GCSE MATHEMATICS (9-1) — TUTORIALS & WORKED EXAMPLES

Grade 8 Mathematics — Tutorials & Examples

(Higher Tier)

Higher Tier Tutorials, Common Mistake Warnings & Step-by-Step Worked Examples

- **Higher Tier Core Concept Notes** & Precision Geometric/Graph Diagrams
- **Exam-Focused "Common Mistake" Warnings** on every single step
- **Model Worked Examples** tagged **Calculator** or **Non-Calculator**
- **Streamlined Higher Revision Guide** (Tutorials & Worked Examples Only)

Student Name: _____

Class / Teacher: _____

Target Grade: _____

SCAN FOR ONLINE PRACTICE & WORKED SOLUTIONS

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Interactive Step-by-Step GCSE Mathematics Mastery

Table of Contents

Click any topic below to jump straight to its Core Concept notes, Common Mistake warnings, and Worked Examples.

ALGEBRA

Completing the Square Page 3
Quadratic Inequalities Page 12
Transforming Graphs Page 17
Quadratic Simultaneous Equations Page 8
Iteration Page 14

RATIO, PROPORTION AND RATES OF CHANGE

Kinematics Page 26
Velocity-Time Graphs Page 30

ALGEBRA — QUADRATICS

Completing the Square

1 Completing the Square

CORE CONCEPT & METHOD

Completing the square allows us to rewrite a quadratic to easily find its turning point.

Halve the middle number (b) and put it in a bracket squared: $(x + \frac{b}{2})^2$

Then **subtract** the square of that number, and add the original c back on the end!

> **COMMON MISTAKE:** When $a \neq 1$ in $ax^2 + bx + c$, factor a out of the first two terms FIRST and remember to multiply the subtracted square by a when expanding the outer bracket.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$

STEP-BY-STEP SOLUTION:

1. Halve the middle number ($8 \div 2 = 4$). Put in a bracket: $(x + 4)^2$

2. Subtract 4^2: $(x + 4)^2 - 16$

3. Bring down the original $+3$:
$(x + 4)^2 - 16 + 3$

$$(x + 4)^2 - 13$$

EXAMPLE 1.2

Non-Calculator

Write $2x^2 + 16x + 9$ in the form $a(x + b)^2 + c$

STEP-BY-STEP SOLUTION:

1. Factor out the 2 from the x terms first:
$2(x^2 + 8x) + 9$

2. Complete the square inside the bracket (halve the 8 to get 4):
$2[(x + 4)^2 - 4^2] + 9$
$2[(x + 4)^2 - 16] + 9$

3. Expand the outer bracket by multiplying the 2 back in:
$2(x + 4)^2 - 32 + 9$

4. Simplify the numbers on the end:

$$2(x + 4)^2 - 23$$

EXAMPLE 1.3

Non-Calculator

Write $x^2 - 10x + 7$ in the form $(x + a)^2 + b$

STEP-BY-STEP SOLUTION:

1. Halve the middle number ($-10 \div 2 = -5$). Put in a bracket: $(x - 5)^2$

2. Subtract $(-5)^2$. (Remember, negative squared is always positive, so we still subtract 25):
$(x - 5)^2 - 25$

3. Bring down the original $+7$:
$(x - 5)^2 - 25 + 7$

$$(x - 5)^2 - 18$$

2 Solving by Completing the Square

CORE CONCEPT & METHOD

To solve a quadratic equation by completing the square, first rewrite the quadratic in completed square form $(x + a)^2 + b = 0$, then make x the subject!

1. Complete the square on the quadratic expression.
2. Move the loose constant to the other side.
3. Square root both sides (don't forget the $\pm$ sign!).
4. Rearrange to get x by itself.

> **COMMON MISTAKE:** When taking the square root of both sides, NEVER forget the $\pm$ sign — omitting the negative root loses the final accuracy mark.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

By completing the square, solve $x^2 - 6x + 2 = 0$. Give your answers to 2 decimal places.

