GCSE MATHEMATICS (9-1) — COMPLETE REVISION & PRACTICE

Grade 7 Mathematics

(Higher Tier)

Higher Tier Tutorials, Common Mistake Warnings, Exam Practice, Mixed Reviews & Answers

- **Higher Tier Concept Notes**, Diagrams & **"Common Mistake"** Warnings
- **Model Worked Examples** & 6 Edexcel Higher Questions per topic
- **Calculator & Non-Calculator Badges** across all examples & questions
- **2 Interleaved Mixed Review Sets** + Full Answer Key Appendix

Student Name: _____

Class / Teacher: _____

Target Grade: _____

SCAN FOR ONLINE PRACTICE & WORKED SOLUTIONS

BreakingDownMaths.com

Interactive Step-by-Step GCSE Mathematics Mastery

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NUMBER — SURDS

Simplifying Surds

1 Simplifying Surds

CORE CONCEPT & METHOD

A **surd** is a square root that doesn't give a nice whole number (like $\sqrt{2}$ or $\sqrt{5}$).

To simplify a surd, find the **largest square number** (4, 9, 16, 25, 36...) that divides into it.

When adding or subtracting surds, treat them like letters! $3x + 2x = 5x$, so $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$.

Note on Adding & Subtracting

To add or subtract surds, the square root part **must be exactly the same**. If the numbers inside the roots are different, you cannot add them directly — you must simplify them first so they match!

COMMON MISTAKE: Never add or subtract numbers under separate square roots ($\sqrt{108} - \sqrt{27} \neq \sqrt{81}$) — simplify each surd to a common root like $6\sqrt{3} - 3\sqrt{3}$ first.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Simplify $\sqrt{20}$

STEP-BY-STEP SOLUTION:

1. Find a square number that goes into 20. (4 goes into 20).
2. Rewrite: $\sqrt{20} = \sqrt{4 \times 5}$
3. Split it: $\sqrt{4} \times \sqrt{5}$
4. Square root the 4: $2 \times \sqrt{5}$

Answer: $2\sqrt{5}$

$$2\sqrt{5}$$

EXAMPLE 1.2

Non-Calculator

Simplify $4\sqrt{5} + 3\sqrt{5}$

STEP-BY-STEP SOLUTION:

The surds match, so just add the numbers outside:

$4 + 3 = 7$

Answer: $7\sqrt{5}$

$$7\sqrt{5}$$

EXAMPLE 1.3

Non-Calculator

Simplify $\sqrt{8} + \sqrt{32}$

STEP-BY-STEP SOLUTION:

The surds don't match! Simplify them first.

1. $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$
2. $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$
3. Now add: $2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$

Answer: $6\sqrt{2}$

$$6\sqrt{2}$$

2 Dividing Surds

CORE CONCEPT & METHOD

When **dividing** surds, divide the numbers inside the roots under a single square root: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (or $\sqrt{a} \div \sqrt{b} = \sqrt{a \div b}$).

When there are whole numbers in front of the surds, divide the **outside numbers** by each other and divide the **inside numbers** separately: $\frac{c\sqrt{a}}{d\sqrt{b}} = \left(\frac{c}{d}\right)\sqrt{\frac{a}{b}}$.

💡 **Always check whether the resulting surd can be simplified further by factoring out a square number!**

COMMON MISTAKE: You can divide numbers INSIDE roots ($\frac{\sqrt{75}}{\sqrt{3}} = \sqrt{25} = 5$), but NEVER divide an outside integer by a number inside a root.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Non-Calculator

Simplify $\frac{\sqrt{72}}{\sqrt{2}}$

STEP-BY-STEP SOLUTION:

1. Combine into a single square root and divide the numbers inside:

$$\sqrt{\frac{72}{2}} = \sqrt{36}$$

2. Evaluate the square root:

$$\sqrt{36} = 6$$

Answer: 6

6

EXAMPLE 2.2 Non-Calculator

Simplify $\frac{\sqrt{90}}{\sqrt{5}}$

STEP-BY-STEP SOLUTION:

1. Divide the numbers inside the roots:

$$\sqrt{\frac{90}{5}} = \sqrt{18}$$

2. Simplify $\sqrt{18}$ using the largest square factor (9):

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

Answer: $3\sqrt{2}$

$3\sqrt{2}$

EXAMPLE 2.3 Non-Calculator

Simplify $\frac{12\sqrt{40}}{3\sqrt{2}}$

STEP-BY-STEP SOLUTION:

1. Divide the numbers outside the roots: $12 \div 3 = 4$
2. Divide the numbers inside the roots: $\sqrt{40 \div 2} = \sqrt{20}$
3. Simplify $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$
4. Multiply with the number outside: $4 \times 2\sqrt{5} = 8\sqrt{5}$

Answer: $8\sqrt{5}$

$8\sqrt{5}$

EDEXCEL GCSE EXAM PRACTICE

Simplifying Surds — Practice Questions

Total: 11 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(1 mark)**

Simplify $9\sqrt{2} - 5\sqrt{2}$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

Simplify $\sqrt{640}$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(2 marks)**

Simplify $\sqrt{108} - \sqrt{27}$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(2 marks)**

Simplify $\dfrac{\sqrt{75}}{\sqrt{3}}$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (2 marks)

Simplify $\frac{18\sqrt{60}}{6\sqrt{3}}$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator (2 marks)

Simplify $\frac{20\sqrt{45}}{4\sqrt{5}}$

WORKING OUT SPACE

Answer: .....

NUMBER — SURDS

Expanding with Surds

CORE CONCEPT & METHOD

When **multiplying** surds, multiply the outside numbers together and multiply the inside numbers together: $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$.

💡 **Tip: A surd multiplied by itself gives the number inside!** $\sqrt{a} \times \sqrt{a} = a$

To expand brackets, use the same rules (like FOIL) as you do in Algebra.

> **COMMON MISTAKE:** When squaring a bracket like $(3 + 7\sqrt{2})^2$, write $(3 + 7\sqrt{2})(3 + 7\sqrt{2})$ in full so you do not miss the two middle terms, and square BOTH parts of $(7\sqrt{2})^2 = 49 \times 2 = 98$.

MODEL WORKED EXAMPLES

EXAMPLE 1

Non-Calculator

Expand $\sqrt{5}(\sqrt{2} + \sqrt{5})$

STEP-BY-STEP SOLUTION:

Multiply both terms inside by $\sqrt{5}$:

1. $\sqrt{5} \times \sqrt{2} = \sqrt{10}$
2. $\sqrt{5} \times \sqrt{5} = 5$

Answer: $\sqrt{10} + 5$

$$\sqrt{10} + 5$$

EXAMPLE 2

Non-Calculator

Expand and simplify $(3 + \sqrt{7})(4 - \sqrt{7})$

STEP-BY-STEP SOLUTION:

Use FOIL (First, Outside, Inside, Last):

F: $3 \times 4 = 12$

O: $3 \times -\sqrt{7} = -3\sqrt{7}$

I: $\sqrt{7} \times 4 = 4\sqrt{7}$

L: $\sqrt{7} \times -\sqrt{7} = -7$

Collect numbers: $12 - 7 = 5$

Collect surds: $4\sqrt{7} - 3\sqrt{7} = \sqrt{7}$

Answer: $5 + \sqrt{7}$

$$5 + \sqrt{7}$$

EXAMPLE 3

Non-Calculator

Expand and simplify $(5+\sqrt{3})(5-\sqrt{3})$

STEP-BY-STEP SOLUTION:

This is a 'Difference of Two Squares'!

F: $5 \times 5 = 25$

O/I: $-5\sqrt{3} + 5\sqrt{3} = 0$

L: $\sqrt{3} \times -\sqrt{3} = -3$

Result: $25 - 3 = 22$

Answer: 22

22

EDEXCEL GCSE EXAM PRACTICE

Expanding with Surds — Practice Questions

Total: 13 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Simplify fully $3\sqrt{2} \times 5\sqrt{6}$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

Expand and simplify $\sqrt{2}(6 + \sqrt{2})$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(2 marks)**

Expand and simplify $(3 + 7\sqrt{2})^2$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(2 marks)**

Expand and simplify $(2 + \sqrt{2})(4 + \sqrt{2})$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (3 marks)

Expand and simplify $(5+3\sqrt{2})^2-(3-2\sqrt{2})^2$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator (2 marks)

Expand and simplify $(6+\sqrt{2})(6-\sqrt{2})$

WORKING OUT SPACE

Answer: .....

NUMBER — SURDS

Rationalising the Denominator

CORE CONCEPT & METHOD

Mathematicians hate leaving a surd (square root) on the bottom (denominator) of a fraction. To **rationalise** it, we multiply the top and bottom by a clever trick to make the bottom a whole number.

- If the bottom is $\sqrt{a}$, multiply top and bottom by $\sqrt{a}$.
- If the bottom is $a + \sqrt{b}$, multiply top and bottom by its **conjugate** $a - \sqrt{b}$. (This creates a difference of two squares!).

COMMON MISTAKE: For a two-term denominator like $3 - \sqrt{2}$, multiply top and bottom by its CONJUGATE $3 + \sqrt{2}$ (flipping only the middle sign), NEVER by $\sqrt{2}$ alone.

MODEL WORKED EXAMPLES

EXAMPLE 1

Non-Calculator

Rationalise $\dfrac{12}{\sqrt{3}}$

STEP-BY-STEP SOLUTION:

Multiply top and bottom by $\sqrt{3}$:

$$\frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12\sqrt{3}}{3}$$

Simplify the whole numbers: $12 \div 3 = 4$

Answer: $4\sqrt{3}$

$$4\sqrt{3}$$

EXAMPLE 2

Non-Calculator

Rationalise $\dfrac{15}{4+\sqrt{3}}$

STEP-BY-STEP SOLUTION:

Multiply top and bottom by the conjugate $(4-\sqrt{3})$:

Top: $15(4-\sqrt{3})$

Bottom (Diff of Squares): $(4+\sqrt{3})(4-\sqrt{3}) = 4^2 - (\sqrt{3})^2$
$= 16 - 3 = 13$

Answer: $\dfrac{15(4-\sqrt{3})}{13}$

$$\frac{15(4-\sqrt{3})}{13}$$

EDEXCEL GCSE EXAM PRACTICE

Rationalising the Denominator — Practice Questions

Total: 16 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Rationalise the denominator and simplify: $\frac{10}{2\sqrt{5}}$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

Rationalise the denominator and simplify: $\frac{7}{3-\sqrt{2}}$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(3 marks)**

Rationalise the denominator and simplify: $\frac{25-\sqrt{40}}{\sqrt{5}}$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(3 marks)**

Rationalise the denominator and simplify: $\dfrac{12}{\sqrt{7}-1}$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator **(3 marks)**

Show that $\dfrac{3+\sqrt{5}}{\sqrt{5}-1}$ can be written in the form $a+\sqrt{b}$, where a and b are integers.

WORKING & PROOF SPACE

Question 6 Non-Calculator **(3 marks)**

Rationalise the denominator and simplify: $\dfrac{(2+3\sqrt{5})(\sqrt{5}-6)}{\sqrt{5}}$

WORKING OUT SPACE

Answer: .....

NUMBER — APPROXIMATION & BOUNDS

Calculating with Bounds

1 Bounds of Shapes

CORE CONCEPT & METHOD

When a measurement has been rounded, it has an **Upper Bound** and a **Lower Bound**.

To find the maximum possible Area of a rectangle, you must multiply the Upper Bound of the length by the Upper Bound of the width ($UB \times UB$).

COMMON MISTAKE: Halve the rounding unit to find the error interval (± 0.5 for nearest integer, ± 0.05 for 1 d.p.), and use the exact unrounded upper bound $x + \text{half-unit}$ (never $x + 0.49$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Calculator

A rectangular garden has length 15 m and width 9 m, both measured to the nearest metre. Calculate the upper bound for its area.

STEP-BY-STEP SOLUTION:

Length upper bound = 15.5
Width upper bound = 9.5
$\text{Area Upper Bound} = 15.5 \times 9.5 = 147.25 \text{ m}^2$

2 Bounds of Formulas

CORE CONCEPT & METHOD

Division is tricky! To find the maximum possible result of a division ($v = \frac{s}{t}$):

You must divide the LARGEST possible numerator by the SMALLEST possible denominator.

Max v = $UB(s) \div LB(t)$

COMMON MISTAKE: For a division $d = \frac{m}{v}$, the UPPER bound uses $\frac{\text{UB}(m)}{\text{LB}(v)}$ and the LOWER bound uses $\frac{\text{LB}(m)}{\text{UB}(v)}$ — never divide upper by upper.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

Given $v = \frac{d}{t}$, where $d = 160$ (nearest 10) and $t = 8$ (nearest integer). Find the upper bound of v.

STEP-BY-STEP SOLUTION:

To maximize a fraction, divide the upper bound of the numerator by the lower bound of the denominator.

$d_{\text{upper}} = 165$

$t_{\text{lower}} = 7.5$

$v_{\text{upper}} = \frac{165}{7.5} = 22$

3 Suitable Degree of Accuracy

CORE CONCEPT & METHOD

If a question asks for a 'suitable degree of accuracy', you must:

1. Calculate the Upper Bound of the final answer.
2. Calculate the Lower Bound of the final answer.
3. Find the most decimal places or significant figures that **match** between both numbers.

> **COMMON MISTAKE:** You MUST calculate both UB and LB in full, then round to the most accurate place value where BOTH bounds give the exact same rounded figure.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Non-Calculator

A calculation gives an upper bound of 7.843 and a lower bound of 7.836. What is a suitable degree of accuracy for the answer?

STEP-BY-STEP SOLUTION:

Compare the upper and lower bounds rounded to increasing degrees of accuracy:

Bound	1 d.p.	2 d.p.	3 d.p.
$\text{UB} = 7.843$	7.8	7.84	7.843
$\text{LB} = 7.836$	7.8	7.84	7.836
Comparison	**Same**	**Same ← Choose**	**Different**

Both bounds agree to 1 d.p. and 2 d.p., but differ at 3 d.p.. We choose the most accurate level where both bounds still match: 7.84 **(to 2 d.p.)**.

EDEXCEL GCSE EXAM PRACTICE

Calculating with Bounds — Practice Questions

Total: 12 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator (2 marks)

A rectangle has a length of 23cm, to the nearest cm, and a width of 6.7cm, to the nearest mm.

Work out the **lower bound** for the area of the rectangle.

WORKING OUT SPACE

Answer: .....

Question 2 Calculator (2 marks)

A rectangle has a length of 17cm, to the nearest cm, and a width of 8.0cm, to the nearest mm.

Work out the **upper bound** for the area of the rectangle.

WORKING OUT SPACE

Answer: .....

Question 3 Calculator (2 marks)

In a right-angled triangle, the hypotenuse $c = 10$ cm (correct to the nearest cm), and one side $a = 6.4$ cm (correct to the nearest mm).

Calculate the **lower bound** for the other side b.
Give your answer to 1 decimal place.

WORKING OUT SPACE

Answer: .....

Question 4 Calculator (2 marks)

$$v = \frac{s}{t}$$

$s = 6.44$ correct to 2 decimal places.
$t = 3.600$ correct to 3 decimal places.

Work out the **upper bound** for v.
Give your answer to 3 decimal places.

WORKING OUT SPACE

Answer: .....

Question 5 Calculator (2 marks)

$$p = \frac{F}{A}$$

$F = 26.00$ correct to 2 decimal places.
$A = 5.50$ correct to 3 significant figures.

By considering bounds, work out the value of p to a suitable degree of accuracy.

WORKING OUT SPACE

Answer: .....

Question 6 Calculator (2 marks)

$$a = \frac{v - u}{t}$$

$v = 36.42$ correct to 2 decimal places.
$u = 11.21$ correct to 2 decimal places.
$t = 7.35$ correct to 2 decimal places.

By considering bounds, work out the value of a to a suitable degree of accuracy.

WORKING OUT SPACE

Answer: .....

ALGEBRA — EXPRESSIONS & FORMULAE

Rearranging Harder Formulae

CORE CONCEPT & METHOD

When the letter you want to make the subject appears more than once, you need to collect all of them on one side and then **factorise**!

> **COMMON MISTAKE:** Whenever the target letter x appears TWICE, collect every x-term on one side and factor x out as a single bracket — your final expression must NEVER still contain x on the right-hand side.

MODEL WORKED EXAMPLES

EXAMPLE 1

Non-Calculator

Make x the subject of the formula: $px = qx + r$

STEP-BY-STEP SOLUTION:

1. Get all the x's on one side! Subtract qx from both sides:

$$\begin{array}{rcl} px & = & qx + r \\ -qx & & -qx \\ \hline \mathbf{px - qx} & \mathbf{=} & \mathbf{r} \end{array}$$

2. The letter x is trapped in two terms. Factorise it out!

$$x(p - q) = r$$

3. Divide by the bracket to get x on its own:

$$\begin{array}{rcl} x(p-q) & = & r \\ \div(p-q) & & \div(p-q) \\ \hline \mathbf{x} & \mathbf{=} & \dfrac{\mathbf{r}}{\mathbf{p-q}} \end{array}$$

$$x = \frac{r}{p - q}$$

EXAMPLE 2

Non-Calculator

Make x the subject of the formula: $y = \dfrac{x+4}{x-5}$

STEP-BY-STEP SOLUTION:

1. Multiply both sides by $(x-5)$ to clear the fraction:

$$\begin{array}{rcl} y & = & \dfrac{x+4}{x-5} \\ \times(x-5) & & \times(x-5) \\ \hline \mathbf{y(x-5)} & = & \mathbf{x+4} \end{array}$$

2. Expand the bracket:

$$yx - 5y = x + 4$$

3. Get all the x's on the left, and everything else on the right. Subtract x:

$$\begin{array}{rcl} yx - 5y & = & x+4 \\ -x & & -x \\ \hline \mathbf{yx - x - 5y} & = & \mathbf{4} \end{array}$$

4. Add $5y$ to the right:

$$\begin{array}{rcl} yx - x - 5y & = & 4 \\ +5y & & +5y \\ \hline \mathbf{yx - x} & = & \mathbf{4 + 5y} \end{array}$$

5. Factorise x out on the left:

$$x(y-1) = 4 + 5y$$

6. Divide by the bracket:

$$\begin{array}{rcl} x(y-1) & = & 4+5y \\ \div(y-1) & & \div(y-1) \\ \hline \mathbf{x} & = & \dfrac{\mathbf{4+5y}}{\mathbf{y-1}} \end{array}$$

$$x = \frac{4+5y}{y-1}$$

EXAMPLE 3 Non-Calculator

Make x the subject of the formula: $y = \dfrac{3x+5}{2x-7}$

STEP-BY-STEP SOLUTION:

1. Multiply both sides by $(2x-7)$ to clear the fraction:

$$y(2x-7) = 3x+5$$

2. Expand the bracket:

$$2xy - 7y = 3x + 5$$

3. Collect all x terms on the left and non-x terms on the right (subtract $3x$ and add $7y$):

$$2xy - 3x = 7y + 5$$

4. Factorise x out on the left:

$$x(2y-3) = 7y+5$$

5. Divide both sides by $(2y-3)$:

$$x = \frac{7y+5}{2y-3}$$

$$\boxed{x = \frac{7y+5}{2y-3}}$$

EDEXCEL GCSE EXAM PRACTICE

Rearranging Harder Formulae — Practice Questions

Total: 18 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Make x the subject of the formula: $V = mx + nx$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(3 marks)**

Make x the subject of the formula: $mx + 3 = dx + 11$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(3 marks)**

Make x the subject of the formula: $m(x + 4) = n(x + 2)$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(3 marks)**

Make x the subject of the formula: $y = \dfrac{x + 5}{x - 2}$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator **(3 marks)**

Make x the subject of the formula: $y = \dfrac{4x + 3}{5x - 2}$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(4 marks)**

Make x the subject of the formula:

$$y = \sqrt{\frac{9 - 2x}{3x - 4}}$$

WORKING OUT SPACE

Answer: .....

ALGEBRA — QUADRATICS

Factorising Harder Quadratics

CORE CONCEPT & METHOD

When the x^2 has a number in front (like $2x^2$), the puzzle changes. Follow these steps:

1. **Multiply** the first number by the last number.
2. **Find factors** of *that* new number which add to the middle term.
3. **Split** the middle x term into those two pieces.
4. **Factorise** each half!

> **COMMON MISTAKE:** In $ax^2 + bx + c$ with $a > 1$, find two numbers that multiply to $a \times c$ (NOT c alone) and add to b, then split the middle bx term before factorising in pairs.

MODEL WORKED EXAMPLES

EXAMPLE 1

Non-Calculator

Factorise $2x^2 + 9x + 4$

STEP-BY-STEP SOLUTION:

1. Multiply first and last: $2 \times 4 = \mathbf{8}$.
2. Find two numbers that **Multiply** to $+8$ and **Add** to $+9$.

Multiply = $+8$	Add = $+9$
1×8	$1 + 8 = +9$ ✔
2×4	$2 + 4 = 6$ ✘

3. Split the middle term using $+8x$ and $+1x$:

$$2x^2 + \mathbf{8x} + \mathbf{1x} + 4$$

4. Factorise the first half ($2x^2 + 8x$):
   $\rightarrow 2x(x + 4)$

5. Factorise the second half ($+1x + 4$):
   $\rightarrow +1(x + 4)$

The brackets match! Pull them together:

$$(2x + 1)(x + 4)$$

EXAMPLE 2

Non-Calculator

Factorise $3x^2 - 5x - 2$

STEP-BY-STEP SOLUTION:

1. Multiply first and last: $3 \times -2 = \mathbf{-6}$.
2. Find two numbers that **Multiply** to -6 and **Add** to -5.

Multiply = -6	Add = -5
-2×3	$-2 + 3 = +1$ ✘
-6×1	$-6 + 1 = -5$ ✔

3. Split the middle term using $-6x$ and $+1x$:

$$3x^2 \mathbf{-6x + 1x} - 2$$

4. Factorise the first half ($3x^2 - 6x$):
   $\rightarrow 3x(x - 2)$

5. Factorise the second half ($+1x - 2$):
   $\rightarrow +1(x - 2)$

The brackets match! Pull them together:

$$(3x + 1)(x - 2)$$

EDEXCEL GCSE EXAM PRACTICE

Factorising Harder Quadratics — Practice Questions

Total: 12 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator (2 marks)

Factorise $2x^2 + 7x + 3$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator (2 marks)

Factorise $3x^2 - 11x + 6$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator (2 marks)

Factorise $2x^2 + 5x - 12$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator (2 marks)

Factorise $3x^2 - 5x - 12$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (2 marks)

Factorise $6x^2 - 11x + 3$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator (2 marks)

Factorise $4x^2 + 8x - 5$

WORKING OUT SPACE

Answer: .....

ALGEBRA — QUADRATICS

The Quadratic Formula

1 Solving Harder Quadratics (a > 1)

CORE CONCEPT & METHOD

When there is a number in front of the x^2 (like $2x^2 + 5x + 3 = 0$), we use the **AC Method** to split the middle term!

