{13}{50} \text{)}.$$

EXAMPLE 3.2

 **Non-Calculator**

A box contains green and yellow marbles. The probability of picking a green marble is 0.25. There are 18 green marbles in the box.

How many marbles are in the box in **total**?

STEP-BY-STEP SOLUTION:

Step 1: Use the formula: Total = Amount $\div$ Probability.

$$\text{Total marbles} = 18 \div 0.25$$

Step 2: Calculate the division.

$$18 \div 0.25 = 72$$

There are 72 marbles in total.

Scatter Graphs

1 Types of Correlation

📖 CORE CONCEPT & METHOD

Positive Correlation: As one goes up, the other goes up (e.g. Temperature and Ice Cream Sales).

Negative Correlation: As one goes up, the other goes down (e.g. TV hours and Test Scores).

No Correlation: No relationship at all (e.g. Shoe size and IQ).

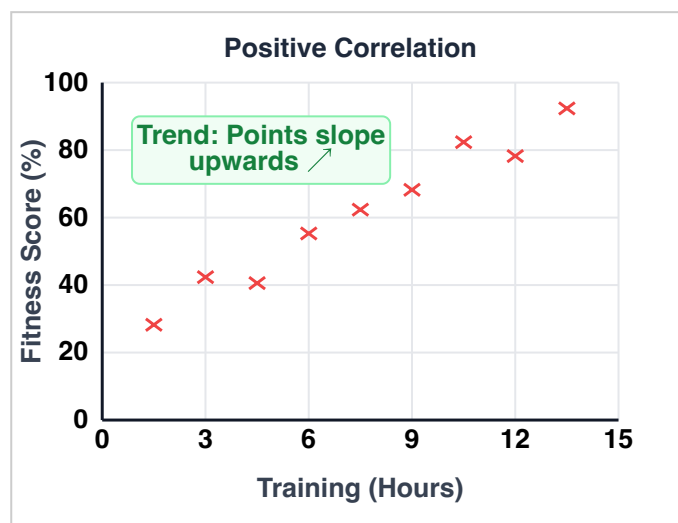
Common Mistake: A line of best fit must follow the trend with roughly equal numbers of points on each side (it does NOT have to pass through $(0, 0)$!), and estimating outside the data range (extrapolation) is unreliable.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

🚫 Non-Calculator

A coach records weekly training hours and fitness test scores. As training hours increase, fitness test scores also increase.



What type of correlation does the scatter graph show?

STEP-BY-STEP SOLUTION:

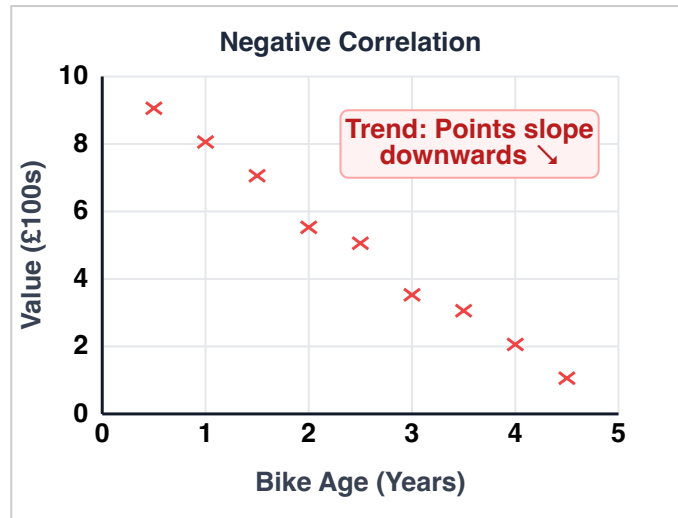
Step 1: Look at the slope of the points.

As training hours increase (move right along the x-axis), the fitness scores also increase (move up along the y-axis).

Step 2: When both variables increase together, the trend slopes upwards from bottom-left to top-right. This is **Positive Correlation**.

EXAMPLE 1.2 **Non-Calculator**

A cycle shop plots the age of a mountain bike against its resale value (£100s).



What type of correlation does this scatter graph show?

STEP-BY-STEP SOLUTION:

Step 1: Look at the direction of the trend.