STEP-BY-STEP SOLUTION:

1. Complete the square on $x^2 - 6x + 2$:
$(x-3)^2 - 9 + 2 = 0 \implies (x-3)^2 - 7 = 0$

2. Move the -7 over:
$(x-3)^2 = 7$

3. Square root both sides:
$x - 3 = \pm\sqrt{7}$

4. Add 3 to both sides:
$x = 3 \pm \sqrt{7}$

Calculate the two values:

$$x = 0.35, \text{ or } x = 5.65$$

EXAMPLE 2.2 Non-Calculator

By completing the square, solve $2x^2 + 16x + 18 = 0$. Give your answers in surd form.

STEP-BY-STEP SOLUTION:

1. Factor out 2 from the x terms and complete the square:
$2[(x+4)^2 - 16] + 18 = 0 \implies 2(x+4)^2 - 32 + 18 = 0$
$2(x+4)^2 - 14 = 0$

2. Move the -14 over and divide by 2:
$2(x+4)^2 = 14 \implies (x+4)^2 = 7$

3. Square root both sides:
$x + 4 = \pm\sqrt{7}$

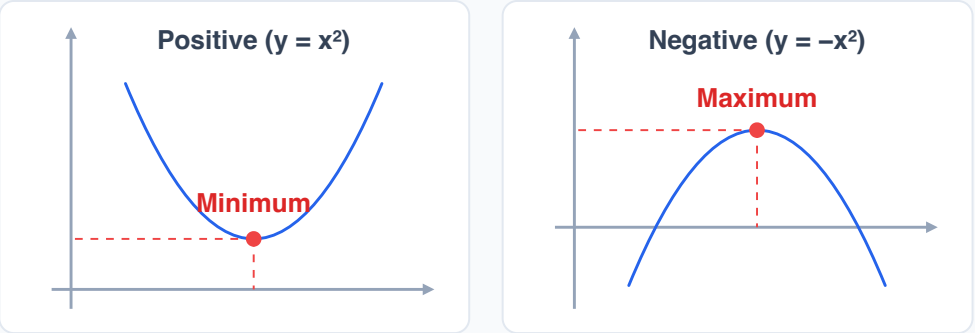
4. Subtract 4 from both sides:

$$x = -4 \pm \sqrt{7}$$

3 Turning Points

CORE CONCEPT & METHOD

Completing the square reveals the **turning point** (the lowest or highest tip) of a quadratic curve!



Once your quadratic is written in completed square form $(x+a)^2+b$, the turning point is at the coordinate $(-a, b)$.

Just **flip the sign** of the number inside the bracket for the x-coordinate, and keep the outside number exactly the same for the y-coordinate!

> **COMMON MISTAKE:** For $y=a(x+p)^2+q$, flip ONLY the sign inside the bracket to get $(-p, q)$, and remember $a<0$ gives a MAXIMUM point, not a minimum.

MODEL WORKED EXAMPLES

EXAMPLE 3.1 — Non-Calculator

By completing the square, find the coordinates of the minimum point of the curve $y=x^2-12x+40$.

STEP-BY-STEP SOLUTION:

1. Complete the square on $x^2-12x+40$:
$y=(x-6)^2-36+40=(x-6)^2+4$

2. Look inside the bracket: -6. Flip the sign to get $+6$. This is the x-coordinate.
3. Look outside the bracket: $+4$. Keep it the same. This is the y-coordinate.

Put them together as a coordinate pair (x, y):

$$(6, 4)$$

EXAMPLE 3.2

Non-Calculator

By completing the square, find the coordinates of the maximum point of the curve $y = 6 + 8x - x^2$.

STEP-BY-STEP SOLUTION:

1. Factor out -1 from the x terms:

$$y = -(x^2 - 8x) + 6$$

2. Complete the square inside the bracket:

$$y = -[(x-4)^2 - 16] + 6 = -(x-4)^2 + 16 + 6 = -(x-4)^2 + 22$$

3. Look inside the bracket: -4. Flip the sign to get $+4$.
4. Look outside the bracket: $+22$. Keep it the same.

Put them together:

$$(4, 22)$$

ALGEBRA — EQUATIONS & SEQUENCES

Quadratic Simultaneous Equations

CORE CONCEPT & METHOD

When one equation has a squared term (like x^2 or y^2), you cannot use the elimination method. Instead, you must use **Substitution**!