1. Multiply the first number (a) by the last number (c).

2. Find two numbers that multiply to make this new number, but still add up to the middle number (b).

3. Split the middle x term into those two numbers.

4. Factorise the first half and the second half separately to get your double brackets!

> **COMMON MISTAKE:** After factorising $(2x + 3)(x - 4) = 0$, set each bracket equal to zero to flip the signs and divide by the x-coefficient ($x = -\frac{3}{2}$, NOT $+3$ or -3).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Solve $2x^2 + 9x + 4 = 0$

STEP-BY-STEP SOLUTION:

1. Multiply $a \times c$: $2 \times 4 = 8$.
2. Numbers that multiply to 8 and add to 9: 8 **and** 1.
3. Split the middle term: $2x^2 + 8x + 1x + 4 = 0$
4. Factorise first half: $2x(x + 4)$
5. Factorise second half: $+1(x + 4)$
6. Brackets: $(2x + 1)(x + 4) = 0$
7. Set each bracket to zero:
   $2x + 1 = 0 \implies x = -0.5$
   $x + 4 = 0 \implies x = -4$

$$x = -0.5, x = -4$$

EXAMPLE 1.2

Non-Calculator

Solve $6x^2 - 5x - 4 = 0$

STEP-BY-STEP SOLUTION:

1. Multiply $a \times c$: $6 \times -4 = -24$.
2. Numbers that multiply to -24 and add to -5: -8 **and** 3.
3. Split the middle term: $6x^2 - 8x + 3x - 4 = 0$
4. Factorise first half: $2x(3x - 4)$
5. Factorise second half: $+1(3x - 4)$
6. Brackets: $(2x + 1)(3x - 4) = 0$
7. Set each bracket to zero:

$2x + 1 = 0 \implies x = -0.5$

$3x - 4 = 0 \implies x = \frac{4}{3}$

$$x = -0.5, x = \frac{4}{3}$$

2 Quadratic Formula

CORE CONCEPT & METHOD

When a quadratic won't factorise easily, you can use the Quadratic Formula!

First, ensure your equation is in the standard form:

$$ax^2 + bx + c = 0$$

This tells you exactly what numbers to substitute for a, b, and c in the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Just substitute a, b, and c carefully. Don't forget that if b is negative, $-b$ becomes positive!

COMMON MISTAKE: When b is negative, wrap it in brackets inside b^2 so $(-5)^2 = +25$ (never -25), and watch double negatives in $-4ac$ when $c < 0$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

Solve $x^2 + 6x + 4 = 0$ using the quadratic formula. Give your answers to 2 decimal places.

STEP-BY-STEP SOLUTION:

Here, $a = 1$, $b = 6$, and $c = 4$.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

Calculate the two values:

$$x = -0.76, \text{ or } x = -5.24$$

EXAMPLE 2.2 Calculator

Solve $2x^2 - 5x - 4 = 0$ using the quadratic formula. Give your answers to 2 decimal places.

STEP-BY-STEP SOLUTION:

Here, $a = 2$, $b = -5$, and $c = -4$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - (-32)}}{4}$$

$$x = \frac{5 \pm \sqrt{57}}{4}$$

Calculate the two values:

$$x = -0.64, \text{ or } x = 3.14$$

EDEXCEL GCSE EXAM PRACTICE

The Quadratic Formula — Practice Questions

Total: 18 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Solve $5x^2 + 15x + 10 = 0$.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 2 Non-Calculator **(2 marks)**

Solve $2x^2 - 5x - 12 = 0$.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 3 Non-Calculator **(2 marks)**

Solve $12x^2 - 23x + 5 = 0$.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 4 Calculator **(4 marks)**

Solve $2x^2 + 9x - 9 = 0$.
Give your answers correct to 2 decimal places.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 5 Calculator (4 marks)

Solve $2x^2 + 6x - 5 = 0$.

Give your answers correct to 2 decimal places.

WORKING OUT SPACE

$x =$ ..... or $x =$ .....

Question 6 Calculator (4 marks)

A rectangle has width x cm and length $(x + 6)$ cm.

The area of the rectangle is 12 cm^2.

Form a quadratic equation and solve it to find the width x of the rectangle.

Give your answer correct to 2 decimal places.

WORKING OUT SPACE

$x =$ ..... cm

ALGEBRA — EQUATIONS & SEQUENCES

Quadratic Sequences

CORE CONCEPT & METHOD

If the first differences are not the same, but the **second** differences are, it's a quadratic sequence!

Every quadratic sequence follows the standard form:

$$an^2 + bn + c$$

There are two main methods to find the nth term. You can choose whichever you prefer!

Method 1: Subtraction

1. Halve the second difference to find a.
2. Subtract your an^2 sequence from the original sequence to get a normal linear sequence.
3. Find the nth term of that linear sequence to give you the $bn + c$ part!

Example: $4, 11, 22, 37, \ldots$

First differences: $7, 11, 15$. Second difference: 4.
Half of 4 is 2, so it starts with $2n^2$.

Original:	4	11	22	37
$2n^2$**:**	2	8	18	32
Subtract:	2	3	4	5

The linear sequence is $2, 3, 4, 5$, which has an nth term of $n + 1$.
Final Answer: $2n^2 + n + 1$

Method 2: Algebraic

Memorise these three equations to find a, b, and c in order:

- $2a$ = second difference
- $3a + b$ = first difference (between 1st & 2nd term)
- $a + b + c$ = first term

Example: $5, 11, 19, 29, 41, \ldots$

$a+b+c \rightarrow$	**5**	11	19	29	41
$3a+b \rightarrow$		**6**	8	10	12
$2a \rightarrow$			**2**	2	2

$2a = 2$

$a = 1$

$3a + b = 6$

$3(1) + b = 6$

$3 + b = 6$

$b = 3$

$a + b + c = 5$

$1 + 3 + c = 5$

$4 + c = 5$

$c = 1$

The sequence is $an^2 + bn + c$

Final Answer: $n^2 + 3n + 1$

COMMON MISTAKE: The coefficient a of n^2 is HALF the second difference ($2a = \text{2nd diff}$), NOT the full second difference — always check your formula against $n = 1, 2, 3$.

MODEL WORKED EXAMPLES

EXAMPLE 1 — Non-Calculator

Find the nth term of the sequence: $6, 15, 28, 45, \ldots$

STEP-BY-STEP SOLUTION:

	6	**15**	**28**	**45**
	\/	\/	\/	
First Difference	**9**	**13**	**17**	
	\/	\/		
Second Difference	**4**	**4**		

Half the second difference = $4 \div 2 = 2$

So the sequence starts with $2n^2$.

Original:	6	15	28	45
$2n^2$**:**	2	8	18	32
Subtract:	4	7	10	13

The nth term of $4, 7, 10, 13$ is $3n + 1$.

Put them together:

$$2n^2 + 3n + 1$$

EXAMPLE 2 — Non-Calculator

Find the nth term of the decreasing sequence: $25, 20, 11, -2, \ldots$

STEP-BY-STEP SOLUTION:

1. Find the first and second differences:
- First differences: $-5, -9, -13$
- Second difference: -4

2. Halve the second difference: $-4 \div 2 = -2$, so the sequence starts with $-2n^2$.

3. Subtract $-2n^2$ from the original sequence:

Original:	25	20	11	-2
$-2n^2$**:**	-2	-8	-18	-32
Subtract:	27	28	29	30

The linear sequence $27, 28, 29, 30$ has nth term $n + 26$.

$$-2n^2 + n + 26$$

EDEXCEL GCSE EXAM PRACTICE

Quadratic Sequences — Practice Questions

Total: 18 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator (3 marks)

Find the nth term of the sequence: $0, 6, 14, 24, \ldots$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator (3 marks)

Find the nth term of the sequence: $5, 19, 39, 65, \ldots$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator (3 marks)

Find the nth term of the sequence: $27, 22, 15, 6, \ldots$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator (3 marks)

Find the nth term of the sequence: $5, 16, 35, 62, \ldots$

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (3 marks)

Find the nth term of the sequence: $3, 14, 29, 48, \ldots$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator (3 marks)

Find the nth term of the sequence: $39, 34, 25, 12, \ldots$

WORKING OUT SPACE

Answer: .....

ALGEBRA — EXPRESSIONS & FORMULAE

Functions

1 Evaluating Functions

CORE CONCEPT & METHOD

Instead of writing $y = 3x - 5$, we can write $f(x) = 3x - 5$.
This is called **function notation**.

- **Evaluate $f(4)$:** Substitute $x = 4$ into the function.
  $f(4) = 3(4) - 5 = 7$.
- **Solve $f(x) = 10$:** Set the function equal to 10 and solve for x.
  $3x - 5 = 10 \Rightarrow 3x = 15 \Rightarrow x = 5$.

COMMON MISTAKE: In $f(-2)$ for $f(x) = 3x^2 - 4x$, square $(-2)^2 = +4$ BEFORE multiplying by 3, and treat $-4(-2)$ as $+8$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Given that $f(x) = 6x + 5$, find $f(4)$.

STEP-BY-STEP SOLUTION:

Substitute $x = 4$ into the function.
$f(4) = 6(4) + 5$
$f(4) = 24 + 5 = 29$

$$29$$

EXAMPLE 1.2

Non-Calculator

Given that $f(x) = 3x - 8$, solve $f(x) = 19$.

STEP-BY-STEP SOLUTION:

Set the function equal to 19 and solve for x.
$3x - 8 = 19$
$3x = 27$
$x = 9$

$$9$$

2 Composite Functions

CORE CONCEPT & METHOD

A composite function is when you put one function **inside** another.

- $fg(x)$ **means** $f(g(x))$**:** You put $g(x)$ inside $f(x)$. Note: You do g FIRST, then f.
- **Evaluating** $fg(3)$**:** First find $g(3)$. Then put that answer into f.

COMMON MISTAKE: Work from the INSIDE out: $fg(x)$ means $f(g(x))$ — substitute the entire expression for $g(x)$ into f, NEVER multiply $f(x) \times g(x)$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Non-Calculator

Given that $f(x) = 4x - 3$ and $g(x) = 2x + 5$, find $fg(x)$.

STEP-BY-STEP SOLUTION:

Substitute $g(x)$ into $f(x)$ wherever there is an x.

$f(g(x)) = 4(g(x)) - 3$

$fg(x) = 4(2x + 5) - 3$

$fg(x) = 8x + 20 - 3 = 8x + 17$

$$8x + 17$$

EXAMPLE 2.2 Non-Calculator

Given that $f(x) = 4x + 5$ and $g(x) = x - 8$, find $fg(6)$.

STEP-BY-STEP SOLUTION:

First, evaluate the inner function $g(6)$.

$g(6) = (6) - 8 = -2$

Now substitute this result into f.

$f(-2) = 4(-2) + 5 = -8 + 5 = -3$

$$-3$$

EXAMPLE 2.3

Non-Calculator

Given that $f(x) = x^2 + 3$ and $g(x) = 2x - 1$, solve the equation $fg(x) = 28$.

STEP-BY-STEP SOLUTION:

1. Substitute $g(x)$ into $f(x)$:
$fg(x) = (2x - 1)^2 + 3$

2. Set $fg(x) = 28$ and solve:
$(2x - 1)^2 + 3 = 28$
$(2x - 1)^2 = 25$
$2x - 1 = \pm 5$
$2x = 6 \implies x = 3$ or $2x = -4 \implies x = -2$

$$x = 3, -2$$

3 Inverse Functions

CORE CONCEPT & METHOD

The inverse function $f^{-1}(x)$ does the exact opposite of $f(x)$. It works backwards!

To find an inverse function algebraically:

1. Write $y = f(x)$.
2. Rearrange the formula to make x the subject.
3. Swap y for x at the very end.

> **COMMON MISTAKE:** $f^{-1}(x)$ means the INVERSE function (rearrange $y = f(x)$ and swap x and y), NOT the reciprocal $\frac{1}{f(x)}$.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Non-Calculator

Given that $f(x) = 5x - 9$, find an expression for $f^{-1}(x)$.

STEP-BY-STEP SOLUTION:

1. Write as $y = 5x - 9$.
2. Rearrange for x:
$y + 9 = 5x$
$\frac{y + 9}{5} = x$

3. Swap y for x:
$f^{-1}(x) = \frac{x + 9}{5}$

$$\frac{x + 9}{5}$$

EXAMPLE 3.2 Non-Calculator

Given that $f(x) = \dfrac{2x+3}{x-4}$, find an expression for $f^{-1}(x)$.

STEP-BY-STEP SOLUTION:

1. Write $y = \dfrac{2x+3}{x-4}$ and multiply both sides by $(x-4)$:

$$y(x-4) = 2x+3 \implies xy - 4y = 2x+3$$

2. Collect x terms on one side and factorise:

$$xy - 2x = 4y+3 \implies x(y-2) = 4y+3 \implies x = \frac{4y+3}{y-2}$$

3. Swap y for x:

$$f^{-1}(x) = \frac{4x+3}{x-2}$$

$$\frac{4x+3}{x-2}$$

EDEXCEL GCSE EXAM PRACTICE

Functions — Practice Questions

Total: 16 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Given that $f(x) = 3x^2 - 4x$ and $g(x) = 2x - 5$, work out the value of $fg(1)$.

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

Given that $f(x) = \dfrac{4x - 7}{3}$, find an expression for $f^{-1}(x)$.

WORKING OUT SPACE

$f^{-1}(x) =$ .....

Question 3 Non-Calculator **(3 marks)**

Given that $f(x) = 2x^2 - 3x + 1$ and $g(x) = 3x - 2$, find and simplify an expression for $fg(x)$.

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(3 marks)**

The functions f and g are defined such that $f(x) = x^2 + 2$ and $g(x) = 2x - 3$.
Solve the equation $fg(x) = gf(x)$.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 5 Non-Calculator **(3 marks)**

The functions f and g are defined such that $f(x) = \dfrac{5x+7}{x-1}$ and $g(x) = 2x - 1$.

Solve the equation $fg(x) = 7$.

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(3 marks)**

Given that $f(x) = \dfrac{3x+5}{x-4}$, find an expression for $f^{-1}(x)$.

WORKING OUT SPACE

$f^{-1}(x) =$ .....

ALGEBRA — COORDINATE GEOMETRY

Trig & Exponential Graphs

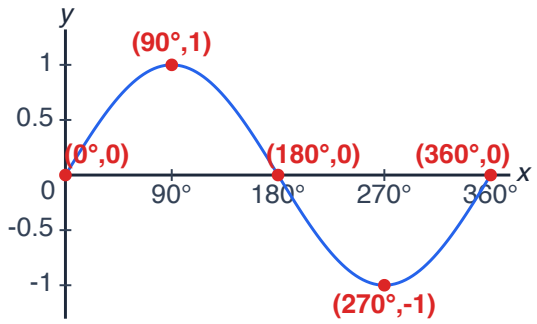
1 Trigonometric Graphs (y = sin x, y = cos x, y = tan x)

CORE CONCEPT & METHOD

Exact Trigonometric Graphs ($0^\circ \leq x \leq 360^\circ$)

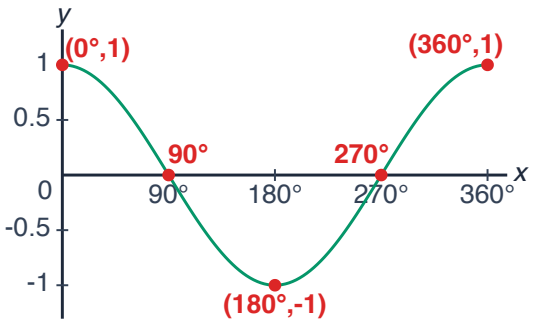
You must be able to recognise, sketch, and state the key coordinates and symmetries of the three trigonometric graphs for $0^\circ \leq x \leq 360^\circ$.

1. The Sine Graph: $y = \sin x^\circ$



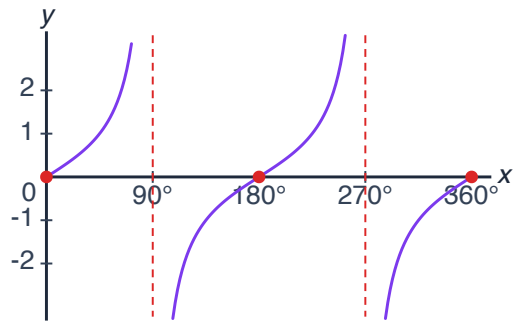
- **Starts at** $(0^\circ, 0)$.
- Maximum at $(90^\circ, 1)$; minimum at $(270^\circ, -1)$.
- Crosses the x-axis at $0^\circ, 180^\circ, 360^\circ$.
- Period = 360°, range $-1 \leq y \leq 1$.

2. The Cosine Graph: $y = \cos x^\circ$



- **Starts at** $(0^\circ, 1)$ (bucket/valley shape).
- Minimum at $(180^\circ, -1)$; maxima at $(0^\circ, 1)$ and $(360^\circ, 1)$.
- Crosses the x-axis at 90° and 270°.
- Symmetric about the line $x = 180^\circ$.

3. The Tangent Graph: $y = \tan x^\circ$



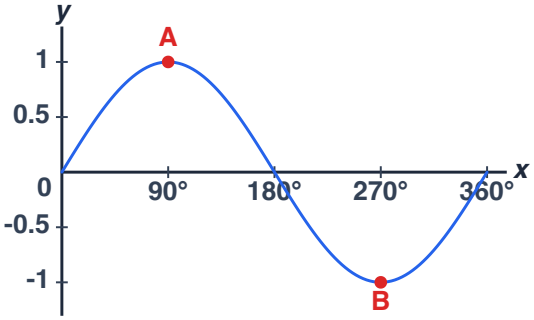
- Passes through $(0^\circ, 0)$, $(180^\circ, 0)$, and $(360^\circ, 0)$.
- Has **vertical asymptotes at $x = 90^\circ$ and $x = 270^\circ$** (where $\tan x$ is undefined).
- Period = 180°, with no maximum or minimum value.

COMMON MISTAKE: $y = \sin x^\circ$ starts at $(0, 0)$, whereas $y = \cos x^\circ$ starts at $(0, 1)$, and $y = \tan x^\circ$ has vertical asymptotes at $x = 90^\circ$ and $x = 270^\circ$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1 Non-Calculator

Here are sketches of $y = \sin x^\circ$ and $y = \cos x^\circ$ for $0 \leq x \leq 360$.



State the coordinates of the maximum point A and the minimum point B on $y = \sin x^\circ$, and state the equations of the vertical asymptotes of $y = \tan x^\circ$ for $0 \leq x \leq 360$.

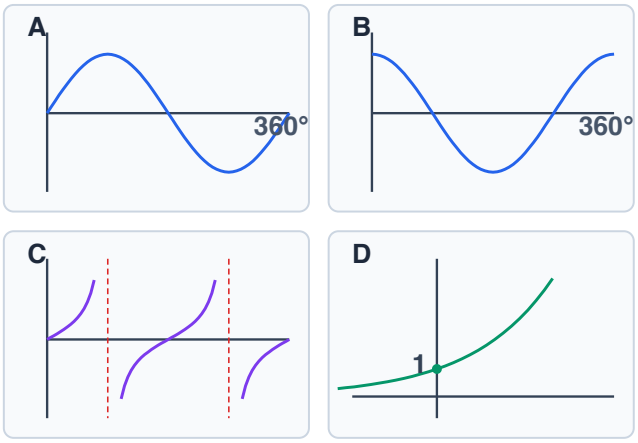
STEP-BY-STEP SOLUTION:

1. On $y = \sin x^\circ$, the curve reaches its maximum value of 1 at $x = 90^\circ$, so $A = (90^\circ, 1)$.
2. It reaches its minimum value of -1 at $x = 270^\circ$, so $B = (270^\circ, -1)$.
3. On $y = \tan x^\circ$, the vertical asymptotes occur at $x = 90^\circ$ and $x = 270^\circ$.

$$A(90^\circ, 1),\ B(270^\circ, -1),\ x = 90^\circ, 270^\circ$$

EXAMPLE 1.2 Non-Calculator

Match each graph A, B, C, D below to its equation from: $y = \sin x°$, $y = \cos x°$, $y = \tan x°$, and $y = 2^x$.



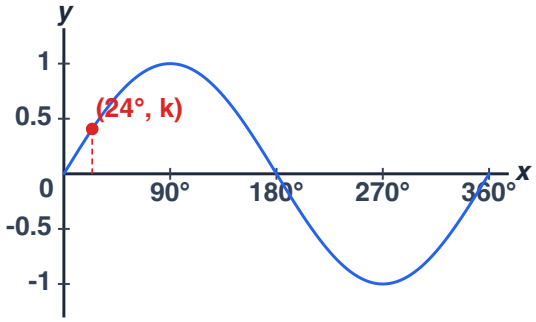
STEP-BY-STEP SOLUTION:

1. **Graph A** starts at $(0°, 0)$, rises to $(90°, 1)$, and completes one wave at $360° \rightarrow y = \sin x°$.
2. **Graph B** starts at $(0°, 1)$ and has a minimum at $(180°, -1) \rightarrow y = \cos x°$.
3. **Graph C** has vertical asymptotes at $90°$ and $270° \rightarrow y = \tan x°$.
4. **Graph D** is an increasing exponential curve passing through $(0, 1) \rightarrow y = 2^x$.

$$A : y = \sin x°,\ B : y = \cos x°,\ C : y = \tan x°,\ D : y = 2^x$$

EXAMPLE 1.3 Non-Calculator

Here is a sketch of $y = \sin x°$ for $0 \leq x \leq 360$.



Given that $\sin 24° = k$, write down in terms of k the values of $\sin 156°$ and $\sin 204°$.

STEP-BY-STEP SOLUTION:

1. By symmetry of the first arch of $y = \sin x°$ about $x = 90°$, since $180° - 24° = 156°$:
$$\sin 156° = \sin 24° = k$$
2. Between $180°$ and $360°$, the arch is inverted below the x-axis. Since $180° + 24° = 204°$:
$$\sin 204° = -\sin 24° = -k$$

$$\sin 156° = k, \quad \sin 204° = -k$$

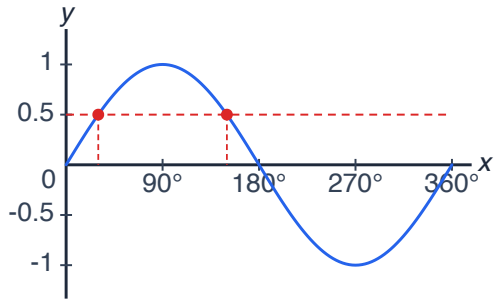
2 Solving Trigonometric Equations & Modelling

CORE CONCEPT & METHOD

Solving Trigonometric Equations & Real-World Models

Because trigonometric graphs repeat and have symmetry, an equation like $\sin x = 0.5$ has **more than one solution** in the interval $0^\circ \leq x \leq 360^\circ$.

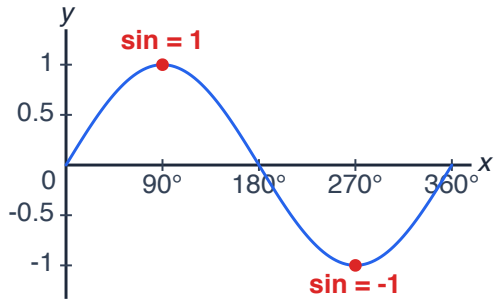
1. Secondary Value Symmetry Rules



Once you find the first solution x_1, use symmetry to find the second solution in $0^\circ \leq x \leq 360^\circ$:

- **Sine ($y = \sin x$):** $x_2 = 180^\circ - x_1$ (or $540^\circ - x_1$ if negative)
- **Cosine ($y = \cos x$):** $x_2 = 360^\circ - x_1$
- **Tangent ($y = \tan x$):** $x_2 = x_1 + 180^\circ$

2. Trigonometric Modelling ($d = a \pm b\sin(kt)^\circ$)



Since $-1 \leq \sin(\theta) \leq 1$ and $-1 \leq \cos(\theta) \leq 1$:

- **Maximum value** $= a + |b|$
- **Minimum value** $= a - |b|$

Set the inside angle equal to 90° or 270° to find when it occurs!