As bike age increases, the resale value decreases — the points slope downwards from top-left to bottom-right.

Step 2: When one variable increases while the other decreases, this is **Negative Correlation**.

2 Lines of Best Fit **CORE CONCEPT & METHOD**

A line of best fit passes through the middle of the points. It does **NOT** have to pass through the origin (0,0)!

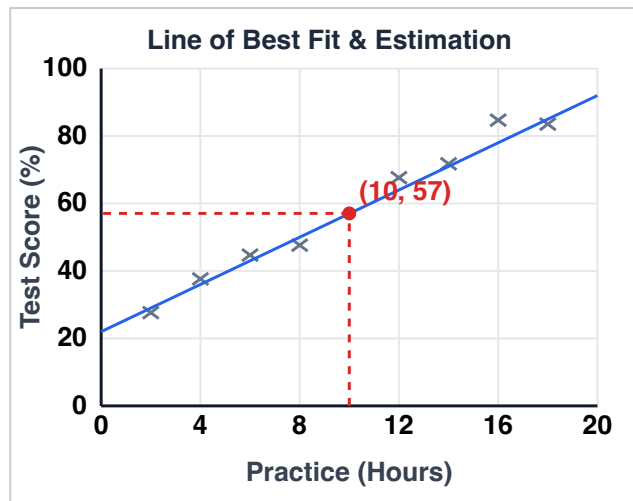
You can use the line to estimate a missing y value for a given x value.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

The scatter graph shows the test scores of students and the number of hours they spent practising.



A student practised for 10 hours. Use the line of best fit to estimate their test score.

STEP-BY-STEP SOLUTION:

Step 1 — Draw a line of best fit:

Draw a straight line that follows the general direction of the points, keeping roughly an equal number of points above and below the line.

Step 2 — Use projection lines:

1. Find 10 on the horizontal axis (Practice Hours).
2. Draw a dashed line straight up until it hits the line of best fit.
3. From that point, read horizontally across to the vertical axis (Test Score).

Answer: The estimated test score is approximately 57% (an exam mark scheme typically accepts $\pm 2\%$, e.g. 55% to 59%).

3 Outliers & Reliability

□ CORE CONCEPT & METHOD

Outliers: Points that do not fit the general trend.

Extrapolation: Predicting values outside the range of your actual data. This is *unreliable*!

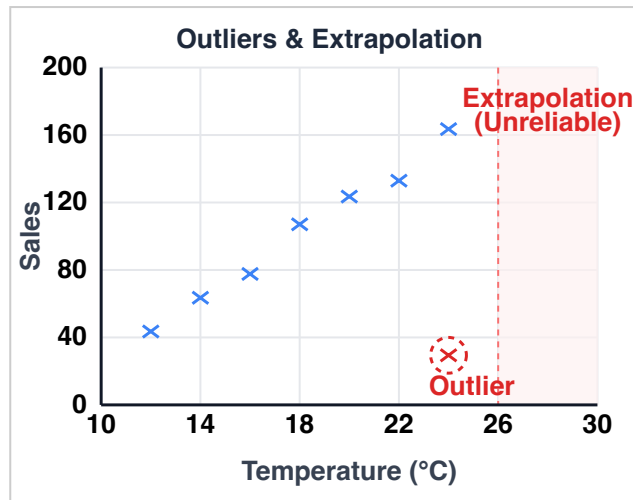
Causation: Just because two things are correlated doesn't mean one *causes* the other.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Non-Calculator

The scatter graph shows the daily temperature and the number of iced coffees sold at a café over 8 days.



(a) Identify the outlier.

(b) Explain why using the graph to estimate iced coffee sales at 32°C would be unreliable.

STEP-BY-STEP SOLUTION:

(a) Identify the outlier:

An outlier is a point that does not fit the general pattern of the data. Look for the point far away from the trend: at 24°C, only 30 iced coffees were sold. The outlier is at (24, 30).

(b) Explain why the estimate at 32°C is unreliable:

The recorded temperature data only goes up to 26°C. Estimating at 32°C requires **extrapolation** (predicting outside the range of known data). We do not know if the linear trend continues at extreme temperatures (e.g. the café might run out of stock), so extrapolation is **unreliable**.