- **Step 1:** Rearrange the linear (simple) equation to get $x =$ or $y =$.
- **Step 2:** Substitute this expression into the quadratic equation.
- **Step 3:** Expand, simplify, and set to 0 to solve the quadratic.
- **Step 4:** Substitute both answers back into the linear equation to find the pairs!

COMMON MISTAKE: Always substitute the linear expression inside brackets — e.g. $(7-3x)^2 = 49 - 42x + 9x^2$, NEVER $49 + 9x^2$ — and pair each x with its matching y.

MODEL WORKED EXAMPLES

EXAMPLE 1 Non-Calculator

Solve the simultaneous equations:

$x^2 + y^2 = 25$

$x = y - 7$

STEP-BY-STEP SOLUTION:

1. The linear equation is already arranged as $x = y - 7$.
We substitute $(y - 7)$ in place of x in the quadratic equation:

$$(y - 7)^2 + y^2 = 25$$

2. Expand the bracket.
(Remember $(y - 7)^2 = (y - 7)(y - 7) = y^2 - 14y + 49$):

$$y^2 - 14y + 49 + y^2 = 25$$

$$2y^2 - 14y + 49 = 25$$

3. Set to 0:

$$2y^2 - 14y + 49 = 25 \quad -25 \qquad -25$$

$$2y^2 - 14y + 24 = 0$$

4. Divide by 2 to make it easier to factorise:

$$2y^2 - 14y + 24 = 0$$

$$\div 2 \quad \div 2$$

$$y^2 - 7y + 12 = 0$$

5. Factorise. Two numbers that multiply to $+12$ and add to -7 are -3 and -4:

$$(y - 3)(y - 4) = 0$$

So $y = 3$ or $y = 4$.

6. Substitute back into the linear equation ($x = y - 7$):
When $y = 3$:
$x = 3 - 7 = -4$
When $y = 4$:
$x = 4 - 7 = -3$

$$x = -4, y = 3 \text{ and } x = -3, y = 4$$

EXAMPLE 2

Non-Calculator

Solve algebraically the simultaneous equations:

$x^2 - 3y^2 = 24$

$2x + 3y = 6$

STEP-BY-STEP SOLUTION:

1. Rearrange the linear equation. Let's make x the subject:

$$\begin{array}{r} 2x + 3y = 6 \quad -3y \qquad -3y \\ \hline 2x = 6 - 3y \end{array}$$

$$x = \frac{6 - 3y}{2}$$

2. Substitute this expression in place of x in the quadratic equation:

$$\left(\frac{6 - 3y}{2}\right)^2 - 3y^2 = 24$$

3. Expand the bracket. Square the top and square the bottom:

$$\frac{(6 - 3y)(6 - 3y)}{4} - 3y^2 = 24$$

$$\frac{36 - 36y + 9y^2}{4} - 3y^2 = 24$$

4. Multiply everything by 4 to remove the fraction:

$$36 - 36y + 9y^2 - 12y^2 = 96$$

$$-3y^2 - 36y + 36 = 96$$

5. Set to 0:

$$\begin{array}{r} -3y^2 - 36y + 36 = 96 \\ -96 \quad -96 \\ \hline -3y^2 - 36y - 60 = 0 \end{array}$$

6. Divide by -3 to make it a positive y^2:

$$\begin{array}{r} -3y^2 - 36y - 60 = 0 \\ \div -3 \quad \div -3 \\ \hline y^2 + 12y + 20 = 0 \end{array}$$

7. Factorise:

$$(y + 10)(y + 2) = 0$$

So $y = -10$ or $y = -2$.

8. Substitute back into $x = \frac{6 - 3y}{2}$:

When $y = -10$:

$$x = \frac{6 - 3(-10)}{2} = \frac{36}{2} = 18$$

When $y = -2$:

$$x = \frac{6 - 3(-2)}{2} = \frac{12}{2} = 6$$

$$x = 6, y = -2 \text{ and } x = 18, y = -10$$

ALGEBRA — EQUATIONS & SEQUENCES

Quadratic Inequalities

CORE CONCEPT & METHOD

To solve a quadratic inequality (like $x^2 + 5x + 6 > 0$):

- **Step 1:** Factorise the quadratic (or use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ if it does not factorise) to find the *critical values* where it equals 0.
- **Step 2:** Sketch a quick U-shaped parabola crossing the x-axis at those critical values.
- **Step 3:** Look at the inequality sign. If it is > 0, you want the parts of the graph *above* the axis (the two separate outer tails). If it is < 0, you want the part *below* the axis (the single connected region in the middle).