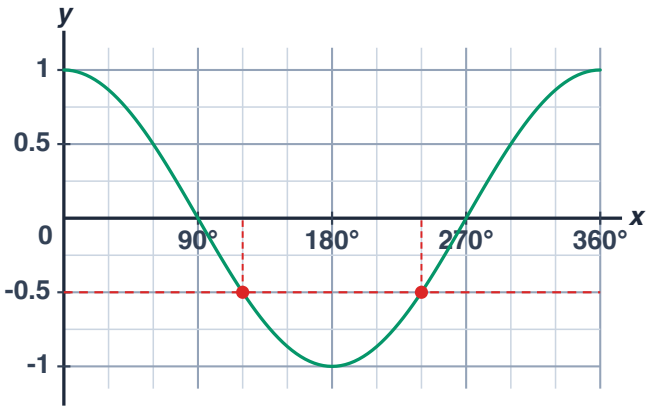
COMMON MISTAKE: Your calculator gives only the principal angle — use symmetry ($180^\circ - \theta$ for sin, $360^\circ - \theta$ for cos, $180^\circ + \theta$ for tan) to find the second root in $0 \leq x \leq 360$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator

Here is the graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$.



Use the graph to solve $\cos x^\circ = -0.5$ for $0 \leq x \leq 360$.

STEP-BY-STEP SOLUTION:

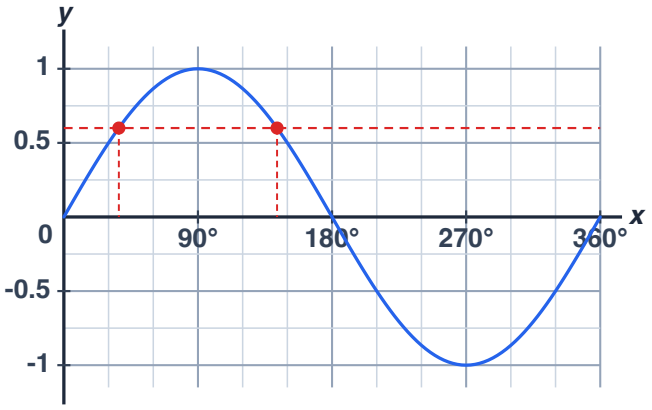
1. Draw the horizontal line $y = -0.5$ across the graph.
2. Read the x-coordinates where $y = -0.5$ intersects $y = \cos x^\circ$: the first intersection is at $x_1 = 120^\circ$.
3. Using symmetry about $x = 180^\circ$, the second solution is $x_2 = 360^\circ - 120^\circ = 240^\circ$.

$$x = 120^\circ,\ 240^\circ$$

EXAMPLE 2.2

Calculator

Solve $\sin x^\circ = 0.6$ for $0 \leq x \leq 360$, giving your answers correct to 1 decimal place.



STEP-BY-STEP SOLUTION:

1. Find the principal acute angle using inverse sine:

$$x_1 = \sin^{-1}(0.6) = 36.9^\circ \text{ (1 d.p.)}$$

2. For $y = \sin x^\circ$, use symmetry about $x = 90^\circ$ ($x_2 = 180^\circ - x_1$):

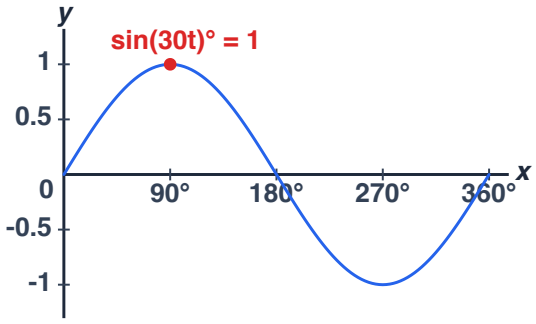
$$x_2 = 180^\circ - 36.9^\circ = 143.1^\circ$$

$$x = 36.9^\circ,\ 143.1^\circ$$

EXAMPLE 2.3

Non-Calculator

The depth of water, d metres, in a harbour is modelled by $d = 6 - 3\sin(30t)°$, where t is the number of hours after midnight ($0 \leq t \leq 12$).



Find the minimum depth of water and the time t when it occurs.

STEP-BY-STEP SOLUTION:

1. Since $-1 \leq \sin(30t)° \leq 1$, the depth $d = 6 - 3\sin(30t)°$ is at its **minimum** when $\sin(30t)° = 1$:

$$d_{\min} = 6 - 3(1) = 3 \text{ m}$$

2. From the sine graph, $\sin(30t)° = 1$ when the angle is $90°$:

$$30t = 90 \implies t = 3 \text{ hours}$$

$$d_{\min} = 3 \text{ m at } t = 3$$

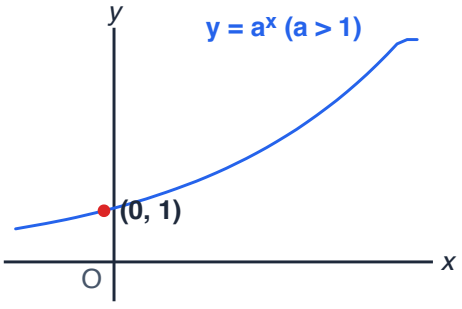
3 Exponential Graphs (y = k^x and y = pq^x)

CORE CONCEPT & METHOD

Exponential Graphs ($y = k^x$ and $y = pq^x$)

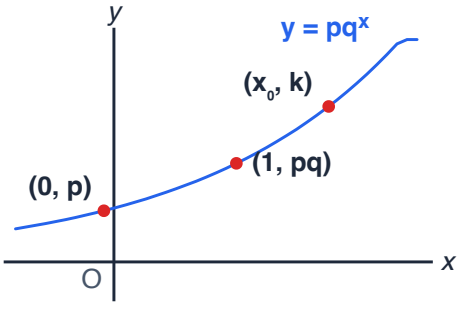
An exponential function has the variable x in the power (exponent), producing rapid growth or decay.

1. Basic Exponential Curve: $y = a^x$



- Because $a^0 = 1$, $y = a^x$ **always crosses the y-axis at $(0, 1)$.**
- The x-axis ($y = 0$) is a **horizontal asymptote** ($a^x > 0$ for all x).
- Growth if $a > 1$; decay if $0 < a < 1$.

2. Finding p, q, and k on $y = pq^x$



- Substitute the y-intercept $(0, y_0)$: since $q^0 = 1$, we immediately get $p = y_0$.
- Substitute the second given point into $y = pq^x$ to solve for q.
- Once p and q are known, substitute the x-coordinate of the third point to calculate k!

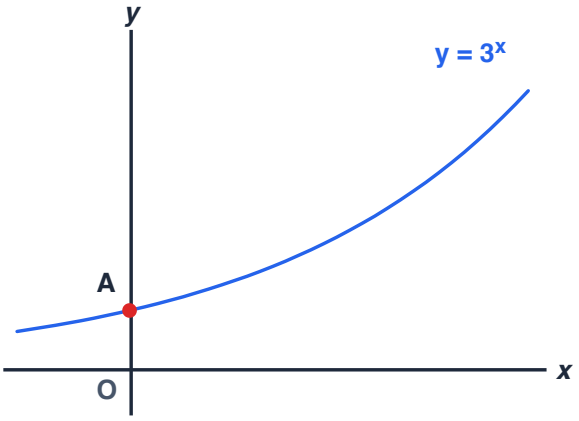
COMMON MISTAKE: In $y = pq^x$, substitute $(0, y_0)$ first to get $p = y_0$ since $q^0 = 1$ (NOT 0), and remember $pq^x = p \times (q^x)$, NEVER $(pq)^x$.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Non-Calculator

The diagram shows a sketch of the curve $y = 3^x$.



Write down the coordinates of the y-intercept A and state the equation of the horizontal asymptote of the curve.

STEP-BY-STEP SOLUTION:

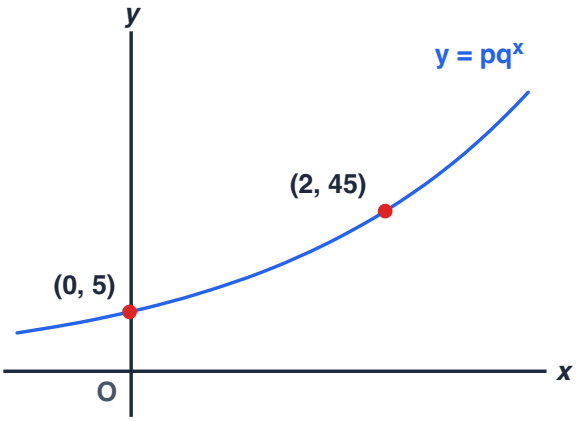
1. On the y-axis, $x = 0$. Since $3^0 = 1$, the y-intercept is $A = (0, 1)$.
2. As $x \to -\infty$, $3^x \to 0$ but never reaches 0. Therefore, the horizontal asymptote is the x-axis, which has equation $y = 0$.

$$A(0, 1), \quad y = 0$$

EXAMPLE 3.2

Non-Calculator

The diagram shows a sketch of the curve $y = pq^x$, where p and q are positive constants.
The curve passes through $(0, 5)$ and $(2, 45)$.



Work out the value of p and the value of q.

STEP-BY-STEP SOLUTION:

1. Substitute $(0, 5)$ into $y = pq^x$:

$$5 = p \times q^0 = p \implies p = 5$$

2. Substitute $(2, 45)$ into $y = 5q^x$:

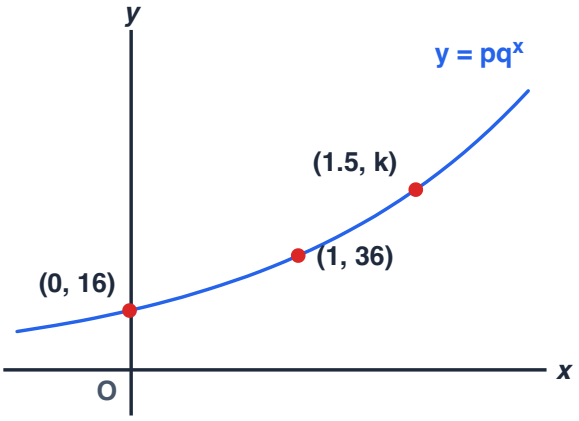
$$45 = 5q^2 \implies q^2 = 9$$

3. Since $q > 0$, $q = 3$.

$$p = 5, \quad q = 3$$

EXAMPLE 3.3 Non-Calculator

The diagram shows a sketch of the curve $y = pq^x$ passing through $(0, 16)$, $(1, 36)$, and $(1.5, k)$.



Work out the value of k.

STEP-BY-STEP SOLUTION:

1. Substitute $(0, 16)$ into $y = pq^x$: $p = 16$.
2. Substitute $(1, 36)$ into $y = 16q^x$:

$$36 = 16q \implies q = \frac{36}{16} = \frac{9}{4} = 2.25$$

3. Substitute $x = 1.5$ into $y = 16 \times \left(\frac{9}{4}\right)^x$:

$$k = 16 \times \left(\frac{9}{4}\right)^{1.5} = 16 \times \left(\frac{3}{2}\right)^3 = 16 \times \frac{27}{8} = 54$$

$$k = 54$$

EDEXCEL GCSE EXAM PRACTICE

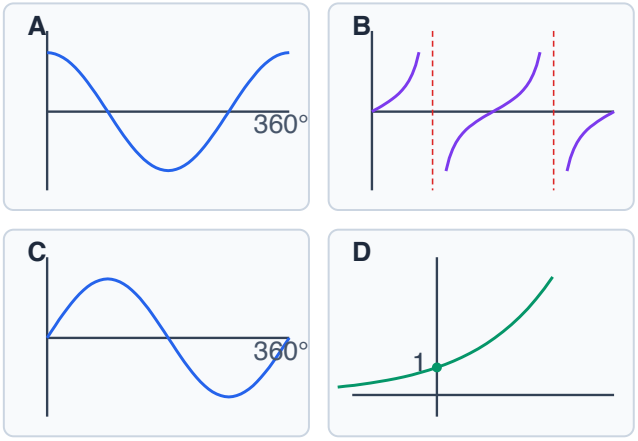
Trig & Exponential Graphs — Practice Questions

Total: 15 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator (2 marks)

Four graphs labeled A, B, C, and D are shown below for $0^\circ \leq x \leq 360^\circ$ (or real x).



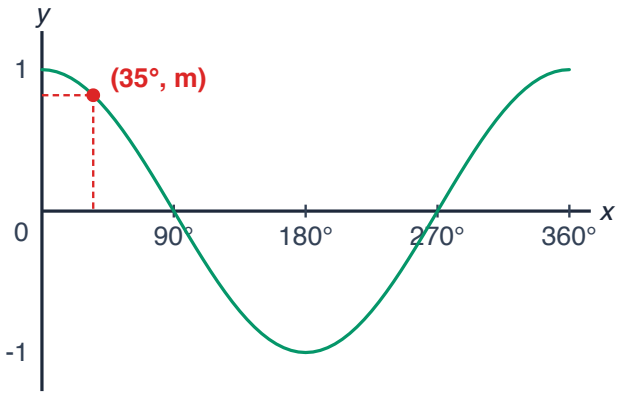
Match the equations $y = \sin x^\circ$, $y = \cos x^\circ$, $y = \tan x^\circ$, and $y = 2^x$ to the letters A, B, C, D in that order.

WORKING OUT SPACE

$y = \sin x^\circ$: ..... $y = \cos x^\circ$: ..... $y = \tan x^\circ$: ..... $y = 2^x$: .....

Question 2 Non-Calculator (2 marks)

Here is a sketch of the curve $y = \cos x^\circ$ for $0 \leq x \leq 360$.



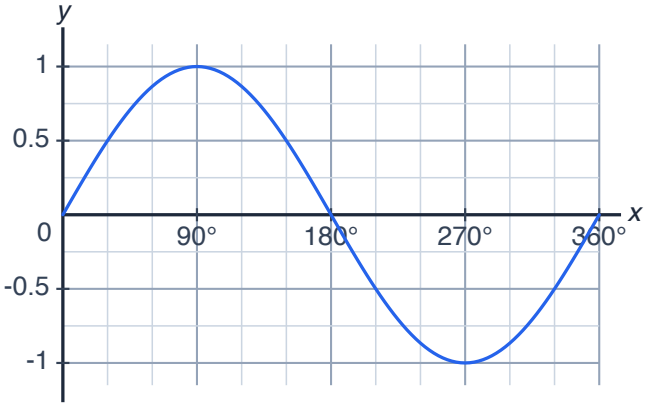
Given that $\cos 35^\circ = m$, write down in terms of m:
(a) the value of $\cos 145^\circ$
(b) the value of $\cos 325^\circ$

WORKING OUT SPACE

(a) ..... **(b)** .....

Question 3 Non-Calculator (2 marks)

Here is the graph of $y = \sin x^\circ$ for $0 \leq x \leq 360$.



Use the graph to find the two solutions to $\sin x^\circ = -0.5$ in the interval $0 \leq x \leq 360$.

WORKING OUT SPACE

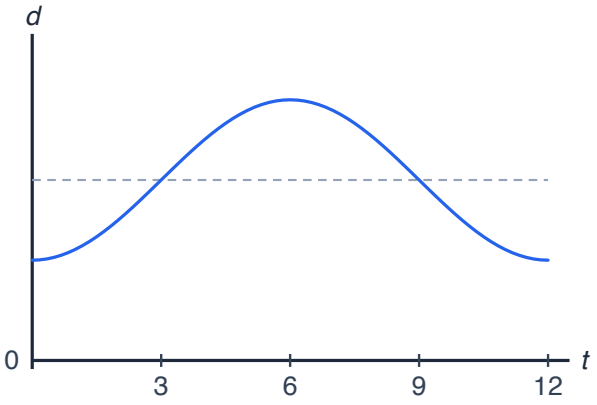
$x =$ ..... ° **and** $x =$ ..... °

Question 4 Non-Calculator (3 marks)

The depth of water, d metres, at the entrance to a harbour is modelled by

$$d = 9 - 4\cos(30t)^\circ$$

where t is the number of hours after midnight for $0 \leq t \leq 12$.



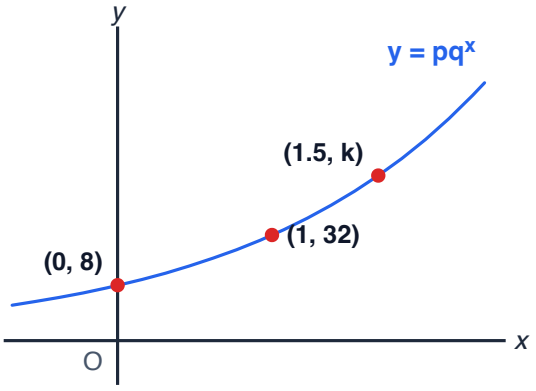
Find:
(a) the maximum depth of water in the harbour (in metres),
(b) the value of t when this maximum depth occurs.

WORKING OUT SPACE

(a) ..... m **(b)** $t =$ .....

Question 5 Non-Calculator (3 marks)

The diagram shows a sketch of the curve $y = pq^x$, where p and q are positive constants.
The curve passes through the points $(0, 8)$, $(1, 32)$, and $(1.5, k)$.



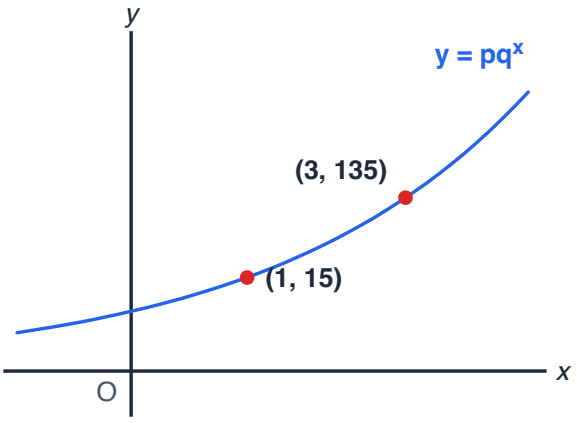
Work out the value of k.

WORKING OUT SPACE

$k =$ .....

Question 6 Non-Calculator **(3 marks)**

A curve has equation $y = pq^x$, where p and q are positive constants.
The curve passes through the points $(1, 15)$ and $(3, 135)$.



Find the value of p and the value of q.

WORKING OUT SPACE

$p =$ ..... $q =$ .....

ALGEBRA — EXPRESSIONS & FORMULAE

Algebraic Fractions

1 Simplifying Fractions

CORE CONCEPT & METHOD

You cannot just cancel out individual terms in an algebraic fraction if they are being added or subtracted!

You MUST **factorise the top and the bottom** first. Only then can you cancel out matching brackets!

COMMON MISTAKE: NEVER cancel individual x^2 or x terms across $+$ or $-$ signs — always factorise numerator and denominator completely first, then cancel matching brackets.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Simplify fully $\dfrac{x^2+2x-15}{x^2-9x+18}$

STEP-BY-STEP SOLUTION:

1. Factorise the top quadratic:
Multiplies to -15, adds to +2. The numbers are -3 and 5.
$(x-3)(x+5)$

2. Factorise the bottom quadratic:
Multiplies to +18, adds to -9. The numbers are -3 and -6.
$(x-3)(x-6)$

3. Write them as a fraction and cancel the matching bracket:

$$\frac{(x-3)(x+5)}{(x-3)(x-6)}$$

$$\frac{x+5}{x-6}$$

EXAMPLE 1.2 Non-Calculator

Simplify fully $\dfrac{4x^2+20x}{x^2-25}$

STEP-BY-STEP SOLUTION:

1. Factorise the top (single bracket!):
Pull out the common factor of $4x$.
$4x(x+5)$

2. Factorise the bottom (Difference of Two Squares!):
x^2-25 becomes $(x+5)(x-5)$

3. Write them as a fraction and cancel the matching bracket:

$$\frac{4x(x+5)}{(x-5)(x+5)}$$

$$\frac{4x}{x-5}$$

EXAMPLE 1.3 Non-Calculator

Simplify fully $\dfrac{2x^2+9x-5}{x^2+4x-5}$

STEP-BY-STEP SOLUTION:

1. Factorise the harder quadratic on top:
$(2x-1)(x+5)$

2. Factorise the bottom quadratic:
$(x-1)(x+5)$

3. Write them as a fraction and cancel the matching bracket:

$$\frac{(2x-1)(x+5)}{(x-1)(x+5)}$$

$$\frac{2x-1}{x-1}$$

2 Adding & Subtracting Fractions

CORE CONCEPT & METHOD

To add or subtract algebraic fractions, you need a **common denominator**, just like normal fractions!

The easiest way is to multiply the two denominators together. Don't forget to cross-multiply the numerators too!

> **COMMON MISTAKE:** When subtracting a fraction, put brackets around its numerator — e.g. $9(2x-1)-4(x+2)=18x-9-4x-8$, distributing the minus sign to BOTH terms.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 — Non-Calculator

Write $\frac{5}{x+3}+\frac{2}{x+4}$ as a single fraction.

STEP-BY-STEP SOLUTION:

Step 1: Make the denominators the same

Multiply the top and bottom of each fraction by the *other* fraction's denominator.

$$\frac{5(x+4)}{(x+3)(x+4)}+\frac{2(x+3)}{(x+4)(x+3)}$$

Step 2: Combine into a single fraction

Now they have the same denominator, we can add the tops!

$$\frac{5(x+4)+2(x+3)}{(x+3)(x+4)}$$

Step 3: Expand and simplify the top

$$=\frac{5x+20+2x+6}{(x+3)(x+4)} \qquad =\frac{7x+26}{(x+3)(x+4)}$$

EXAMPLE 2.2

Non-Calculator

Write $\frac{5}{x-3} - \frac{2}{x+4}$ as a single fraction.

STEP-BY-STEP SOLUTION:

Step 1: Make the denominators the same

$$\frac{5(x+4)}{(x-3)(x+4)} - \frac{2(x-3)}{(x+4)(x-3)}$$

Step 2: Combine into a single fraction

$$\frac{5(x+4) - 2(x-3)}{(x-3)(x+4)}$$

Step 3: Expand and simplify the top

Watch out for the negative sign when expanding $-2(x-3)$!

$$= \frac{5x + 20 - 2x + 6}{(x-3)(x+4)} \qquad = \frac{3x + 26}{(x-3)(x+4)}$$

3 Dividing Algebraic Fractions

CORE CONCEPT & METHOD

To divide algebraic fractions, use the same rule as normal fractions: **Keep, Change, Flip (KFC)**!

Once it's a multiplication, factorise every part and cancel the matching brackets from the top and bottom.

> **COMMON MISTAKE:** Flip ONLY the second fraction (the divisor) and change ÷ to ×, then factorise every quadratic before multiplying.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Non-Calculator

Simplify fully $\dfrac{4x+12}{x-5} \div \dfrac{2x^2+11x+15}{x^2-5x}$

STEP-BY-STEP SOLUTION:

Step 1: Keep, Change, Flip

Keep the first fraction, change $\div$ to $\times$, and flip the second fraction upside down:

$$\frac{4x+12}{x-5} \times \frac{x^2-5x}{2x^2+11x+15}$$

Step 2: Factorise EVERYTHING

Factorise all four parts (top left, bottom left, top right, bottom right):

$$\frac{4(x+3)}{x-5} \times \frac{x(x-5)}{(2x+5)(x+3)}$$

Step 3: Cancel brackets

Since it's multiplication, any bracket on the top can cancel the same bracket on the bottom.

$$\frac{4(x+3)}{(x-5)} \times \frac{x(x-5)}{(2x+5)(x+3)}$$

Result!

$$\frac{4x}{2x+5}$$

EXAMPLE 3.2

Non-Calculator

Simplify fully $\frac{3x-6}{x+4} \div \frac{x^2-7x+10}{2x^2+11x+12}$

STEP-BY-STEP SOLUTION:

Step 1: Keep, Change, Flip

$$\frac{3x-6}{x+4} \times \frac{2x^2+11x+12}{x^2-7x+10}$$

Step 2: Factorise EVERYTHING

$$\frac{3(x-2)}{x+4} \times \frac{(2x+3)(x+4)}{(x-2)(x-5)}$$

Step 3: Cancel brackets

The $(x-2)$ brackets cancel, and the $(x+4)$ brackets cancel.

$$\frac{3(x-2)}{(x+4)} \times \frac{(2x+3)(x+4)}{(x-2)(x-5)}$$

Result!

$$\frac{3(2x+3)}{x-5}$$

4 Solving Algebraic Fractions

CORE CONCEPT & METHOD

When an algebraic fraction has an equals sign, your goal is to solve for x!

Step 1: Combine the fractions into a single fraction (just like Step 2).
Step 2: Multiply both sides by the denominator to get rid of it.
Step 3: You will usually be left with a quadratic equation. Make it equal to 0, factorise, and solve!

COMMON MISTAKE: When multiplying through by the common denominator $(x-2)(x+3)$, remember to multiply the integer on the right-hand side by $(x-2)(x+3)$ as well.

MODEL WORKED EXAMPLES

EXAMPLE 4.1 Non-Calculator

Solve $\frac{2}{2x-1}+\frac{8}{x+2}=2$

STEP-BY-STEP SOLUTION:

Step 1: Combine over common denominator $(2x-1)(x+2)$

$$\frac{2(x+2)+8(2x-1)}{(2x-1)(x+2)}=2 \implies \frac{18x-4}{(2x-1)(x+2)}=2$$

Step 2: Multiply both sides by $(2x-1)(x+2)$ and expand

$$18x-4=2(2x^2+3x-2)=4x^2+6x-4$$

Step 3: Rearrange to $=0$ and factorise

$$4x^2-12x=0 \implies 4x(x-3)=0 \implies x=0 \text{ or } x=3$$

EXAMPLE 4.2 Non-Calculator

Solve $\frac{9}{x+2}-\frac{4}{2x-1}=1$

STEP-BY-STEP SOLUTION:

Step 1: Combine over common denominator $(x+2)(2x-1)$

$$\frac{9(2x-1)-4(x+2)}{(x+2)(2x-1)}=1 \implies \frac{14x-17}{(x+2)(2x-1)}=1$$

Step 2: Multiply both sides by $(x+2)(2x-1)$ and expand

$$14x-17=2x^2+3x-2$$

Step 3: Rearrange to $=0$ and factorise

$$2x^2-11x+15=0 \implies (2x-5)(x-3)=0 \implies x=\frac{5}{2} \text{ or } x=3$$

EDEXCEL GCSE EXAM PRACTICE

Algebraic Fractions — Practice Questions

Total: 14 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

Simplify fully $\dfrac{3x^2 - 9x - 12}{x^2 - 6x + 8}$

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

Write as a single fraction: $\dfrac{6}{x+5} - \dfrac{3}{x+4}$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(2 marks)**

Write as a single fraction: $\dfrac{3}{x+3} + \dfrac{5}{x-2}$

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator (2 marks)

Simplify fully $\frac{4x}{x+4} \div \frac{x^2+2x}{x^2+7x+12}$

WORKING OUT SPACE

Answer: .....

Question 5 Calculator (3 marks)

Solve $\frac{6}{x-3} - \frac{4}{2x+5} = 3$

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 6 Non-Calculator (3 marks)

Solve $\frac{7}{x+2} + \frac{4}{3x-5} = 2$

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

RATIO, PROPORTION AND RATES OF CHANGE — PROPORTION

Algebraic Proportion

1 Direct Proportion

CORE CONCEPT & METHOD

If $y \propto x$, it means $y = kx$ for some constant k. Substitute the given values to find k, then use the formula.

COMMON MISTAKE: Always write $y = kx$ and substitute the given pair to find k FIRST — do not guess additive patterns.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

y is directly proportional to x.

When $x = 6$, $y = 42$.

Find y when $x = 9$.

STEP-BY-STEP SOLUTION:

Direct proportion equation: $y = kx$
Substitute known values: $42 = k \times 6$
$k = 42 \div 6 = 7$
So, $y = 7x$
When $x = 9$, $y = 7 \times 9 = 63$.

2 Inverse Proportion

CORE CONCEPT & METHOD

If $y \propto \frac{1}{x}$, it means $y = \frac{k}{x}$. Find k by multiplying x and y together.

COMMON MISTAKE: Inverse proportion means $y = \frac{k}{x}$ so $k = x \times y$ (multiply the pair to find k), NEVER $k = \frac{y}{x}$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator

y is inversely proportional to x.

When $x = 3$, $y = 12$.

Find y when $x = 9$.

STEP-BY-STEP SOLUTION:

Inverse proportion equation: $y = \frac{k}{x}$

Substitute known values: $12 = \frac{k}{3}$

$k = 12 \times 3 = 36$

So, $y = \frac{36}{x}$

When $x = 9$, $y = \frac{36}{9} = 4$.

3 Proportion with Powers & Roots

CORE CONCEPT & METHOD

The steps are the same, just be careful to square, cube, or root the value of x first before finding k!

> **COMMON MISTAKE:** Read the power or root carefully: "square of x" is x^2, "square root of x" is $\sqrt{x}$, and apply the power ONLY to x ($y = kx^2$, NOT $(kx)^2$).

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Non-Calculator

y is directly proportional to the square of x.

When $x = 4$, $y = 48$.

Find y when $x = 6$.

STEP-BY-STEP SOLUTION:

Equation: $y = kx^2$

Substitute known values: $48 = k(4^2)$

$48 = 16k$

$k = 48 \div 16 = 3$

So, $y = 3x^2$

When $x = 6$, $y = 3(6^2) = 3(36) = 108$.

EXAMPLE 3.2

Non-Calculator

y is directly proportional to the cube root of x.

When $x = 27$, $y = 18$.

Find y when $x = 125$.

STEP-BY-STEP SOLUTION:

Equation: $y = k\sqrt[3]{x}$

Substitute known values: $18 = k(\sqrt[3]{27})$

$18 = k(3)$

$k = 18 \div 3 = 6$

So, $y = 6\sqrt[3]{x}$

When $x = 125$, $y = 6(\sqrt[3]{125}) = 6(5) = 30$.

4 Finding Relationships from Tables

CORE CONCEPT & METHOD

To find the formula, test which calculation gives a constant k. Try $k = \frac{y}{x}$, then try $k = \frac{y}{x^2}$, etc.

COMMON MISTAKE: Test $\frac{y}{x}$, $\frac{y}{x^2}$, and $\frac{y}{x^3}$ across BOTH columns of the table — the relationship holds only when the constant k is identical for every pair.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

Non-Calculator

A table shows values for x and y, where $y \propto x$. Find the missing value.

x	4	7
y	24	?

STEP-BY-STEP SOLUTION:

Since $y \propto x$, we know $y = kx$. Use the complete pair $(4, 24)$ to find k:

$$24 = k(4) \implies k = 24 \div 4 = 6$$

The formula is $y = 6x$.

Now substitute $x = 7$ into the formula to find the missing value:

$$y = 6(7) = 42$$

EDEXCEL GCSE EXAM PRACTICE

Algebraic Proportion — Practice Questions

Total: 14 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(2 marks)**

c is directly proportional to q.

When $c = 84, q = 12$.

Find the value of c when $q = 3$.

WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator **(2 marks)**

p is inversely proportional to q.

When $p = 9, q = 4$.

Find the value of p when $q = 3$.

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(2 marks)**

y is inversely proportional to the square of x.

When $y = 9, x = 4$.

Find the value of y when $x = 3$.

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(2 marks)**

The table shows pairs of values for x and y.

x	4	5
y	80	125

(i) Is $y \propto x$, $y \propto x^2$, or $y \propto x^3$?
(ii) Write a formula for y in terms of x.

WORKING OUT SPACE

(i) ..... **(ii)** $y =$ .....

Question 5 Non-Calculator **(3 marks)**

p is inversely proportional to the square root of q.

When $p = 6, q = 16$.

Find the value of q when $p = 3$.

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(3 marks)**

y is directly proportional to the cube of x.

When $x = 2, y = 40$.

Find the value of x when $y = 135$.

WORKING OUT SPACE

Answer: .....

GEOMETRY AND MEASURES — 3D SHAPES

Similar Shapes: Area, Volume & Frustums

1 Linear, Area & Volume Scale Factors

CORE CONCEPT & METHOD

If two 3D shapes are mathematically similar:

- **Linear Scale Factor (LSF)** = Ratio of Lengths
- **Area Scale Factor (ASF)** = LSF²
- **Volume Scale Factor (VSF)** = LSF³

Why?

Length is 1D (e.g. L)

Area is 2D (e.g. $L \times L = L^2$)

Volume is 3D (e.g. $L \times L \times L = L^3$)

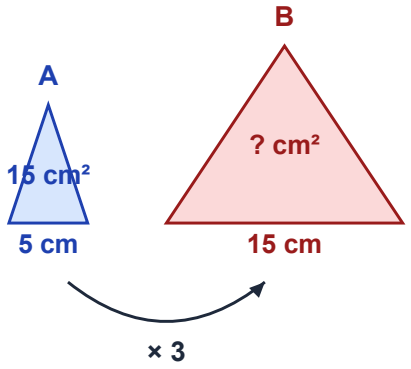
> **COMMON MISTAKE:** If the length scale factor is k, multiply areas by k^2 and volumes by k^3 — never multiply area or volume directly by k.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Triangle A has an area of $15\ \text{cm}^2$ and a base of $5\ \text{cm}$. Triangle B is mathematically similar with a base of $15\ \text{cm}$. What is the area of Triangle B?



STEP-BY-STEP SOLUTION:

Step 1: Find Linear Scale Factor (LSF)

$$\text{LSF} = \frac{\text{New Length}}{\text{Old Length}} = \frac{15}{5} = 3$$

Step 2: Find Area Scale Factor (ASF)

$$\text{ASF} = \text{LSF}^2 = 3^2 = 9$$

Step 3: Find Area of B

$$\text{Area}_B = 15 \times 9 = 135\ \text{cm}^2$$

2 Working Backwards

CORE CONCEPT & METHOD

If you are given the Areas, divide them to find the **ASF**. Then square root it to find the **LSF**!

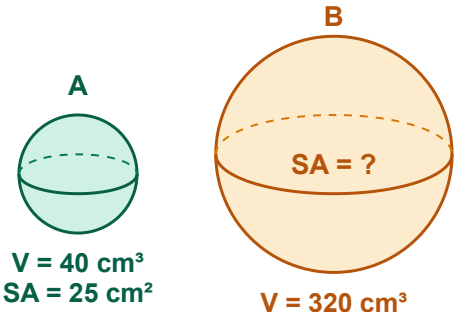
If you are given Volumes, divide them to find the **VSF**, then cube root it to find the **LSF**.

> **COMMON MISTAKE:** When given two areas or two volumes, take the square root ($\sqrt{\text{ASF}}$) or cube root ($\sqrt[3]{\text{VSF}}$) FIRST to find the linear scale factor k before converting.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 — Non-Calculator

Two solid spheres are mathematically similar. The smaller sphere has a volume of $40\ \text{cm}^3$ and a surface area of $25\ \text{cm}^2$. The larger sphere has a volume of $320\ \text{cm}^3$. Find the surface area of the larger sphere.



STEP-BY-STEP SOLUTION:

Step 1: Find Volume Scale Factor (VSF)

$$\text{VSF} = \frac{320}{40} = 8$$

Step 2: Work Backwards to Linear Scale Factor (LSF)

$$\text{LSF} = \sqrt[3]{\text{VSF}} = \sqrt[3]{8} = 2$$

Step 3: Find Area Scale Factor (ASF)

$$\text{ASF} = \text{LSF}^2 = 2^2 = 4$$

Step 4: Find Surface Area of B

$$\text{SA}_B = 25 \times 4 = 100\ \text{cm}^2$$

3 Frustums

CORE CONCEPT & METHOD

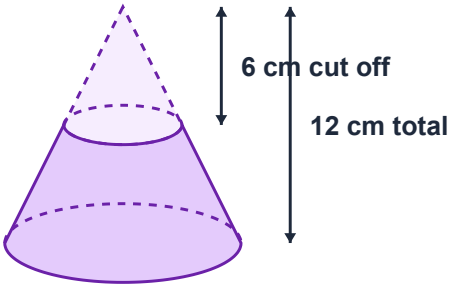
A **frustum** is a cone with the top chopped off. To find its volume, calculate the volume of the **Large Cone** and subtract the volume of the **Small Cone** (the chopped part).

> **COMMON MISTAKE:** A frustum is NOT a prism — always use similarity on the full large cone and removed small cone, then compute $V_{\text{frustum}} = V_{\text{large}} - V_{\text{small}}$.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 — Non-Calculator

A cone has a total height of 12 cm and a volume of 640 cm^3. The top 6 cm of the cone is cut off to form a frustum. Find the volume of the frustum.



STEP-BY-STEP SOLUTION:

Step 1: Volumes of Similar Shapes

The small cone (top) is mathematically similar to the large cone (entire shape).

$$\text{LSF} = \frac{\text{Large Height}}{\text{Small Height}} = \frac{12}{6} = 2$$

$$\text{VSF} = 2^3 = 8$$

Step 2: Volume of Small Cone

Since the large cone is 8 times bigger, the small cone is $\frac{1}{8}$ of the large volume:

$$\text{Small Vol} = \frac{640}{8} = 80 \text{ cm}^3$$

Step 3: Volume of Frustum

The frustum is what remains when the small cone is removed from the large cone.

$$\text{Frustum Vol} = \text{Large Vol} - \text{Small Vol}$$

$$\text{Frustum Vol} = 640 - 80 = 560 \text{ cm}^3$$

EDEXCEL GCSE EXAM PRACTICE

Similar Shapes: Area, Volume & Frustums — Practice Questions

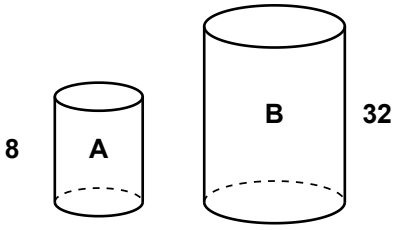
Total: 19 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator

(3 marks)

Two solid shapes, A and B, are mathematically similar.
The length of shape A is 8 cm and the length of shape B is 32 cm.
The volume of shape A is 440 cm³.
Work out the volume of shape B.

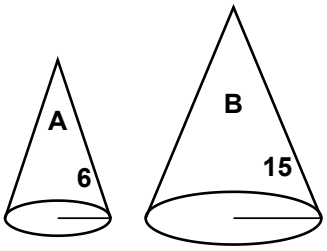


WORKING OUT SPACE

Answer: .....

Question 2 Calculator (3 marks)

Two solid shapes, A and B, are mathematically similar.
The radius of A is 6 cm and the radius of B is 15 cm.
The surface area of shape A is 290 cm².
Work out the surface area of shape B.

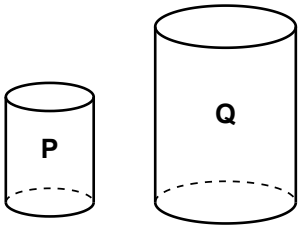


WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator (3 marks)

Two solid shapes, P and Q, are mathematically similar.
The volume of shape P is 50 cm³.
The volume of shape Q is 400 cm³.
The length of shape P is 6 cm.
Work out the length of shape Q.

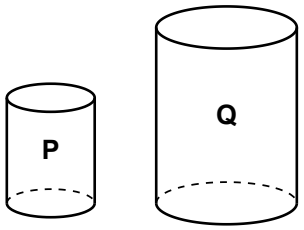


WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator (3 marks)

Two solid shapes, P and Q, are mathematically similar.
The volume of shape P is 24 cm³ and the volume of shape Q is 192 cm³.
The total surface area of shape P is 30 cm².
Work out the total surface area of shape Q.



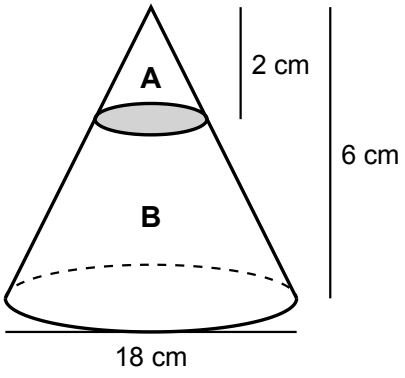
WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (4 marks)

The diagram represents a large cone of height 6 cm and base diameter 18 cm.
The large cone is made by placing a small cone A of height 2 cm on top of a frustum B.
Calculate the volume of the frustum B.

Give your answer in terms of π. *(Volume of a cone* $= \frac{1}{3}\pi r^2 h$*)*

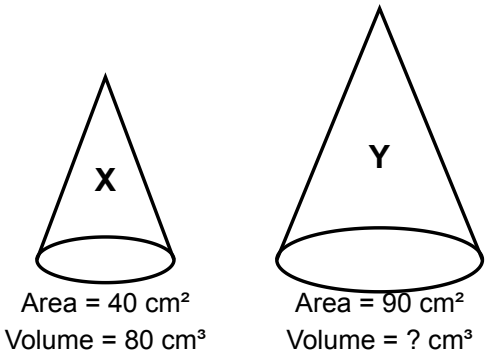


WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(3 marks)**

Two solid shapes, X and Y, are mathematically similar.
The total surface area of shape X is 40 cm² and the total surface area of shape Y is 90 cm².
The volume of shape X is 80 cm³.
Work out the volume of shape Y.



Area = 40 cm² Area = 90 cm²
Volume = 80 cm³ Volume = ? cm³

WORKING OUT SPACE

Answer: .....

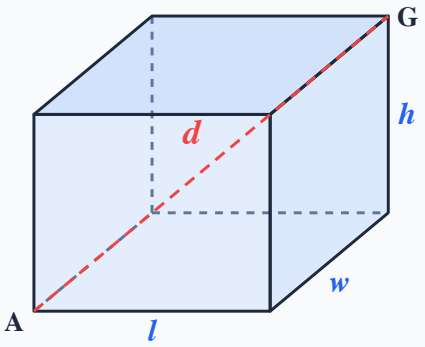
GEOMETRY AND MEASURES — RIGHT-ANGLED TRIANGLES

3D Pythagoras & Trig

1 3D Pythagoras in Cuboids

CORE CONCEPT & METHOD

To find the space diagonal of a cuboid, you can use 3D Pythagoras: $d^2 = l^2 + w^2 + h^2$

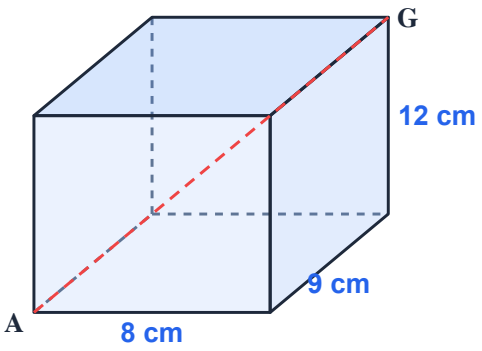


COMMON MISTAKE: Distinguish the 2D base diagonal $\sqrt{l^2 + w^2}$ from the 3D space diagonal $\sqrt{l^2 + w^2 + h^2}$, and keep the surd exact until the final step.

MODEL WORKED EXAMPLES

EXAMPLE 1.1 Non-Calculator

A cuboid has length 8 cm, width 9 cm, and height 12 cm. Find the length of the space diagonal AG.



STEP-BY-STEP SOLUTION:

Step 1: Identify the length, width, and height.

$l = 8, w = 9, h = 12$

Step 2: Use the 3D Pythagoras formula.

$$d^2 = l^2 + w^2 + h^2$$

$$AG^2 = 8^2 + 9^2 + 12^2$$

$$AG^2 = 64 + 81 + 144$$

$$AG^2 = 289$$

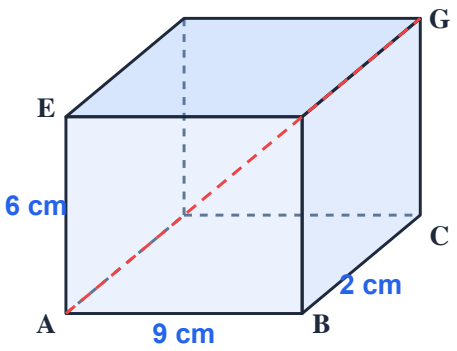
Step 3: Square root to find the space diagonal.

$$AG = \sqrt{289} = 17 \text{ cm}$$

EXAMPLE 1.2

Calculator

A cuboid ABCDEFGH has AB = 9 cm, BC = 2 cm, AE = 6 cm. Calculate the length of the space diagonal AG, correct to 1 decimal place.



STEP-BY-STEP SOLUTION:

Step 1: Identify dimensions from the letters.

Length (AB) = 9, Width (BC) = 2, Height (AE) = 6

Step 2: Use the 3D Pythagoras formula.

$$d^2 = l^2 + w^2 + h^2$$

$$AG^2 = 9^2 + 2^2 + 6^2$$

$$AG^2 = 81 + 4 + 36$$

$$AG^2 = 121$$

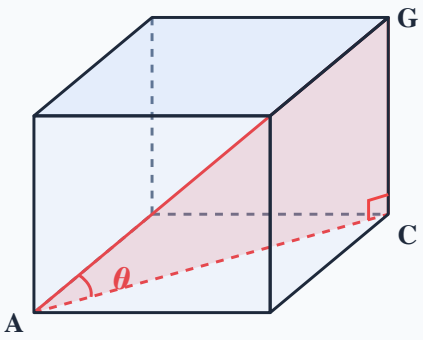
Step 3: Square root to find the space diagonal.

$$AG = \sqrt{121} = 11 \text{ cm}$$

2 3D Trigonometry

CORE CONCEPT & METHOD

First, find the 2D diagonal using Pythagoras. Then, use SOH CAH TOA with the vertical height to find the angle of elevation.

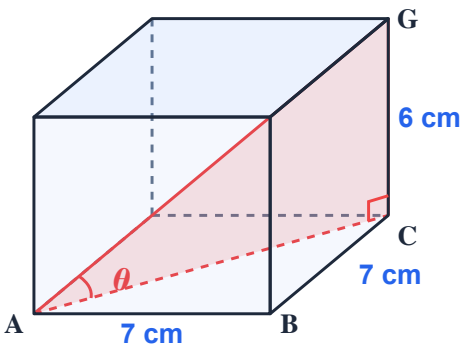


COMMON MISTAKE: Draw out the flat 2D right-angled triangle inside the solid and label its right angle clearly before choosing $\sin$, $\cos$, or $\tan$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

A cuboid ABCDEFGH has AB = 7 cm, BC = 7 cm, and AE = 6 cm. Find the angle the space diagonal AG makes with the base ABCD.



STEP-BY-STEP SOLUTION:

Step 1: Find the 2D diagonal of the base (AC).

Look at the base $ABCD$. It's a 7×7 rectangle.

Use Pythagoras:

$$AC^2 = 7^2 + 7^2 = 98$$

$$AC = \sqrt{98}\ \text{cm}$$

Step 2: Draw out the vertical right-angled triangle ACG.

We want the angle of elevation θ at A.

- Adjacent (base AC) = $\sqrt{98}$
- Opposite (height CG) = 6

Step 3: Use SOH CAH TOA.

$$\tan(\theta) = \frac{\text{Opp}}{\text{Adj}}$$

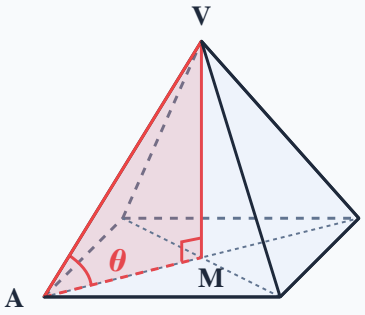
$$\tan(\theta) = \frac{6}{\sqrt{98}}$$

$$\theta = \tan^{-1}\left(\frac{6}{\sqrt{98}}\right) = 31.2^\circ\ (1\ \text{d.p.})$$

3 Prisms and Pyramids

CORE CONCEPT & METHOD

For pyramids, find the diagonal of the base first, then halve it to find the distance to the center. Then use right-angled triangles with the vertical height.