COMMON MISTAKE: Never write a split outer region like $x < -3$ or $x > 5$ as a single combined statement $-3 > x > 5$ — Edexcel awards zero final marks for invalid inequality notation.

MODEL WORKED EXAMPLES

EXAMPLE 1

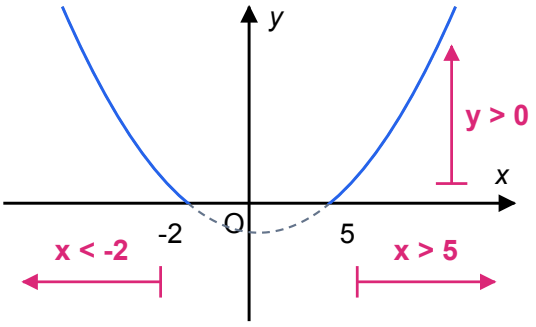
Non-Calculator

Solve the inequality $x^2 - 3x - 10 > 0$

STEP-BY-STEP SOLUTION:

1. Factorise to find the critical values:
$(x - 5)(x + 2) = 0$
The critical values are $x = 5$ and $x = -2$.

2. Think of a U-shaped graph crossing at -2 and 5:



3. The inequality is > 0, which means we want the parts of the graph ABOVE the x-axis. These are the two outer tails going outwards in opposite directions.
So $x < -2$ or $x > 5$.

$$x < -2 \text{ or } x > 5$$

EXAMPLE 2

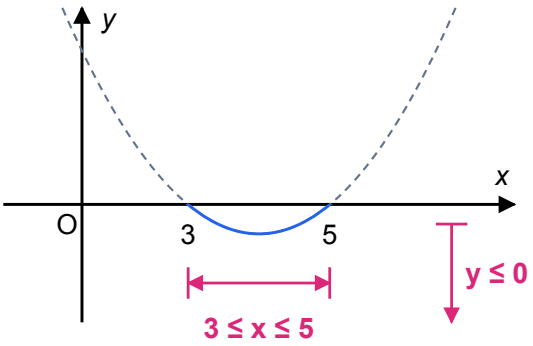
Non-Calculator

Solve the inequality $x^2 - 8x + 15 \leq 0$

STEP-BY-STEP SOLUTION:

1. Factorise to find the critical values:
$(x - 5)(x - 3) = 0$
The critical values are $x = 3$ and $x = 5$.

2. Think of a U-shaped graph crossing at 3 and 5:



3. The inequality is ≤ 0, which means we want the part of the graph BELOW the x-axis. This is the single continuous dip connected between 3 and 5.
So $3 \leq x \leq 5$.

$$3 \leq x \leq 5$$

ALGEBRA — EQUATIONS & SEQUENCES

Iteration

1 Using Iteration Formulae

CORE CONCEPT & METHOD

What is Iteration?

Iteration means repeating a process. In maths, we take an initial value (usually called x_0) and plug it into a formula to get the next value, x_1. We then plug x_1 back into the formula to get x_2, and so on.

Calculator Tip:

Type your x_0 value and press **=**. Then, type the formula using the **ANS** button instead of x_n. Every time you press **=**, you get the next term in the sequence!

COMMON MISTAKE: Use your calculator ANS key with the FULL unrounded display at each step — rounding x_1 or x_2 early causes drift in the 3rd decimal place of x_3.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Calculator

Using $x_{n+1} = 2 + \dfrac{8}{x_n^2}$ with $x_0 = 2$. Find the value of x_1, x_2, x_3.

STEP-BY-STEP SOLUTION:

$$x_1 = 2 + \frac{8}{2^2} = 2 + 2 = 4$$

$$x_2 = 2 + \frac{8}{4^2} = 2 + 0.5 = 2.5$$

$$x_3 = 2 + \frac{8}{2.5^2} = 2 + 1.28 = 3.28$$

2 Estimating Roots & Rearranging

CORE CONCEPT & METHOD

Rearranging into an Iteration Formula

Iteration is a powerful tool for solving complex equations (like cubics) that don't easily factorise. To do this, we rearrange the equation to make x the subject on one side, and then turn it into an iteration formula.