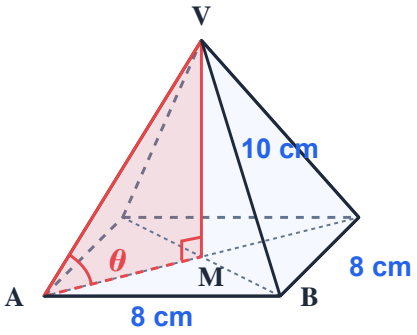


COMMON MISTAKE: In a right pyramid, the vertical height drops to the CENTRE M of the base, so use HALF the base diagonal ($AM = \frac{1}{2}AC$) with the sloping edge VA.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Calculator

A pyramid has a square base ABCD of side length 8 cm. The vertical height of the pyramid from the center of the base (M) to the apex (V) is 10 cm. Find the angle the edge AV makes with the base ABCD.



STEP-BY-STEP SOLUTION:

Step 1: Find the full diagonal of the square base (AC).

The base is an 8×8 square.

$$AC^2 = 8^2 + 8^2 = 64 + 64 = 128$$

$$AC = \sqrt{128}\ \text{cm}$$

Step 2: Find the distance from the corner to the center (AM).

The center M is exactly halfway along the diagonal AC.

$$AM = \frac{\sqrt{128}}{2}\ \text{cm}$$

Step 3: Use SOH CAH TOA in the vertical triangle AMV.

We want the angle θ at A.

- Adjacent (AM) = $\dfrac{\sqrt{128}}{2}$
- Opposite (height VM) = 10

$$\tan(\theta) = \frac{\text{Opp}}{\text{Adj}} = \frac{10}{\frac{\sqrt{128}}{2}}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.6568\ldots}\right) = 60.5^\circ\ (1\ \text{d.p.})$$

EDEXCEL GCSE EXAM PRACTICE

3D Pythagoras & Trig — Practice Questions

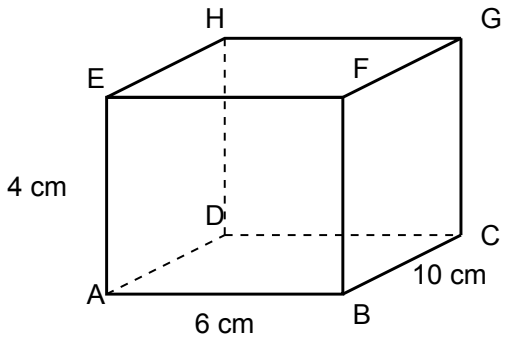
Total: 20 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator (3 marks)

The diagram shows a cuboid ABCDEFGH.
AB = 6 cm
BC = 10 cm
AE = 4 cm

Calculate the length of AG.
Give your answer correct to 1 decimal place if it is a decimal.



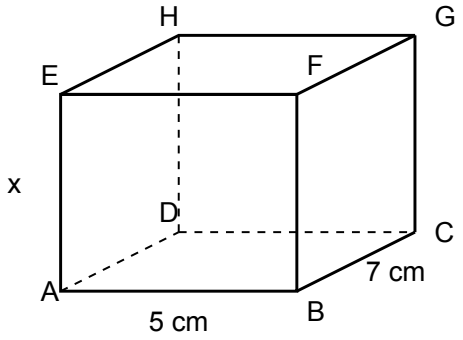
WORKING OUT SPACE

Answer: .....

Question 2 Calculator **(3 marks)**

The diagram shows a cuboid ABCDEFGH.
AB = 5 cm
BC = 7 cm
AG = 10 cm

Calculate the length of AE (x).
Give your answer correct to 1 decimal place.



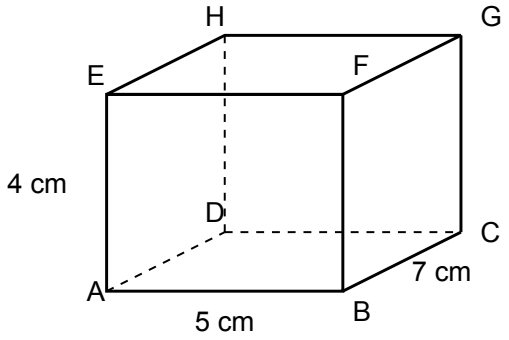
WORKING OUT SPACE

Answer: .....

Question 3 Calculator **(3 marks)**

The diagram shows a cuboid ABCDEFGH.
AB = 5 cm
BC = 7 cm
AE = 4 cm

Calculate the size of angle ECA.
Give your answer correct to 1 decimal place.



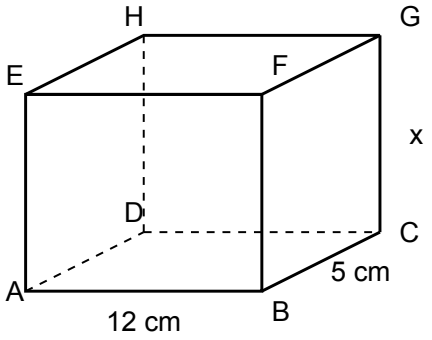
WORKING OUT SPACE

Answer: .....

Question 4 Calculator **(3 marks)**

The diagram shows a cuboid ABCDEFGH.
AB = 12 cm
BC = 5 cm
Angle GAC = 32°

Calculate the vertical height CG (x) of the cuboid.
Give your answer correct to 1 decimal place.



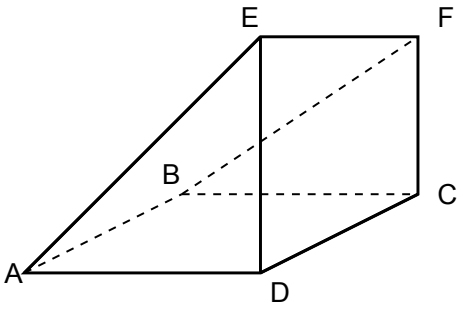
WORKING OUT SPACE

Answer: .....

Question 5 Calculator **(4 marks)**

The diagram shows a triangular prism.
The base ABCD is a rectangle.
AD = 30 cm
CD = 22 cm
Angle FDC = 44°
Triangle FDC is right-angled at C.

Calculate the size of the angle the line AF makes with the plane ABCD.
Give your answer correct to 1 decimal place.



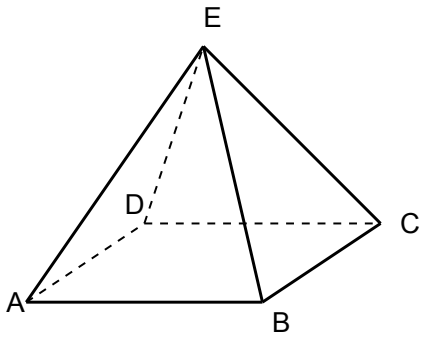
WORKING OUT SPACE

Answer: .....

Question 6 Calculator **(4 marks)**

The diagram shows a pyramid.
The base ABCD is a square.
AB = 11 cm
Angle EAC = 71° (where E is the peak directly above the center of the base).

Calculate the volume of the pyramid.
Give your answer correct to 1 decimal place.



WORKING OUT SPACE

Answer: .....

GEOMETRY AND MEASURES — NON-RIGHT-ANGLED TRIANGLES

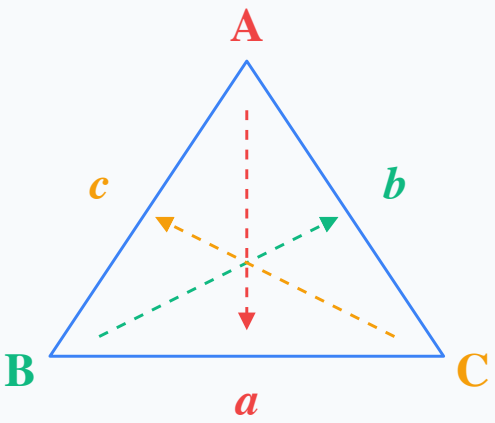
The Sine Rule

1 Labelling the Triangle

CORE CONCEPT & METHOD

Before using the Sine, Cosine, or Area Rules, you must know how to label the triangle correctly.

- **Angles** are labelled with capital letters: A, B, C.
- **Sides** are labelled with lower-case letters: a, b, c.
- **Crucial Rule:** Side a is ALWAYS exactly opposite angle A. Side b is opposite angle B, and side c is opposite angle C.



COMMON MISTAKE: Lowercase side a is ALWAYS directly opposite uppercase angle A — mislabelling adjacent sides breaks both the Sine and Cosine Rules.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

If you are given angle B, which side is exactly opposite to it?

STEP-BY-STEP SOLUTION:

Side b is always opposite angle B.

2 Finding Missing Sides

CORE CONCEPT & METHOD

The Sine Rule states: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$.

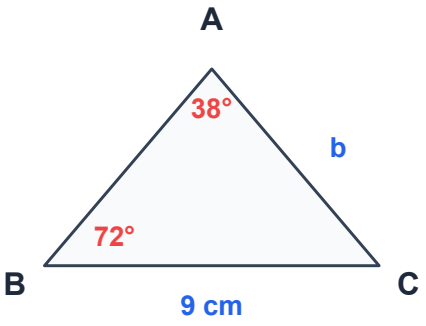
Use this to find a missing side when you know a side and its opposite angle, and another angle.

COMMON MISTAKE: Use the Sine Rule only when you have a complete matching opposite pair (an angle and its opposite side); put the unknown side on TOP ($\frac{a}{\sin A} = \frac{b}{\sin B}$).

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

In triangle ABC, angle A = 38°, angle B = 72°, and side *a* = 9 cm. Find side *b*.



STEP-BY-STEP SOLUTION:

We know side a opposite angle A, and we want side b opposite angle B.
Write the Sine Rule:

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$

Substitute the values:

$$\frac{b}{\sin(72^\circ)} = \frac{9}{\sin(38^\circ)}$$

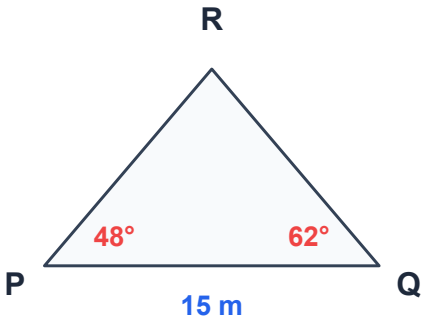
Multiply both sides by $\sin(72^\circ)$:

$$b = \frac{9 \times \sin(72^\circ)}{\sin(38^\circ)} = \frac{9 \times 0.9511\ldots}{0.6157\ldots} = 13.9 \text{ cm (1 d.p.)}$$

EXAMPLE 2.2

Calculator

Triangle PQR has angle P = 48°, angle Q = 62°, and PQ = 15 m. Find side QR.



STEP-BY-STEP SOLUTION:

Side QR is opposite angle P (= 48°). Side PQ is opposite angle R.

First find angle R:

$$R = 180^\circ - 48^\circ - 62^\circ = 70^\circ$$

Now apply the Sine Rule:

$$\frac{QR}{\sin(48^\circ)} = \frac{15}{\sin(70^\circ)}$$

$$QR = \frac{15 \times \sin(48^\circ)}{\sin(70^\circ)} = \frac{15 \times 0.7431\ldots}{0.9397\ldots} = 11.9 \text{ m } (1 \text{ d.p.})$$

3 Finding Missing Angles

CORE CONCEPT & METHOD

Flip the rule to find angles: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$.

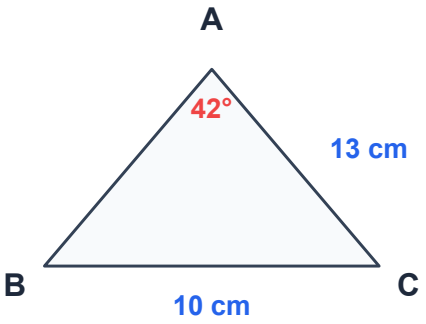
Remember, if the question says the angle is **obtuse**, subtract your calculator answer from 180°!

COMMON MISTAKE: Put $\sin x$ on TOP when finding an angle, and if the question states the angle is OBTUSE, subtract your acute $\sin^{-1}$ answer from 180°.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Calculator

In triangle ABC, side *a* = 10 cm, side *b* = 13 cm, and angle A = 42°. Find angle B.



STEP-BY-STEP SOLUTION:

Use the flipped Sine Rule:

$$\frac{\sin(B)}{b} = \frac{\sin(A)}{a}$$

Substitute:

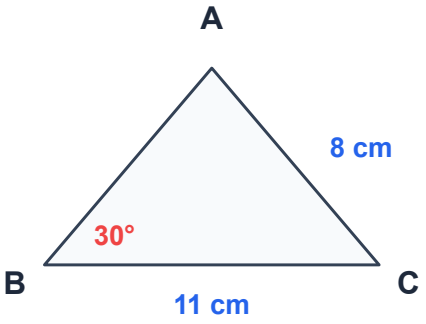
$$\frac{\sin(B)}{13} = \frac{\sin(42^\circ)}{10}$$

$$\sin(B) = \frac{13 \times \sin(42^\circ)}{10} = \frac{13 \times 0.6691\ldots}{10} = 0.8699\ldots$$

$$B = \sin^{-1}(0.8699\ldots) = 60.4^\circ \text{ (1 d.p.)}$$

EXAMPLE 3.2 Calculator

In triangle ABC, side *a* = 11 cm, side *b* = 8 cm, and angle B = 30°. Angle A is obtuse. Find angle A.



STEP-BY-STEP SOLUTION:

Use the Sine Rule:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\sin(A) = \frac{11 \times \sin(30^\circ)}{8} = \frac{11 \times 0.5}{8} = 0.6875$$

The calculator gives $\sin^{-1}(0.6875) = 43.4^\circ$, but angle A is **obtuse**, so:

$$A = 180^\circ - 43.4^\circ = 136.6^\circ \text{ (1 d.p.)}$$

4 Problem Solving

CORE CONCEPT & METHOD

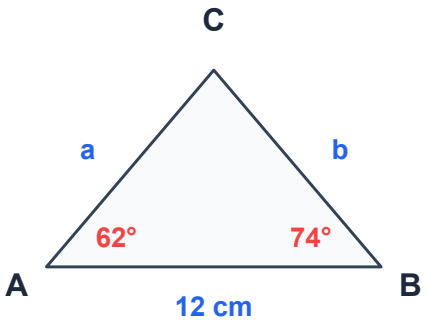
Combine the Sine Rule with the Area of a Triangle formula ($\frac{1}{2}ab\sin(C)$), Pythagoras, or angles in a triangle adding to 180°.

> **COMMON MISTAKE:** Remember angles in a triangle sum to 180° — you often need to subtract the two given angles from 180° first to unlock a matching opposite pair.

MODEL WORKED EXAMPLES

EXAMPLE 4.1 Calculator

Triangle ABC has angle A = 62°, angle B = 74°, and side AB = 12 cm. Find the perimeter of the triangle.



STEP-BY-STEP SOLUTION:

Step 1 — Find angle C:

$$C = 180^\circ - 62^\circ - 74^\circ = 44^\circ$$

Step 2 — Side AB (= c) is opposite angle C = 44°. Use Sine Rule to find BC (= a, opposite A = 62°):

$$\frac{a}{\sin(62^\circ)} = \frac{12}{\sin(44^\circ)} \implies a = \frac{12\sin(62^\circ)}{\sin(44^\circ)} = 15.25\ldots \text{ cm}$$

Step 3 — Find AC (= b, opposite B = 74°):

$$b = \frac{12\sin(74^\circ)}{\sin(44^\circ)} = 16.61\ldots \text{ cm}$$

Step 4 — Perimeter:

$$12 + 15.25\ldots + 16.61\ldots = 43.9 \text{ cm (1 d.p.)}$$

EDEXCEL GCSE EXAM PRACTICE

The Sine Rule — Practice Questions

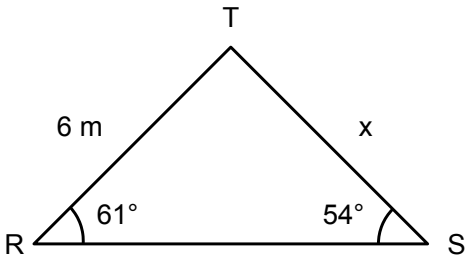
Total: 21 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator

(3 marks)

Work out the value of x.
Give your answer to 1 decimal place.



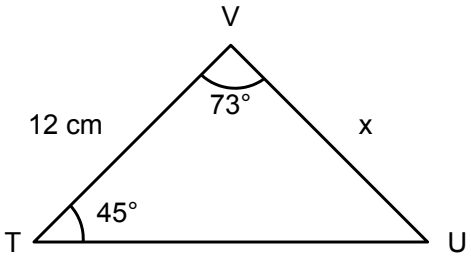
WORKING OUT SPACE

Answer: .....

Question 2 Calculator

(3 marks)

Work out the value of x.
Give your answer to 1 decimal place.

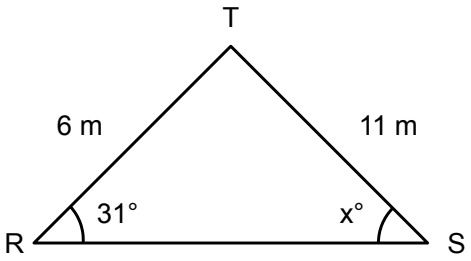


WORKING OUT SPACE

Answer: .....

Question 3 Calculator (3 marks)

Work out the size of angle x.
Give your answer to 1 decimal place.

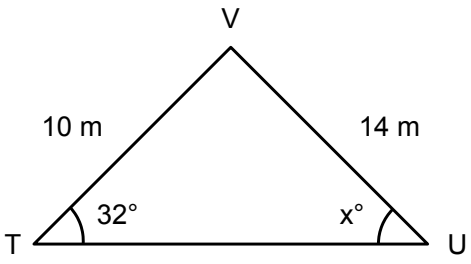


WORKING OUT SPACE

Answer: .....

Question 4 Calculator (3 marks)

Work out the size of angle x.
Give your answer to 1 decimal place.

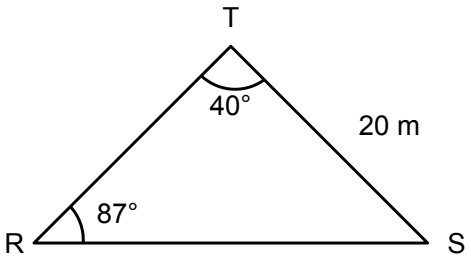


WORKING OUT SPACE

Answer: .....

Question 5 Calculator **(5 marks)**

Work out the area of triangle RST.
Give your answer to 1 decimal place.

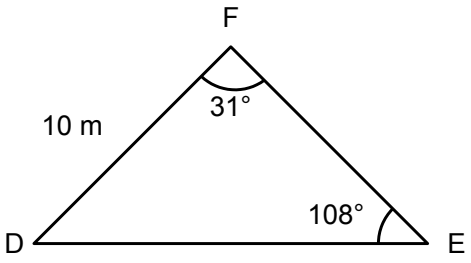


WORKING OUT SPACE

Answer: .....

Question 6 Calculator **(4 marks)**

Work out the perimeter of triangle DEF.
Give your answer to 1 decimal place.



WORKING OUT SPACE

Answer: .....

GEOMETRY AND MEASURES — NON-RIGHT-ANGLED TRIANGLES

The Cosine Rule

1 Finding Missing Sides

CORE CONCEPT & METHOD

The Cosine Rule states: $a^2 = b^2 + c^2 - 2bc\cos(A)$.

Use this to find a missing side when you know two sides and the *included angle* (the angle between them).

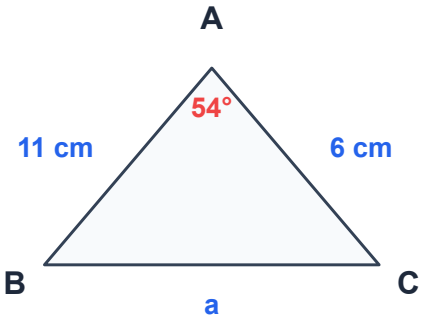
COMMON MISTAKE: In $a^2 = b^2 + c^2 - 2bc\cos A$, remember to take the SQUARE ROOT at the end, and note that when $A > 90°$, $\cos A$ is negative so $-2bc\cos A$ ADDS to $b^2 + c^2$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Calculator

A triangle has sides *b* = 6 cm and *c* = 11 cm with an included angle A = 54°. Find side *a*.



STEP-BY-STEP SOLUTION:

Apply the Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

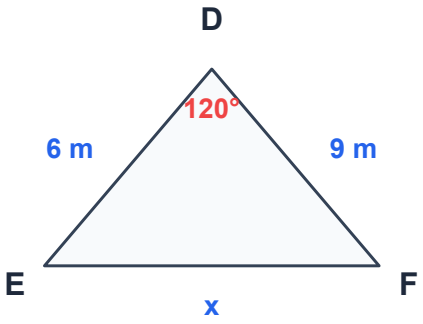
Substitute the values:

$$a^2 = 6^2 + 11^2 - 2(6)(11)\cos(54°)$$
$$a^2 = 36 + 121 - 132 \times 0.5878\ldots$$
$$a^2 = 157 - 77.59\ldots = 79.41\ldots$$
$$a = \sqrt{79.41\ldots} = 8.9 \text{ cm } (1 \text{ d.p.})$$

EXAMPLE 1.2

Calculator

In triangle DEF, DE = 6 m, DF = 9 m, and angle D = 120°. Find EF.



STEP-BY-STEP SOLUTION:

Here $b = 9$, $c = 6$, and the included angle is $D = 120°$:

$$d^2 = 9^2 + 6^2 - 2(9)(6)\cos(120°)$$

Since $\cos(120°) = -0.5$:

$$d^2 = 81 + 36 - 108 \times (-0.5) = 117 + 54 = 171$$

$$d = \sqrt{171} = 13.1 \text{ m } (1 \text{ d.p.})$$

2 Finding Missing Angles

CORE CONCEPT & METHOD

Rearrange the rule to find angles: $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$.

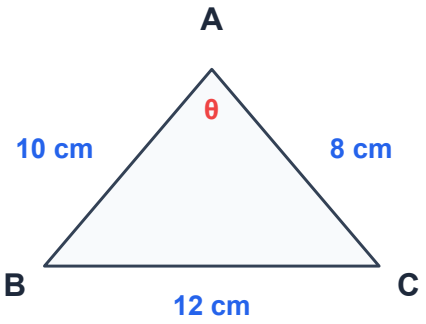
Use this when you know all three sides of the triangle.

COMMON MISTAKE: In $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, the subtracted side a^2 MUST be the side opposite the angle A you are trying to find.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

A triangle has sides *a* = 12 cm, *b* = 8 cm, and *c* = 10 cm. Find angle A.



STEP-BY-STEP SOLUTION:

Use the rearranged Cosine Rule:

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

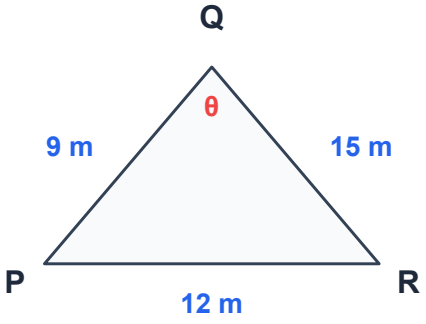
Substitute:

$$\cos(A) = \frac{8^2 + 10^2 - 12^2}{2 \times 8 \times 10} = \frac{64 + 100 - 144}{160} = \frac{20}{160} = 0.125$$

$$A = \cos^{-1}(0.125) = 82.8^\circ \text{ (1 d.p.)}$$

EXAMPLE 2.2 Calculator

Triangle PQR has PQ = 9 m, QR = 15 m, PR = 12 m. Find angle Q.



STEP-BY-STEP SOLUTION:

Angle Q is between sides PQ and QR, so the side opposite is PR = 12 m. Let $a = PR = 12$, $b = PQ = 9$, $c = QR = 15$:

$$\cos(Q) = \frac{9^2 + 15^2 - 12^2}{2 \times 9 \times 15} = \frac{81 + 225 - 144}{270} = \frac{162}{270} = 0.6$$

$$Q = \cos^{-1}(0.6) = 53.1^\circ \text{ (1 d.p.)}$$

3 Problem Solving

CORE CONCEPT & METHOD

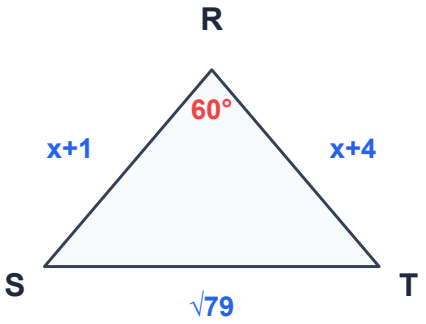
You may need to form quadratic equations if the sides are given algebraically, or combine the Cosine Rule with the Area formula ($\frac{1}{2}ab\sin(C)$).

COMMON MISTAKE: When sides are algebraic expressions like $(2x-4)$ and $(x+1)$ with 60°, substitute $\cos 60^\circ = \frac{1}{2}$ so $-2bc\cos 60^\circ$ simplifies cleanly to $-bc$.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Non-Calculator

A triangle has sides $(x+1)$ cm, $(x+4)$ cm and $\sqrt{79}$ cm. The angle between the first two sides is 60°. Find x.



STEP-BY-STEP SOLUTION:

The side opposite the 60° angle is $\sqrt{79}$. Apply the Cosine Rule:

$$(\sqrt{79})^2 = (x+1)^2 + (x+4)^2 - 2(x+1)(x+4)\cos(60^\circ)$$

Since $\cos(60^\circ) = 0.5$:

$$79 = (x^2+2x+1) + (x^2+8x+16) - (x+1)(x+4)$$
$$79 = 2x^2+10x+17 - (x^2+5x+4)$$
$$79 = x^2+5x+13$$
$$x^2+5x-66 = 0$$
$$(x+11)(x-6) = 0$$

Since lengths must be positive, $x = 6$.

EDEXCEL GCSE EXAM PRACTICE

The Cosine Rule — Practice Questions

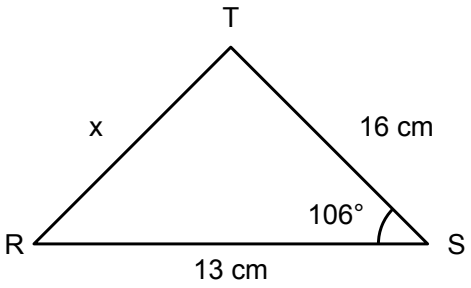
Total: 20 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator

(3 marks)

Work out the value of x.
Give your answer to 1 decimal place.



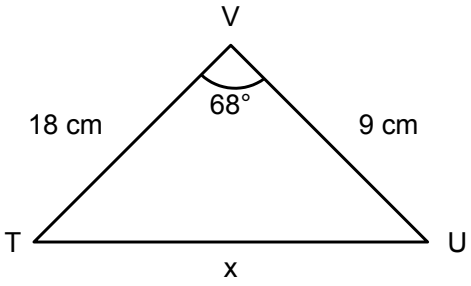
WORKING OUT SPACE

Answer: .....

Question 2 Calculator

(3 marks)

Work out the value of x.
Give your answer to 1 decimal place.

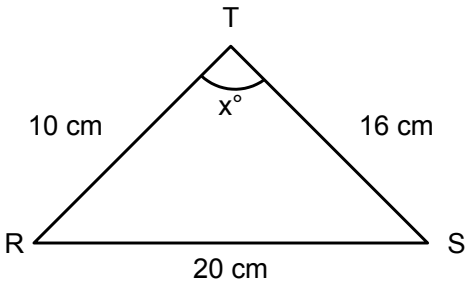


WORKING OUT SPACE

Answer: .....

Question 3 Calculator (3 marks)

Work out the size of angle x.
Give your answer to 1 decimal place.

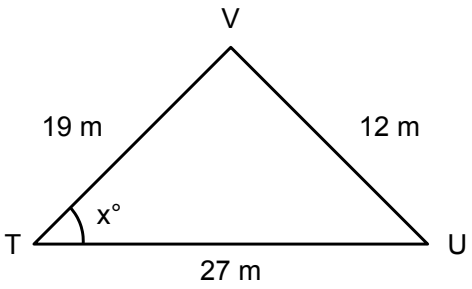


WORKING OUT SPACE

Answer: .....

Question 4 Calculator (3 marks)

Work out the size of angle x.
Give your answer to 1 decimal place.

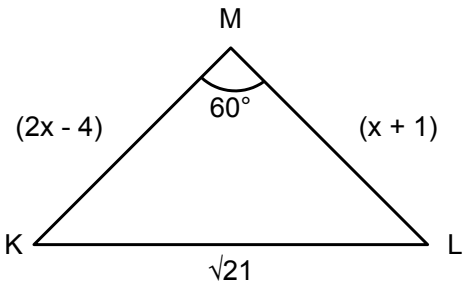


WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator **(4 marks)**

Work out the value of x.

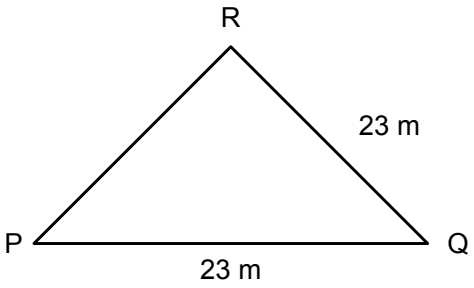


WORKING OUT SPACE

Answer: .....

Question 6 Calculator **(4 marks)**

The area of the triangle is $193\ \text{m}^2$.
Calculate the perimeter of the triangle.
Give your answer to 1 decimal place.



WORKING OUT SPACE

Answer: .....

GEOMETRY AND MEASURES — NON-RIGHT-ANGLED TRIANGLES

Area of Any Triangle

1 Finding the Area

CORE CONCEPT & METHOD

To find the area of *any* triangle, use the formula: $\text{Area} = \frac{1}{2}ab\sin(C)$.

You need two sides (a and b) and the *included angle* (C) between them.

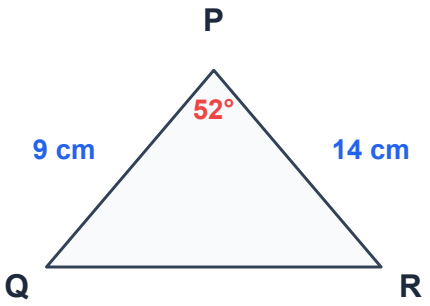
COMMON MISTAKE: In $\text{Area} = \frac{1}{2}ab\sin C$, angle C MUST be the included angle sandwiched BETWEEN sides a and b — never use $\frac{1}{2} \times \text{base} \times \text{slanted side}$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Calculator

A triangle has two sides of length 9 cm and 14 cm with an included angle of 52°. Find its area.



STEP-BY-STEP SOLUTION:

Use the formula $\text{Area} = \frac{1}{2}ab\sin(C)$.

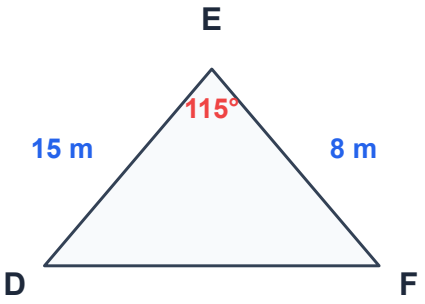
Substitute $a = 9$, $b = 14$, $C = 52°$:

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 9 \times 14 \times \sin(52°) \\ &= \frac{1}{2} \times 9 \times 14 \times 0.7880\ldots \\ &= 49.6 \text{ cm}^2 \text{ (1 d.p.)}\end{aligned}$$

EXAMPLE 1.2

Calculator

Triangle DEF has DE = 15 m, EF = 8 m, and angle E = 115°. Find the area.



STEP-BY-STEP SOLUTION:

$a = DE = 15$, $b = EF = 8$, included angle $E = 115^\circ$:

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 15 \times 8 \times \sin(115^\circ) \\ &= \frac{1}{2} \times 120 \times 0.9063\ldots \\ &= 54.4 \text{ m}^2 \text{ (1 d.p.)}\end{aligned}$$

2 Finding a Missing Angle

CORE CONCEPT & METHOD

If you know the area and two sides, you can find the included angle:

$$\sin(C) = \frac{\text{Area}}{\frac{1}{2}ab}.$$

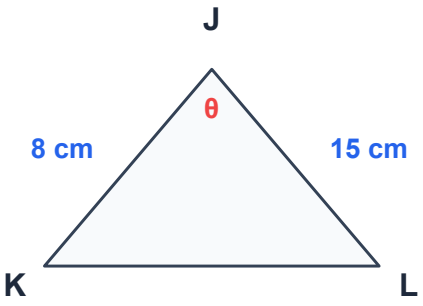
Don't forget to use inverse sine ($\sin^{-1}$) at the end!

COMMON MISTAKE: After rearranging to $\sin C = \dfrac{2 \times \text{Area}}{ab}$, do not forget to apply $\sin^{-1}$ to obtain the angle in degrees.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

A triangle has sides of 8 cm and 15 cm. Its area is 42 cm². Find the included angle between the two sides.



STEP-BY-STEP SOLUTION:

Start with the formula:

$$\text{Area} = \frac{1}{2}ab\sin(C)$$

Substitute the known values:

$$42 = \frac{1}{2} \times 8 \times 15 \times \sin(C)$$
$$42 = 60\sin(C)$$
$$\sin(C) = \frac{42}{60} = 0.7$$
$$C = \sin^{-1}(0.7) = 44.4^\circ \text{ (1 d.p.)}$$

EXAMPLE 2.2 Calculator

Two sides of a triangle measure 12 m and 10 m. The area of the triangle is 45 m². Find the angle between these sides.

STEP-BY-STEP SOLUTION:

$$45 = \frac{1}{2} \times 12 \times 10 \times \sin(C)$$
$$45 = 60\sin(C)$$
$$\sin(C) = \frac{45}{60} = 0.75$$
$$C = \sin^{-1}(0.75) = 48.6^\circ \text{ (1 d.p.)}$$

3 Problem Solving

CORE CONCEPT & METHOD

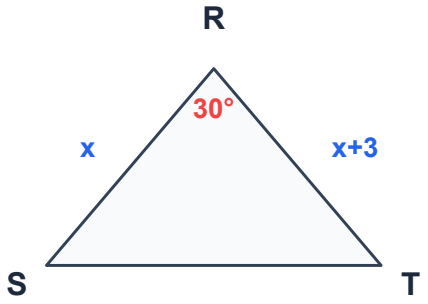
Sometimes the sides are given as algebraic expressions or ratios. You'll need to set up a quadratic or linear equation and solve for x!

COMMON MISTAKE: Use exact trig values ($\sin 30^\circ = \frac{1}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$) when forming a quadratic from the area, and reject any negative root since side lengths must be positive.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Non-Calculator

The area of a triangle is 10 cm². Two sides are (x) cm and $(x+3)$ cm, and the included angle is 30°. Find x.



STEP-BY-STEP SOLUTION:

Use the Area formula:

$$10 = \frac{1}{2} \times x \times (x+3) \times \sin(30^\circ)$$

Substitute $\sin(30^\circ) = 0.5$:

$$10 = \frac{1}{2} \times x(x+3) \times 0.5 = \frac{1}{4}x(x+3)$$

Multiply both sides by 4:

$$x(x+3) = 40$$
$$x^2 + 3x - 40 = 0$$
$$(x+8)(x-5) = 0$$

Since lengths must be positive, $x = 5$ cm.

EDEXCEL GCSE EXAM PRACTICE

Area of Any Triangle — Practice Questions

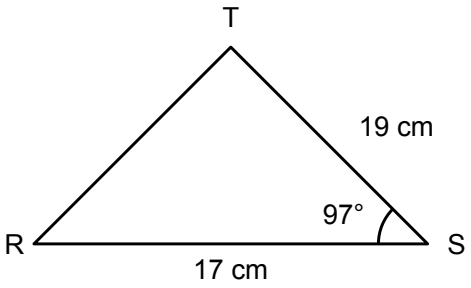
Total: 19 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Calculator

(2 marks)

Work out the area of the triangle.
Give your answer to 1 decimal place.



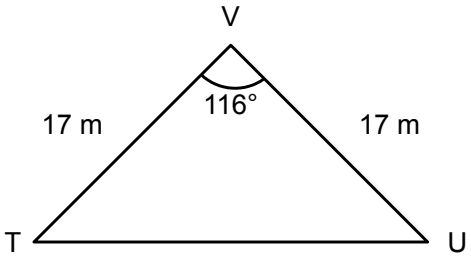
WORKING OUT SPACE

Answer: .....

Question 2 Calculator

(2 marks)

Work out the area of the triangle.
Give your answer to 1 decimal place.

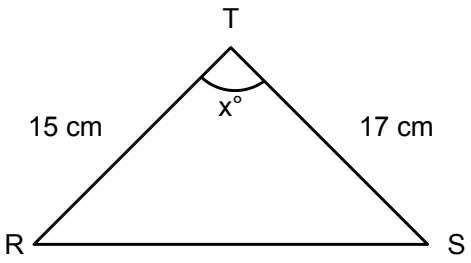


WORKING OUT SPACE

Answer: .....

Question 3 Calculator **(3 marks)**

The area of the triangle is 84 cm².
Work out the value of x.
Give your answer to 1 decimal place.

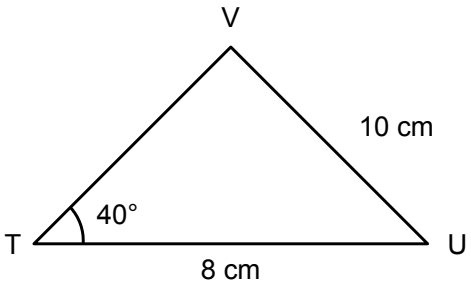


WORKING OUT SPACE

Answer: .....

Question 4 Calculator **(4 marks)**

The diagram shows triangle TUV.
Work out the area of triangle TUV.
Give your answer to 1 decimal place.



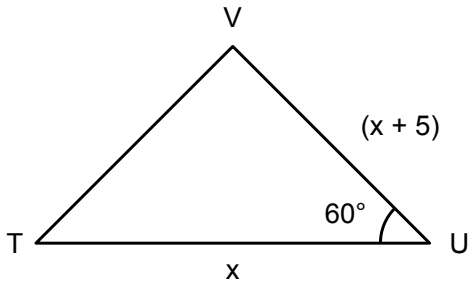
WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator **(4 marks)**

The area of the triangle is $\frac{50\sqrt{3}}{4}$ cm^2.

Work out the value of x.

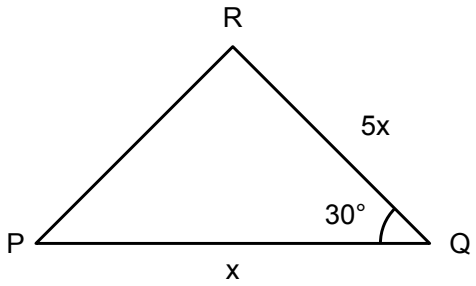


WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(4 marks)**

The ratio of the lengths of two sides of the triangle is 5:1.
The angle between them is $30°$.
The area of the triangle is 125 cm^2.
Work out the length of the shorter side, x.



WORKING OUT SPACE

Answer: .....

PROBABILITY — DEPENDENT EVENTS

Conditional Probability

1 Without Replacement (Items)

CORE CONCEPT & METHOD

When an item is taken and **not replaced**, the probabilities change for the second pick!

If you take a Red counter from a bag of 10, there are now only 9 counters left in total, and 1 fewer Red counter.

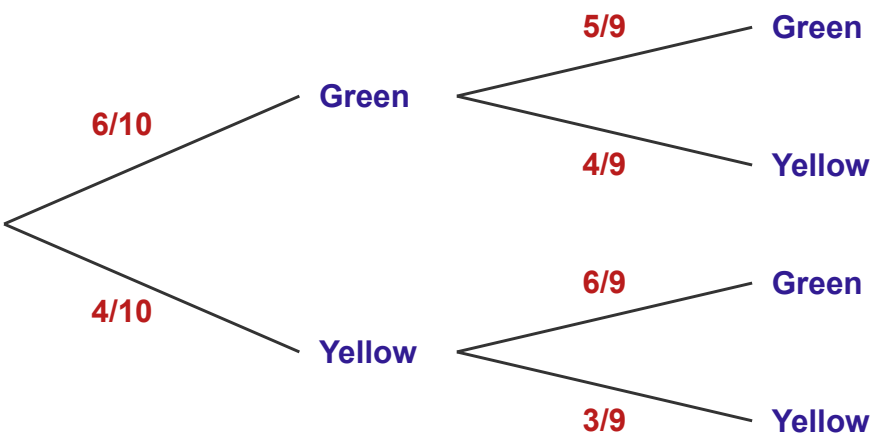
> **COMMON MISTAKE:** Without replacement, reduce the total denominator by 1 on EVERY second-pick branch, and reduce the numerator by 1 ONLY for the colour already picked first.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

A box contains 6 green marbles and 4 yellow marbles. Two marbles are taken out without replacement. Find the probability that both marbles are green.



STEP-BY-STEP SOLUTION:

First pick: There are 10 marbles in total, 6 of which are green.

$$P(\text{Green}_1) = \frac{6}{10}$$

Second pick: One green marble has been removed, so there are now 5 green marbles left out of 9 total.

$$P(\text{Green}_2 \mid \text{Green}_1) = \frac{5}{9}$$

Both green: Multiply along the branches:

$$P(\text{both green}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \mathbf{\frac{1}{3}}$$

2 Given Probabilities

CORE CONCEPT & METHOD

Sometimes you are explicitly told how the probability changes. For example: 'If it rains today, the probability it rains tomorrow is 0.8. If it doesn't, the probability is 0.3'.

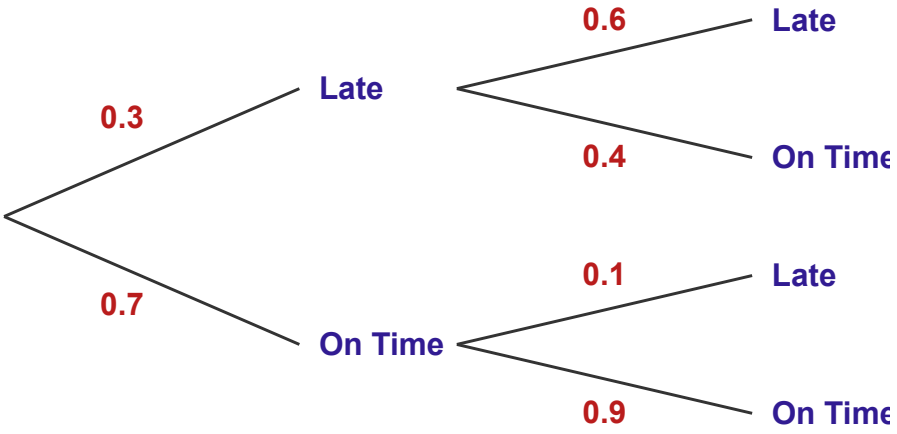
Use these specific numbers on the branches of your tree diagram!

> **COMMON MISTAKE:** Each pair of branches from a single node must add up to 1 — check the top second-stage pair and bottom second-stage pair separately.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 — Non-Calculator

The probability that a bus is late on Wednesday is 0.3. If it is late on Wednesday, the probability it is late on Thursday is 0.6. If it is on time on Wednesday, the probability it is late on Thursday is 0.1.
Find the probability that the bus is late on both days.



STEP-BY-STEP SOLUTION:

Draw a tree diagram with two branches on Wednesday (Late / On Time), then two branches on Thursday for each case.

P(Late Wednesday AND Late Thursday) — multiply along the *Late, Late* branch:

$$0.3 \times 0.6 = \mathbf{0.18}$$

3 Without Replacement (Numbers)

CORE CONCEPT & METHOD

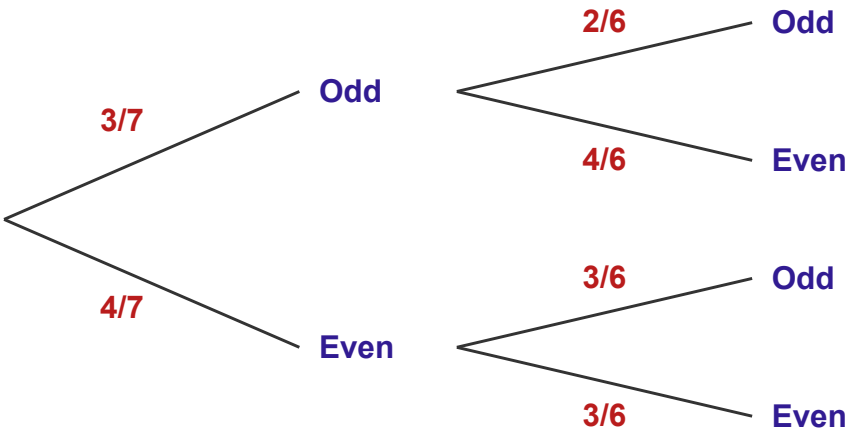
If you take a number card and don't replace it, the pool of available numbers shrinks. You must count how many successful outcomes are left for the second pick.

> **COMMON MISTAKE:** Multiply along branches for "AND", then ADD the separate branch products for "different colours" or use $1 - P(\text{none})$ for "at least one".

MODEL WORKED EXAMPLES

EXAMPLE 3.1 Non-Calculator

A set of tiles has the numbers 2, 3, 4, 5, 6, 7, 8. Two tiles are drawn at random without replacement. Find the probability that both numbers drawn are ODD.



STEP-BY-STEP SOLUTION:

First draw: There are 7 tiles in total. The odd numbers are 3, 5, and 7 (so there are 3 odd tiles).

$$P(\text{Odd}_1) = \frac{3}{7}$$

Second draw: One odd tile is gone. There are 2 odd tiles left out of 6 remaining tiles.

$$P(\text{Odd}_2 \mid \text{Odd}_1) = \frac{2}{6}$$

Both odd: Multiply along the branches:

$$P(\text{both odd}) = \frac{3}{7} \times \frac{2}{6} = \frac{6}{42} = \mathbf{\frac{1}{7}}$$

EDEXCEL GCSE EXAM PRACTICE

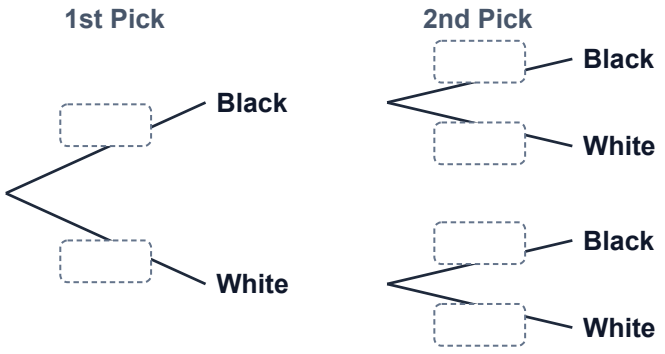
Conditional Probability — Practice Questions

Total: 19 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator (2 marks)

A bag contains 5 Black counters and 5 White counters. Two counters are taken at random **without replacement**.
Complete the probability tree diagram and find the probability of picking Black both times.