Finding the original equation:

If you are given $x_{n+1} = \frac{5}{x_n^2} + 2$, you can find the original equation by replacing x_{n+1} and x_n with just x, and rearranging to equal zero:

$$x = \frac{5}{x^2} + 2$$
$$x - 2 = \frac{5}{x^2}$$
$$x^3 - 2x^2 = 5$$
$$x^3 - 2x^2 - 5 = 0$$

COMMON MISTAKE: Computing $f(a)$ and $f(b)$ alone is NOT enough — you must explicitly state "change of sign (and $f(x)$ is continuous), so a root lies between a and b".

MODEL WORKED EXAMPLES

EXAMPLE 2.1 — Non-Calculator

Show that $x^3 - 3x^2 - 2 = 0$ has a root (or solution) between $x = 3$ and $x = 4$.

STEP-BY-STEP SOLUTION:

Let $f(x) = x^3 - 3x^2 - 2$

$f(3) = 3^3 - 3(3)^2 - 2 = 27 - 27 - 2 = -2$

$f(4) = 4^3 - 3(4)^2 - 2 = 64 - 48 - 2 = 14$

Because there is a **change of sign** between $f(3)$ (negative) and $f(4)$ (positive), a root (or solution) must exist between 3 and 4.

EXAMPLE 2.2

Non-Calculator

Show that the equation $4x^3 - x^2 - 7 = 0$ can be rearranged to give $x = \sqrt{\dfrac{7}{4x - 1}}$.

STEP-BY-STEP SOLUTION:

1. Add 7 to both sides to isolate the x terms:
$4x^3 - x^2 = 7$

2. Factorise x^2 out of the left-hand side:
$x^2(4x - 1) = 7$

3. Divide both sides by $(4x - 1)$:
$x^2 = \dfrac{7}{4x - 1}$

4. Square root both sides:
$x = \sqrt{\dfrac{7}{4x - 1}}$

ALGEBRA — COORDINATE GEOMETRY

Transforming Graphs

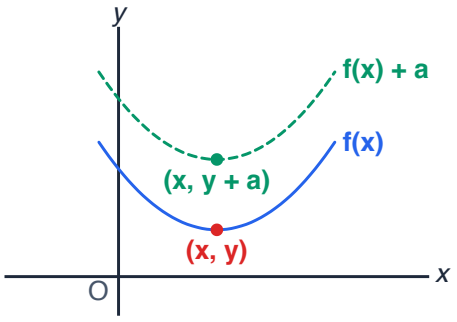
1 Translations of Graphs: $y = f(x) \pm a$ and $y = f(x \pm a)$

CORE CONCEPT & METHOD

Translations of Graphs: $y = f(x) \pm a$ and $y = f(x \pm a)$

Adding or subtracting a constant shifts (translates) the entire curve without changing its shape or orientation.

1. Outside the Bracket: $y = f(x) \pm a$ (Vertical)



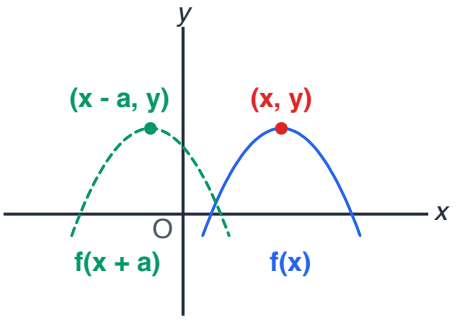
• Affects the y-coordinates only ("does what it says"):

$y = f(x) + a$ shifts **UP by** a: $\begin{pmatrix} 0 \\ a \end{pmatrix}$

$y = f(x) - a$ shifts **DOWN by** a: $\begin{pmatrix} 0 \\ -a \end{pmatrix}$

• Rule: $(x, y) \to (x, y \pm a)$.