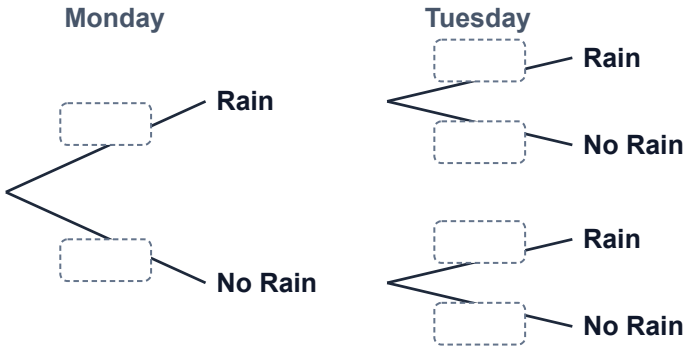


WORKING OUT SPACE

Answer: .....

Question 2 Non-Calculator (3 marks)

The probability that it rains on Monday is 0.2.
If it rains on Monday, the probability that it rains on Tuesday is 0.65.
If it does not rain on Monday, the probability that it rains on Tuesday is 0.15.
Complete the probability tree diagram and work out the probability that it rains on **exactly one** of the two days.

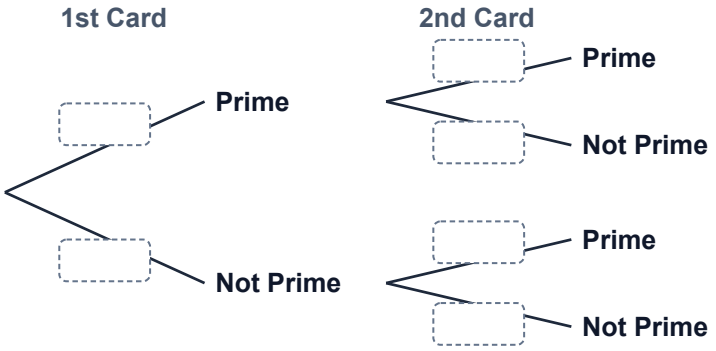


WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator (3 marks)

A set of 8 cards has the numbers $1, 2, 3, 4, 5, 6, 7, 8$. Two cards are drawn at random **without replacement**. Complete the probability tree diagram and work out the probability that **at least one** of the cards drawn is a prime number.

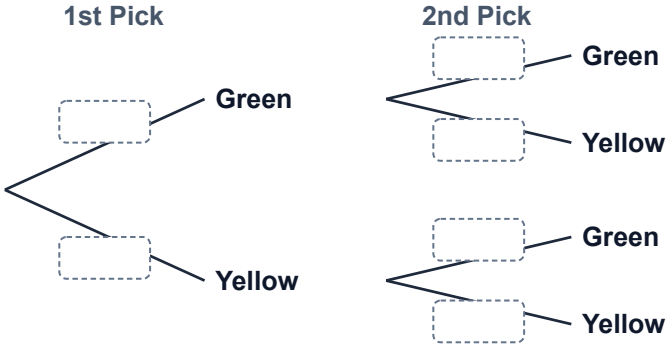


WORKING OUT SPACE

Answer: .....

Question 4 Calculator (3 marks)

A bag contains 7 Green counters and 5 Yellow counters. Two counters are taken at random from the bag **without replacement**.
Complete the probability tree diagram and work out the probability that the two counters taken are of **different** colours.

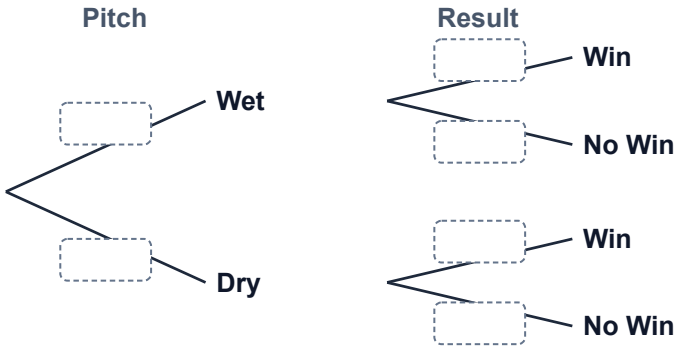


WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator (4 marks)

The probability that a rugby team plays on a wet pitch is 0.3.
When the pitch is wet, the probability that the team wins is 0.6.
When the pitch is dry, the probability that the team wins is 0.8.
(a) Complete the probability tree diagram.



(b) Work out the overall probability that the team wins the match.

WORKING OUT SPACE

(b) .....

Question 6 Calculator (4 marks)

A bag contains 4 Red counters, 5 Blue counters, and 3 Green counters.
Two counters are taken at random from the bag **without replacement**.
Work out the probability that both counters taken are the **same** colour. Give your answer as a fraction in its simplest form.

WORKING OUT SPACE

Answer: .....

STATISTICS — DATA REPRESENTATION

Histograms

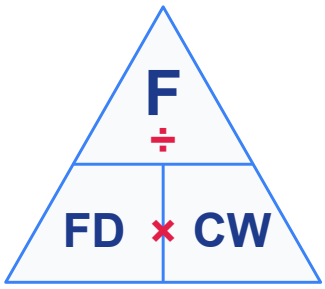
1 Frequency Density

CORE CONCEPT & METHOD

Unlike bar charts, histograms are used for continuous data with **unequal class widths**.

To make it fair, we plot **Frequency Density (FD)** on the y-axis, not frequency!

Remember the FD triangle! Cover up the one you want to find.



Frequency = **FD** × **C**lass **W**idth

FD = **F**requency ÷ **C**lass **W**idth

Class **W**idth = **F**requency ÷ **FD**

COMMON MISTAKE: Always plot $\text{Frequency Density} = \dfrac{\text{Frequency}}{\text{Class Width}}$ on the vertical axis — NEVER plot raw frequency when class widths are unequal.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

A grouped frequency table has the interval $30 < x \leq 50$ with a frequency of 84. Calculate the frequency density for this bar.

STEP-BY-STEP SOLUTION:

Step 1 — Find the class width:

Class width $= 50 - 30 = 20$

Step 2 — Apply the formula:

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}} = \frac{84}{20} = 4.2$$

Answer: The bar for this interval has height **4.2** on the frequency density axis.

2 Frequencies from Areas

CORE CONCEPT & METHOD

On a histogram, the **AREA** of the bar represents the Frequency.

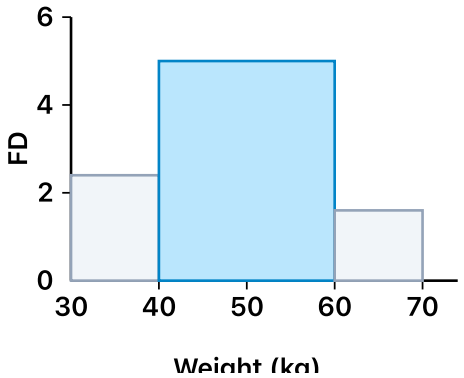
$$\text{Frequency} = \text{Frequency Density} \times \text{Class Width}$$

COMMON MISTAKE: In a histogram, $\text{Frequency} = \text{Area of bar} = \text{Class Width} \times \text{Frequency Density}$ — if the y-axis is unnumbered, use a bar with known frequency to find the scale first.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Non-Calculator

A histogram shows the distribution of weights.



Find the frequency for the interval $40 < w \leq 60$.

STEP-BY-STEP SOLUTION:

Step 1 — Identify Class Width (Width of the bar):
The bar starts at 40 and ends at 60.
Class Width = 60 - 40 = 20.

Step 2 — Identify Frequency Density (Height of the bar):
The bar goes up to 5 on the y-axis.
FD = 5

Step 3 — Calculate Frequency (Area of the bar):

$$\text{Frequency} = \text{FD} \times \text{Class Width}$$
$$\text{Frequency} = 5 \times 20 = 100$$

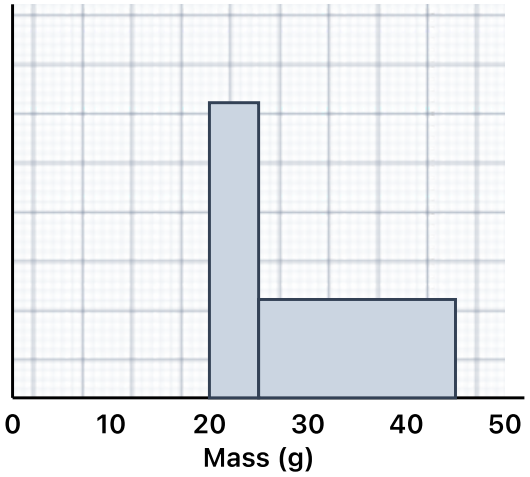
Answer: There are 100 items in this interval.

EXAMPLE 2.2

Non-Calculator

A partially completed frequency table and histogram show the mass of some pears. The y-axis has no scale.

Mass, m (g)	Frequency
$0 < m \leq 10$	24
$10 < m \leq 20$	60
$20 < m \leq 25$	?
$25 < m \leq 45$	40
$45 < m \leq 50$	16



Calculate the missing frequency for the interval 20 < m ≤ 25.

STEP-BY-STEP SOLUTION:

Step 1 — Deduce the scale from the known bar (25 < m ≤ 45):

Frequency = 40

Class Width = $45 - 25 = 20$

$$\text{FD} = \text{Frequency} \div \text{Class Width} = 40 \div 20 = 2$$

On the graph, this bar is exactly **2 large squares** tall. So, 2 large squares = an FD of 2, meaning **1 large square = an FD of 1**.

Step 2 — Find the FD for the missing bar (20 < m ≤ 25):

On the graph, this bar is **6 large squares** tall.

$$\text{FD} = 6 \times 1 = 6$$

Step 3 — Calculate the missing frequency:

Class Width = $25 - 20 = 5$

$$\text{Frequency} = \text{FD} \times \text{Class Width} = 6 \times 5 = 30$$

Answer: The missing frequency is 30.

3 Estimating Proportions

CORE CONCEPT & METHOD

If a bar goes from 10 to 20 with a frequency of 40, we assume those 40 items are spread evenly.

To estimate how many are between 15 and 20 (which is half the bar's width), you take half the frequency (20).

> **COMMON MISTAKE:** When splitting a bar at a value (or finding the median), take that fraction of the class width multiplied by its Frequency Density, assuming values are evenly distributed across the class.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 — Non-Calculator

A histogram bar for the interval $10 < t \leq 50$ has a frequency of 180.

Estimate the frequency of items between 20 and 30.

STEP-BY-STEP SOLUTION:

Step 1 — Find the total width of the bar:
Class Width = 50 - 10 = 40.

Step 2 — Find the width of the target section:
We want the section from 20 to 30.
Target Width = 30 - 20 = 10.

Step 3 — Find the proportion:
The target section is $\frac{10}{40}$ (or one-quarter) of the entire bar.

Step 4 — Calculate the estimated frequency:
Assuming the 180 items are spread evenly, we take one-quarter of them.

$$180 \div 4 = 45$$

Answer: The estimated frequency is 45.

EDEXCEL GCSE EXAM PRACTICE

Histograms — Practice Questions

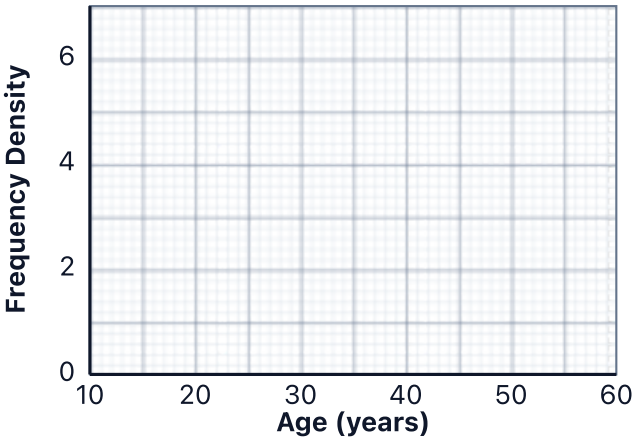
Total: 23 marks

Check the Calculator or Non-Calculator label on each question before you start. Show your working clearly in the box.

Question 1 Non-Calculator **(3 marks)**

The table shows information about the ages of 90 members at a tennis club.

Age, a (years)	Frequency
$10 < a \leq 20$	14
$20 < a \leq 25$	18
$25 < a \leq 30$	22
$30 < a \leq 45$	24
$45 < a \leq 60$	12



On the grid, draw a histogram for the information in the table, and state the **frequency density** for the interval $25 < a \leq 30$.

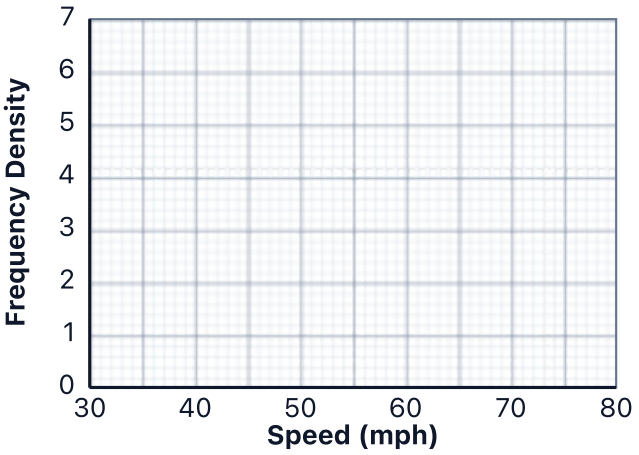
WORKING OUT SPACE

Frequency density = ...

Question 2 Calculator **(4 marks)**

The table shows information about the speed, in mph, of some vans passing a speed camera.

Speed, s (mph)	Frequency
$30 < s \leq 45$	9
$45 < s \leq 50$	14
$50 < s \leq 55$	32
$55 < s \leq 65$	36
$65 < s \leq 80$	9



(a) On the grid, draw a histogram for the information in the table.
(b) Work out an estimate for the number of vans travelling at over $60\ \text{mph}$.

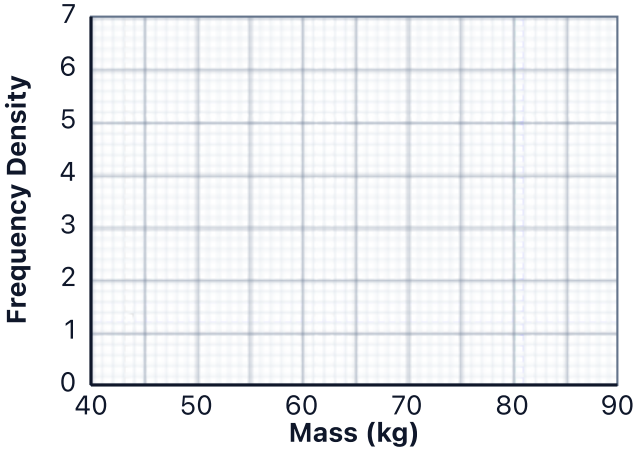
WORKING OUT SPACE

(b) .....

Question 3 Calculator **(4 marks)**

The table shows information about the mass, in kg, of 80 sheep on a farm.

Mass, m (kg)	Frequency
$40 < m \leq 55$	12
$55 < m \leq 65$	20
$65 < m \leq 70$	32
$70 < m \leq 90$	16



(a) On the grid, draw a histogram for the information in the table.
(b) Find an estimate for the median mass.

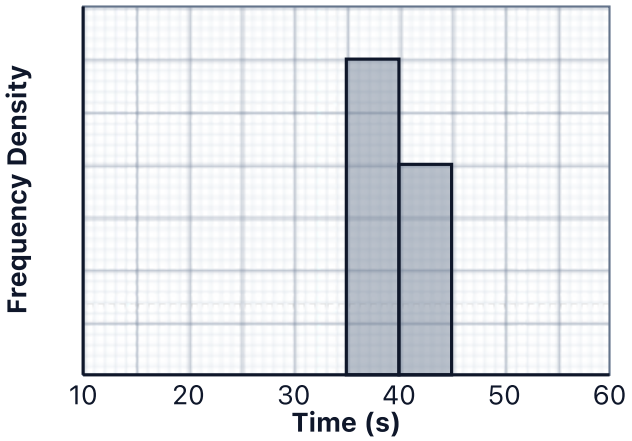
WORKING OUT SPACE

(b) ..... kg

Question 4 Non-Calculator **(4 marks)**

The table and histogram show information about the time, in seconds, taken by some students to solve a logic puzzle.

Time, t (s)	Frequency
$10 < t \leq 25$	18
$25 < t \leq 35$	34
$35 < t \leq 40$	30
$40 < t \leq 45$	
$45 < t \leq 60$	12



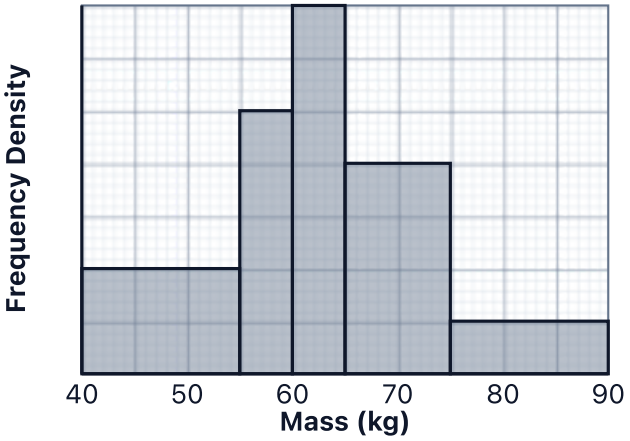
(a) Use the information in the table to complete the histogram.
(b) Use the histogram to find the missing frequency for the interval $40 < t \leq 45$ to complete the table.

WORKING OUT SPACE

(b) Missing frequency = ...

Question 5 Non-Calculator (4 marks)

The histogram shows information about the mass, in kg, of some calves on a farm.



30 calves have a mass between $40\ \text{kg}$ and $55\ \text{kg}$.

(a) Work out an estimate for the number of calves with a mass greater than $70\ \text{kg}$.

(b) Explain why your answer to part (a) is only an estimate.

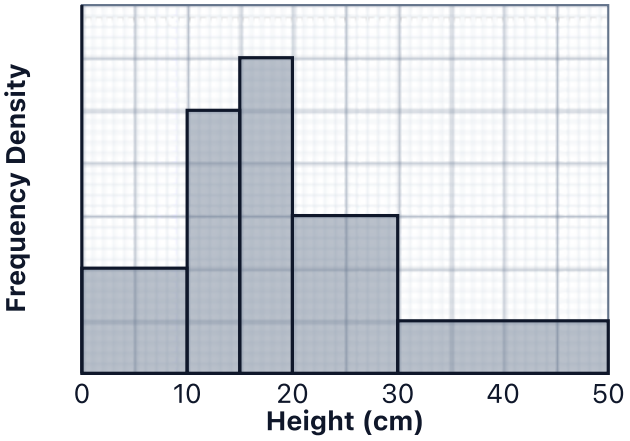
WORKING OUT SPACE

(a) .....

(b) Reason: .....

Question 6 Non-Calculator (4 marks)

The histogram shows information about the height, in cm, of some seedlings in a greenhouse.



(a) Work out an estimate for the proportion of seedlings that are over $25\ \text{cm}$ tall.

(b) Explain why your answer to part (a) is only an estimate.

WORKING OUT SPACE

(a) .....

(b) Reason: .....

EDEXCEL GCSE HIGHER PAPER 1 — NON-CALCULATOR (GRADE 7)

Mixed Review 1 — Choosing the Right Method (Non-Calculator)

Total: 45 marks

These 15 unseen Grade 7 questions are mixed across topics with no topic headings — just like a real Edexcel Paper 1 exam. Identify the underlying method first, set out full algebraic or geometric working, and give exact simplified answers.

Question 1 — Non-Calculator — (2 marks)

Simplify fully:

$$\sqrt{75} + \frac{12\sqrt{24}}{3\sqrt{2}}$$

Give your answer in the form $k\sqrt{3}$, where k is an integer.

WORKING OUT SPACE

Answer: .....

Question 2 — Non-Calculator — (2 marks)

Factorise fully:

$$6x^2 + 7x - 20$$

WORKING OUT SPACE

Answer: .....

Question 3 Non-Calculator **(3 marks)**

Here are the first five terms of a quadratic sequence:

$$2, \quad 12, \quad 28, \quad 50, \quad 78$$

Find an expression, in terms of n, for the nth term of this sequence.

WORKING OUT SPACE

Answer: .....

Question 4 Non-Calculator **(3 marks)**

y is inversely proportional to the square of x. When $x = 2$, $y = 9$.
Find the positive value of x when $y = 4$.

WORKING OUT SPACE

Answer: .....

Question 5 Non-Calculator **(3 marks)**

Expand and simplify fully:

$$(4 + \sqrt{3})^2 - (2 - \sqrt{3})(2 + \sqrt{3})$$

WORKING OUT SPACE

Answer: .....

Question 6 Non-Calculator **(3 marks)**

A bag contains 5 red counters and 4 blue counters. Two counters are taken at random from the bag without replacement.
Work out the probability that the two counters taken are of different colours. Give your answer as a fraction in its simplest form.

WORKING OUT SPACE

Answer: .....

Question 7 Non-Calculator **(3 marks)**

Make x the subject of the formula:

$$y = \frac{3x - 8}{x + 2}$$

WORKING OUT SPACE

Answer: .....

Question 8 Non-Calculator **(3 marks)**

Two solid cones, A and B, are mathematically similar. The total surface area of cone A is $36\ \text{cm}^2$ and the total surface area of cone B is $100\ \text{cm}^2$.
The volume of cone A is $54\ \text{cm}^3$. Work out the volume of cone B in cm^3.

WORKING OUT SPACE

Answer: .....

Question 9 Non-Calculator **(3 marks)**

Rationalise the denominator and simplify fully:

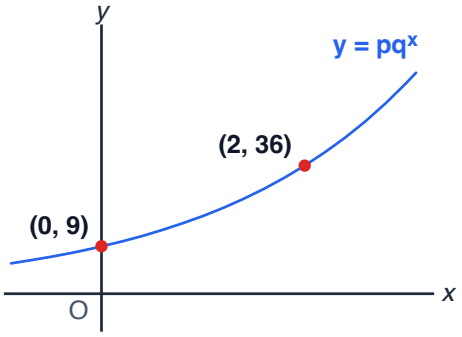
$$\frac{14}{3-\sqrt{2}}$$

WORKING OUT SPACE

Answer: .....

Question 10 Non-Calculator **(3 marks)**

The diagram shows a sketch of the exponential curve $y = pq^x$, where p and q are positive constants. The curve passes through $(0, 9)$ and $(2, 36)$.



(a) Find the value of p and the value of q. (b) Find the value of y when $x = 3$.

WORKING OUT SPACE

(a) $p =$ ..... $q =$ ..... **(b)** $y =$ .....

Question 11 Non-Calculator **(3 marks)**

The functions f and g are defined such that $f(x) = 2x - 5$ and $g(x) = x^2 + 4$.
Solve the equation $gf(x) = 29$.

WORKING OUT SPACE

$x =$ ..... or $x =$ .....

Question 12 Non-Calculator **(3 marks)**

A cuboid $ABCDEFGH$ has base length $AB = 6$ cm, base width $BC = 8$ cm, and space diagonal $AG = 26$ cm.
Work out the vertical height AE of the cuboid in cm.

WORKING OUT SPACE

Answer: .....