2. Inside the Bracket: $y = f(x \pm a)$ (Horizontal)



• Affects the x-coordinates in the opposite direction:

$y = f(x + a)$ shifts **LEFT by** a: $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

$y = f(x - a)$ shifts **RIGHT by** a: $\begin{pmatrix} a \\ 0 \end{pmatrix}$

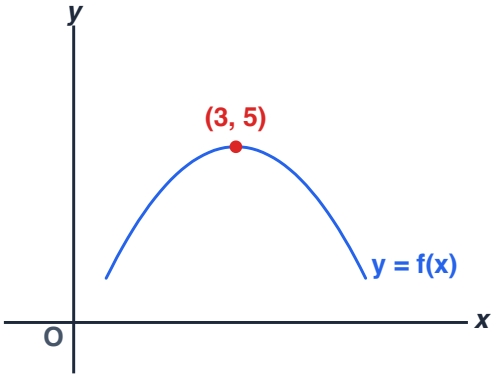
• Rule: $(x, y) \to (x \mp a, y)$.

COMMON MISTAKE: Changes INSIDE $f(x + a)$ shift horizontally the OPPOSITE way to the sign (left by a, leaving y unchanged), whereas $f(x) + a$ shifts vertically (x unchanged).

MODEL WORKED EXAMPLES

EXAMPLE 1.1 Non-Calculator

The diagram shows the curve $y = f(x)$ with a maximum point at $(3, 5)$.



Write down the coordinates of the maximum point on:
(a) $y = f(x) - 4$
(b) $y = f(x + 2)$

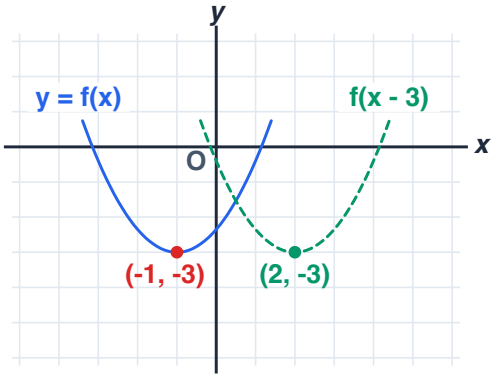
STEP-BY-STEP SOLUTION:

1. For $y = f(x) - 4$, the -4 is **outside** the bracket so it translates the curve 4 units downwards:
$$(3, 5) \rightarrow (3, 5 - 4) = (3, 1)$$
2. For $y = f(x + 2)$, the $+2$ is **inside** the bracket so it translates the curve 2 units to the left:
$$(3, 5) \rightarrow (3 - 2, 5) = (1, 5)$$

$$\text{(a) } (3, 1), \quad \text{(b) } (1, 5)$$

EXAMPLE 1.2 Non-Calculator

The curve $y = f(x)$ shown on the grid has a minimum turning point at $(-1, -3)$.



Find the coordinates of the minimum turning point of $y = f(x - 3)$.

STEP-BY-STEP SOLUTION:

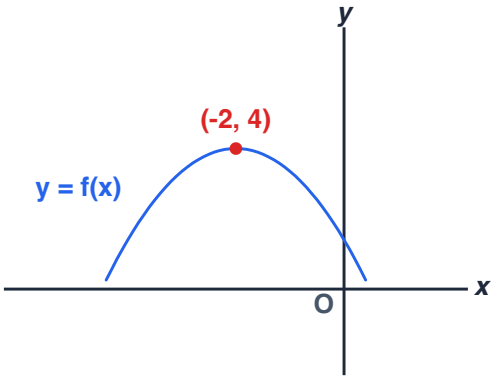
1. The -3 is inside the bracket, which translates the graph 3 units to the **right** (add 3 to the x-coordinate).
2. Adding 3 to the x-coordinate gives:
$$(-1 + 3, -3) = (2, -3)$$

$$(2, -3)$$

EXAMPLE 1.3

Non-Calculator

The curve $y = f(x)$ has a turning point at $(-2, 4)$.



Find the coordinates of the turning point of $y = f(x - 2) + 5$.