Question 13 Non-Calculator **(3 marks)**

Simplify fully:

$$\frac{2x^2 + 5x - 3}{x^2 - 9} \div \frac{4x - 2}{x^2 + 2x - 15}$$

WORKING OUT SPACE

Answer: .....

Question 14 Non-Calculator **(4 marks)**

In triangle PQR, $PQ = x$ cm, $PR = (x + 4)$ cm, and angle $QPR = 30°$.
The area of triangle PQR is 8 cm^2. Form and solve a quadratic equation to find the value of x.

WORKING OUT SPACE

Answer: .....

Question 15 Non-Calculator **(4 marks)**

The table shows information about the time, t minutes, spent exercising by 100 students.

Time, t (minutes)	$0 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 40$	$40 < t \leq 60$
Frequency	16	24	36	24

(a) Calculate the frequency density for each of the four intervals.
(b) Work out an estimate for the fraction of the students who exercised for between 25 minutes and 50 minutes. Give your fraction in its simplest form.

WORKING OUT SPACE

(a) FDs: ..... **(b)** .....

EDEXCEL GCSE HIGHER PAPER 2 / 3 — CALCULATOR (GRADE 7)

Mixed Review 2 — Choosing the Right Method (Calculator)

Total: 52 marks

These 15 unseen Grade 7 questions are interleaved across topics to test method selection under Paper 2/3 conditions. Write down every formula, substitution, or bound clearly before rounding your final answer.

Question 1 Calculator (3 marks)

$d = \frac{m}{v}$, where $m = 45.6$ correct to 1 decimal place and $v = 8.20$ correct to 2 decimal places.
Work out the upper bound for d. Give your answer correct to 2 decimal places.

WORKING OUT SPACE

Answer: .....

Question 2 Calculator (3 marks)

Solve $3x^2 - 8x - 5 = 0$. Give your answers correct to 2 decimal places.

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 3 Calculator (3 marks)

In triangle ABC, $BC = 11$ cm, $AC = 15$ cm, and angle $BAC = 38°$. Angle ABC is acute.
Work out the size of angle ABC. Give your answer correct to 1 decimal place.

WORKING OUT SPACE

Answer: .....

Question 4 Calculator (3 marks)

P is directly proportional to the cube root of Q. When $Q = 64$, $P = 28$.
(a) Find a formula for P in terms of Q. (b) Work out the value of Q when $P = 35$.

WORKING OUT SPACE

(a) $P =$ ..... **(b)** $Q =$ .....

Question 5 Calculator (3 marks)

In triangle PQR, $PQ = 14$ cm, $PR = 19$ cm, and angle $QPR = 104°$.
Calculate the length of QR. Give your answer correct to 1 decimal place.

WORKING OUT SPACE

Answer: .....

Question 6 Calculator (3 marks)

Solve $\cos x° = 0.35$ for $0 \leq x \leq 360$. Give both of your answers correct to 1 decimal place.

WORKING OUT SPACE

$x =$ ..... ° **and** $x =$ ..... °

Question 7 Calculator (3 marks)

The function f is defined by $f(x) = \dfrac{5x - 2}{x + 3}$ for $x \neq -3$.

Find an expression for the inverse function $f^{-1}(x)$.

WORKING OUT SPACE

$f^{-1}(x) =$ .....

Question 8 Calculator (4 marks)

A frustum is made by removing a small cone of height 4 cm from the top of a mathematically similar large cone of height 12 cm and base radius 9 cm.

Calculate the volume of the frustum. Give your answer correct to 1 decimal place. *(Volume of a cone* $= \dfrac{1}{3}\pi r^2 h$*)*

WORKING OUT SPACE

Answer: .....

Question 9 Calculator (3 marks)

$a = \dfrac{v^2}{2s}$, where $v = 18.4$ correct to 3 significant figures and $s = 12.8$ correct to 3 significant figures.

By considering bounds, work out the value of a to a suitable degree of accuracy. You must show all your working and give a reason for your answer.

WORKING OUT SPACE

$a =$ ..... **Reason:** .....

Question 10 Calculator **(4 marks)**

A cuboid $ABCDEFGH$ has a horizontal rectangular base $ABCD$ with $AB = 14$ cm and $BC = 9$ cm. The vertical height of the cuboid is $CG = 7$ cm.
Calculate the size of the angle that the space diagonal AG makes with the horizontal plane $ABCD$. Give your answer correct to 1 decimal place.

WORKING OUT SPACE

Answer: .....

Question 11 Calculator **(4 marks)**

Solve algebraically:

$$\frac{6}{x-2} - \frac{2}{x+3} = 1$$

WORKING OUT SPACE

$x =$ ..... **or** $x =$ .....

Question 12 Calculator **(4 marks)**

A box contains 6 milk chocolates, 5 dark chocolates, and 4 white chocolates. Two chocolates are taken at random from the box without replacement.
Work out the probability that both chocolates taken are of the same type. Give your answer as a fraction in its simplest form.

WORKING OUT SPACE

Answer: .....

Question 13 Calculator **(4 marks)**

In triangle ABC, $AB = 9$ cm, $BC = 14$ cm, and $AC = 17$ cm.
(a) Calculate the size of angle ABC, giving your answer correct to 1 decimal place.
(b) Hence calculate the area of triangle ABC, giving your answer correct to 1 decimal place.

WORKING OUT SPACE

(a) ..... ° **(b)** ..... cm^2

Question 14 Calculator **(4 marks)**

A right pyramid $VABCD$ has a rectangular horizontal base $ABCD$ with $AB = 10$ cm and $BC = 6$ cm. The apex V is directly above the centre M of the base, and each sloping edge $VA = VB = VC = VD = 13$ cm.
Calculate the vertical height VM of the pyramid, and hence work out its volume, giving each answer correct to 1 decimal place. *(Volume of a pyramid* $= \frac{1}{3} \times \text{base area} \times \text{height}$*)*

WORKING OUT SPACE

$VM =$ ..... cm **Volume** $=$ ..... cm^3

Question 15 Calculator **(4 marks)**

A histogram shows the distribution of the mass, m grams, of 120 apples in an orchard:

Mass, m (grams)	$80 < m \leq 100$	$100 < m \leq 115$	$115 < m \leq 125$	$125 < m \leq 155$
Frequency Density	1.0	2.0	4.0	1.0

Use the histogram information to calculate an estimate for the median mass of the apples in grams.

WORKING OUT SPACE

Answer: .....

APPENDIX

Answers & Mark Schemes

Complete Edexcel Higher answers, completed solution diagrams, and Mixed Review topic guides.

Topic 1: Simplifying Surds

Q1 (Non-Calc):	$4\sqrt{2}$	[1m]
Q2 (Non-Calc):	$8\sqrt{10}$	[2m]
Q3 (Non-Calc):	$3\sqrt{3}$	[2m]
Q4 (Non-Calc):	5	[2m]
Q5 (Non-Calc):	$6\sqrt{5}$	[2m]
Q6 (Non-Calc):	15	[2m]

Topic 2: Expanding with Surds

Q1 (Non-Calc):	$30\sqrt{3}$	[2m]
Q2 (Non-Calc):	$2+6\sqrt{2}$	[2m]
Q3 (Non-Calc):	$107+42\sqrt{2}$	[2m]
Q4 (Non-Calc):	$10+6\sqrt{2}$	[2m]
Q5 (Non-Calc):	$26+42\sqrt{2}$	[3m]
Q6 (Non-Calc):	34	[2m]

Topic 3: Rationalising the Denominator

Q1 (Non-Calc):	$\sqrt{5}$	[2m]
Q2 (Non-Calc):	$3+\sqrt{2}$	[2m]
Q3 (Non-Calc):	$5\sqrt{5}-2\sqrt{2}$	[3m]
Q4 (Non-Calc):	$2+2\sqrt{7}$	[3m]
Q5 (Non-Calc):	$\frac{(3+\sqrt{5})(\sqrt{5}+1)}{4}=2+\sqrt{5}$	[3m]
Q6 (Non-Calc):	$\frac{3\sqrt{5}-80}{5}$	[3m]

Topic 4: Calculating with Bounds

Q1 (Calc):	**149.625**	[2m]
Q2 (Calc):	**140.875**	[2m]
Q3 (Calc):	**7.0**	[2m]
Q4 (Calc):	**1.791**	[2m]
Q5 (Calc):	**4.7**	[2m]
Q6 (Calc):	**3.43**	[2m]

Topic 5: Rearranging Harder Formulae

Q1 (Non-Calc):	$x=\frac{V}{m+n}$	[2m]
Q2 (Non-Calc):	$x=\frac{8}{m-d}$	[3m]
Q3 (Non-Calc):	$x=\frac{2n-4m}{m-n}$	[3m]
Q4 (Non-Calc):	$x=\frac{2y+5}{y-1}$	[3m]
Q5 (Non-Calc):	$x=\frac{2y+3}{5y-4}$	[3m]
Q6 (Non-Calc):	$x=\frac{4y^2+9}{3y^2+2}$	[4m]

Topic 6: Factorising Harder Quadratics

Q1 (Non-Calc):	$(2x+1)(x+3)$	[2m]
Q2 (Non-Calc):	$(3x-2)(x-3)$	[2m]
Q3 (Non-Calc):	$(2x-3)(x+4)$	[2m]
Q4 (Non-Calc):	$(3x+4)(x-3)$	[2m]
Q5 (Non-Calc):	$(3x-1)(2x-3)$	[2m]
Q6 (Non-Calc):	$(2x-1)(2x+5)$	[2m]

Topic 7: The Quadratic Formula

Q1 (Non-Calc): $x = -2,\ -1$ [2m]

Q2 (Non-Calc): $x = -\frac{3}{2},\ 4$ [2m]

Q3 (Non-Calc): $x = \frac{1}{4},\ \frac{5}{3}$ [2m]

Q4 (Calc): $x = -5.34,\ 0.84$ [4m]

Q5 (Calc): $x = -3.68,\ 0.68$ [4m]

Q6 (Calc): $x^2 + 6x - 12 = 0 \Rightarrow 1.58\text{ cm}$ [4m]

Topic 8: Quadratic Sequences

Q1 (Non-Calc): $n^2 + 3n - 4$ [3m]

Q2 (Non-Calc): $3n^2 + 5n - 3$ [3m]

Q3 (Non-Calc): $-n^2 - 2n + 30$ [3m]

Q4 (Non-Calc): $4n^2 - n + 2$ [3m]

Q5 (Non-Calc): $2n^2 + 5n - 4$ [3m]

Q6 (Non-Calc): $-2n^2 + n + 40$ [3m]

Topic 9: Functions

Q1 (Non-Calc): **39** [2m]

Q2 (Non-Calc): $f^{-1}(x) = \frac{3x + 7}{4}$ [2m]

Q3 (Non-Calc): $18x^2 - 33x + 15$ [3m]

Q4 (Non-Calc): $x = 1,\ 5$ [3m]

Q5 (Non-Calc): $x = 4$ [3m]

Q6 (Non-Calc): $f^{-1}(x) = \frac{4x + 5}{x - 3}$ [3m]

Topic 10: Trig & Exponential Graphs

Q1 (Non-Calc): $\text{C},\ \text{A},\ \text{B},\ \text{D}$ [2m]

Q2 (Non-Calc): **(a)** $-m$, **(b)** m [2m]

Q3 (Non-Calc): $210°,\ 330°$ [2m]

Q4 (Non-Calc): **(a)** 13 m, **(b)** $t = 6$ [3m]

Q5 (Non-Calc): $k = 64$ [3m]

Q6 (Non-Calc): $p = 5,\ q = 3$ [3m]

Topic 11: Algebraic Fractions

Q1 (Non-Calc): $\frac{3(x + 1)}{x - 2}$ [2m]

Q2 (Non-Calc): $\frac{3x + 9}{(x + 5)(x + 4)}$ [2m]

Q3 (Non-Calc): $\frac{8x + 9}{(x + 3)(x - 2)}$ [2m]

Q4 (Non-Calc): $\frac{4(x + 3)}{x + 2}$ [2m]

Q5 (Calc): $x = -3,\ \frac{29}{6}$ [3m]

Q6 (Non-Calc): $x = \frac{1}{3},\ \frac{7}{2}$ [3m]

Topic 12: Algebraic Proportion

Q1 (Non-Calc): **21** [2m]

Q2 (Non-Calc): **12** [2m]

Q3 (Non-Calc): **16** [2m]

Q4 (Non-Calc): **(i)** $y \propto x^2$, **(ii)** $y = 5x^2$ [2m]

Q5 (Non-Calc): **64** [3m]

Q6 (Non-Calc): **3** [3m]

Topic 13: Similar Shapes: Area, Volume & Frustums

Q1 (Calc):	$28{,}160\ \text{cm}^3$	[3m]
Q2 (Calc):	$1812.5\ \text{cm}^2$	[3m]
Q3 (Non-Calc):	$12\ \text{cm}$	[3m]
Q4 (Non-Calc):	$120\ \text{cm}^2$	[3m]
Q5 (Non-Calc):	$156\pi\ \text{cm}^3$	[4m]
Q6 (Non-Calc):	$270\ \text{cm}^3$	[3m]

Topic 14: 3D Pythagoras & Trig

Q1 (Calc):	**12.3**	[3m]
Q2 (Calc):	**5.1**	[3m]
Q3 (Calc):	**24.9**	[3m]
Q4 (Calc):	**8.1**	[3m]
Q5 (Calc):	**29.7**	[4m]
Q6 (Calc):	**911.1**	[4m]

Topic 15: The Sine Rule

Q1 (Calc):	**6.5**	[3m]
Q2 (Calc):	**9.6**	[3m]
Q3 (Calc):	**16.3**	[3m]
Q4 (Calc):	**22.2**	[3m]
Q5 (Calc):	**102.8**	[5m]
Q6 (Calc):	**22.3**	[4m]

Topic 16: The Cosine Rule

Q1 (Calc):	**23.2**	[3m]
Q2 (Calc):	**16.8**	[3m]
Q3 (Calc):	**97.9**	[3m]
Q4 (Calc):	**22.8**	[3m]
Q5 (Non-Calc):	**4**	[4m]
Q6 (Calc):	**64.3**	[4m]

Topic 17: Area of Any Triangle

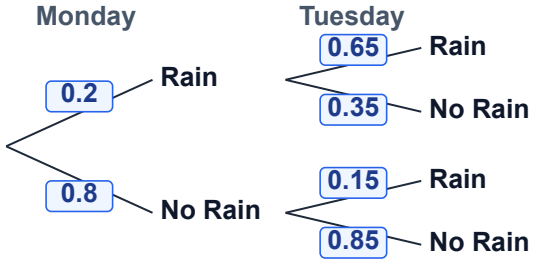
Q1 (Calc):	**160.3**	[2m]
Q2 (Calc):	**129.9**	[2m]
Q3 (Calc):	**41.2**	[3m]
Q4 (Calc):	**37.8**	[4m]
Q5 (Non-Calc):	**5**	[4m]
Q6 (Non-Calc):	**10**	[4m]

Topic 18: Conditional Probability

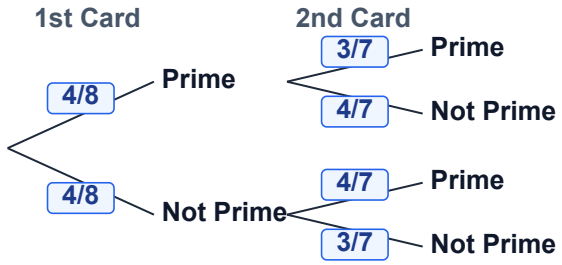
Q1 (Non-Calc):	$\frac{2}{9}$ **(see completed tree below)**	[2m]
Q2 (Non-Calc):	0.19 **(see completed tree below)**	[3m]
Q3 (Non-Calc):	$\frac{11}{14}$ **(see completed tree below)**	[3m]
Q4 (Calc):	$\frac{35}{66}$ **(see completed tree below)**	[3m]
Q5 (Non-Calc):	**(a) See completed tree below, (b)** 0.74	[4m]
Q6 (Calc):	$\frac{19}{66}$	[4m]

Q1 — Completed Probability Tree

Q2 — Completed Probability Tree

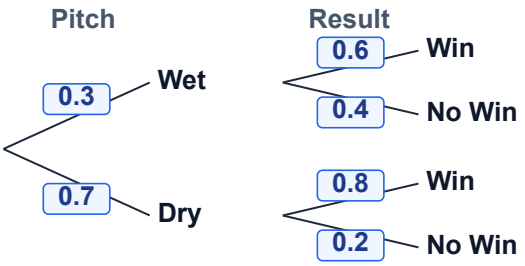


Q3 — Completed Probability Tree



Q4 — Completed Probability Tree

Q5 — Completed Probability Tree



Topic 19: Histograms

Q1 (Non-Calc): $\text{FD} = 4.4$ **(bars:** $1.4, 3.6, 4.4, 1.6, 0.8$ **— see histogram below)** [3m]

Q2 (Calc): **(a) Bars:** $0.6, 2.8, 6.4, 3.6, 0.6$ **(see below), (b)** 27 [4m]

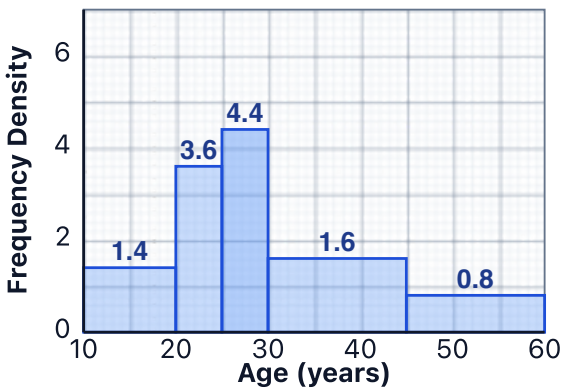
Q3 (Calc): **(a) Bars:** $0.8, 2.0, 6.4, 0.8$ **(see below), (b)** $66.25\ \text{kg}$ [4m]

Q4 (Non-Calc): **(a) Bars:** $1.2, 3.4, 0.8$ **(see below), (b)** 20 [4m]

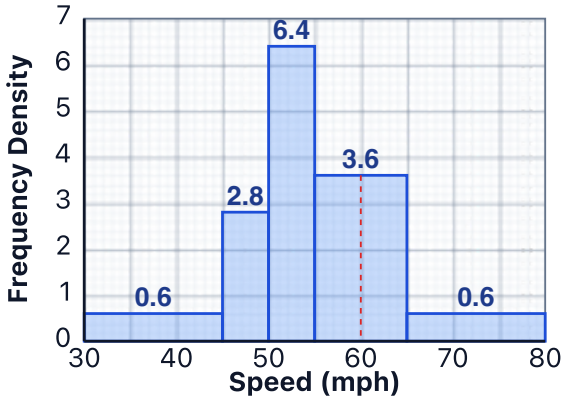
Q5 (Non-Calc): **(a)** 35**, (b) Data is grouped; uniform distribution assumed in interval** [4m]

Q6 (Non-Calc): **(a)** $\frac{7}{25}$**, (b) Data is grouped; uniform distribution assumed in interval** [4m]

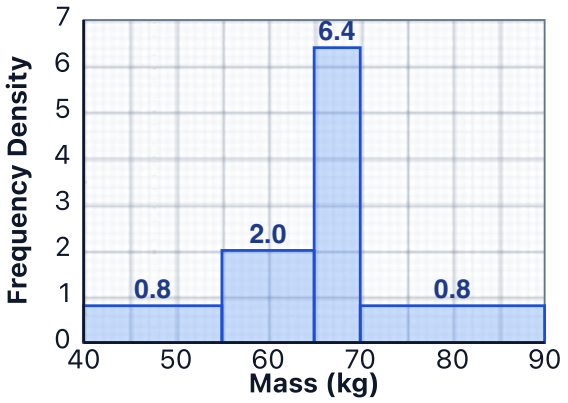
Q1 — Completed Histogram



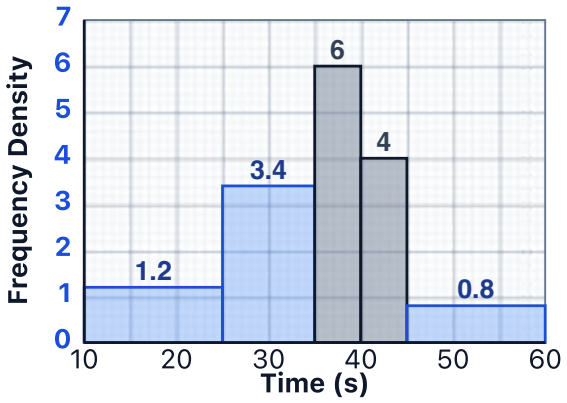
Q2 — Completed Histogram



Q3 — Completed Histogram



Q4 — Completed Histogram



Mixed Review 1 — Choosing the Right Method (Non-Calculator) — Answers & Topic Guide

Q1 (Simplifying Surds, Non-Calc): $13\sqrt{3}$ [2m]

Q2 (Factorising Quadratics, Non-Calc): $(3x-4)(2x+5)$ [2m]

Q3 (Quadratic Sequences, Non-Calc): $3n^2+n-2$ [3m]

Q4 (Algebraic Proportion, Non-Calc): $x=3$ [3m]

Q5 (Expanding with Surds, Non-Calc): $18+8\sqrt{3}$ [3m]

Q6 (Conditional Probability, Non-Calc): $\frac{5}{9}$ [3m]

Q7 (Rearranging Formulae, Non-Calc): $x=\frac{2y+8}{3-y}$ [3m]

Q8 (Similar Shapes, Non-Calc): 250 cm^3 [3m]

Q9 (Rationalising Denominator, Non-Calc): $6+2\sqrt{2}$ [3m]

Q10 (Exponential Graphs, Non-Calc): **(a)** $p=9,\ q=2$, **(b)** 72 [3m]

Q11 (Composite Functions, Non-Calc): $x=0,\ 5$ [3m]

Q12 (3D Pythagoras, Non-Calc): 24 cm [3m]

Q13 (Algebraic Fractions, Non-Calc): $\frac{x+5}{2}$ [3m]

Q14 (Area of Any Triangle, Non-Calc): $x=4$ [4m]

Q15 (Histograms, Non-Calc): **(a)** $0.8, 2.4, 3.6, 1.2$, **(b)** $\frac{3}{5}$ [4m]

Mixed Review 2 — Choosing the Right Method (Calculator) — Answers & Topic Guide

Q1 (Calculating with Bounds, Calc): 5.57 [3m]

Q2 (The Quadratic Formula, Calc): $x = -0.52,\ 3.19$ [3m]

Q3 (The Sine Rule, Calc): $57.1°$ [3m]

Q4 (Algebraic Proportion, Calc): **(a)** $P = 7\sqrt[3]{Q}$, **(b)** 125 [3m]

Q5 (The Cosine Rule, Calc): 26.2 cm [3m]

Q6 (Trig Equations, Calc): $69.5°,\ 290.5°$ [3m]

Q7 (Inverse Functions, Calc): $f^{-1}(x) = \dfrac{3x+2}{5-x}$ [3m]

Q8 (Frustum Volume, Calc): 980.2 cm^3 [4m]

Q9 (Bounds Accuracy, Calc): 13 ($\text{LB} = 13.10,\ \text{UB} = 13.35$) [3m]

Q10 (3D Trigonometry, Calc): $22.8°$ [4m]

Q11 (Algebraic Fractions, Calc): $x = -4,\ 7$ [4m]

Q12 (Conditional Probability, Calc): $\dfrac{31}{105}$ [4m]

Q13 (Cosine & Area Rules, Calc): **(a)** $92.7°$, **(b)** 62.9 cm^2 [4m]

Q14 (3D Pyramid, Calc): $11.6\text{ cm},\ 232.4\text{ cm}^3$ [4m]

Q15 (Histograms Median, Calc): 117.5 g [4m]