STEP-BY-STEP SOLUTION:

1. Inside the bracket, $x - 2$ translates the curve 2 units to the right: $x = -2 + 2 = 0$.
2. Outside the bracket, $+5$ translates the curve 5 units upwards: $y = 4 + 5 = 9$.
3. The new turning point is $(0, 9)$.

$$(0, 9)$$

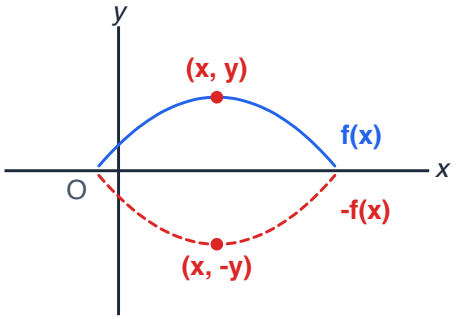
2 Reflections of Graphs: $y = -f(x)$ and $y = f(-x)$

CORE CONCEPT & METHOD

Reflections of Graphs: $y = -f(x)$ and $y = f(-x)$

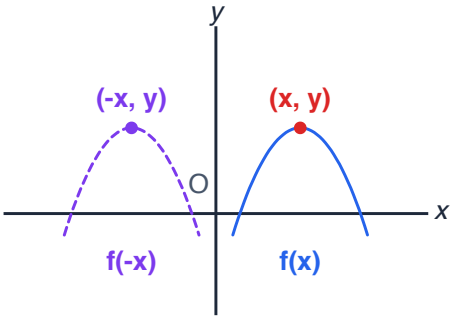
A negative sign outside or inside the function reflects the graph across one of the coordinate axes.

1. Outside the Bracket: $y = -f(x)$ (Reflect in x-axis)



- Multiplies every y-**coordinate by** -1.
- Reflection in the x-**axis** $(y = 0)$.
- Rule: $(x, y) \rightarrow (x, -y)$ (a maximum point becomes a minimum point!).

2. Inside the Bracket: $y = f(-x)$ (Reflect in y-axis)



- Replaces x with $-x$, multiplying every x-**coordinate by** -1.
- Reflection in the y-**axis** $(x = 0)$.
- Rule: $(x, y) \rightarrow (-x, y)$.

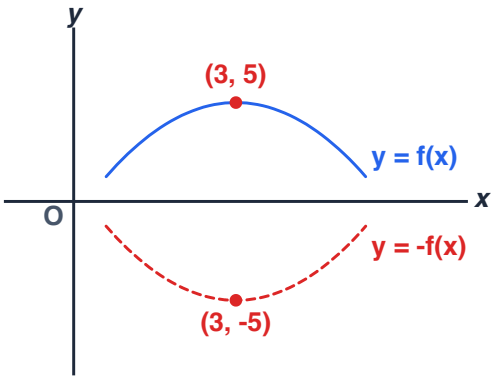
COMMON MISTAKE: Do not mix up the axes: $y = -f(x)$ negates y-coordinates (reflection in the x-axis), whereas $y = f(-x)$ negates x-coordinates (reflection in the y-axis).

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator

The diagram shows the curve $y = f(x)$ with a maximum point at $(3, 5)$.



Write down the coordinates of the turning point of the curve $y = -f(x)$.

STEP-BY-STEP SOLUTION:

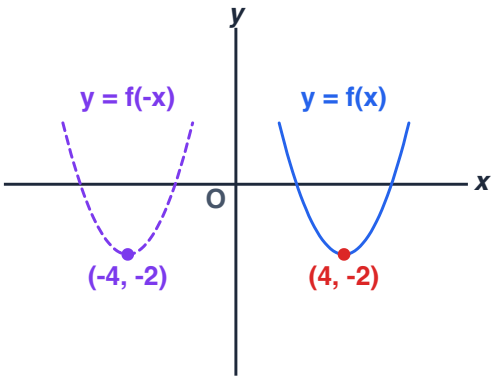
1. The negative sign is **outside** the bracket, which reflects the graph in the x**-axis**.
2. Under a reflection in the x-axis, $(x, y) \rightarrow (x, -y)$, so the maximum at $(3, 5)$ becomes a minimum at $(3, -5)$.

$$(3, -5)$$

EXAMPLE 2.2

Non-Calculator

The diagram shows the curve $y = f(x)$ with a minimum point at $(4, -2)$.



Write down the coordinates of the minimum point of the curve $y = f(-x)$.

STEP-BY-STEP SOLUTION:

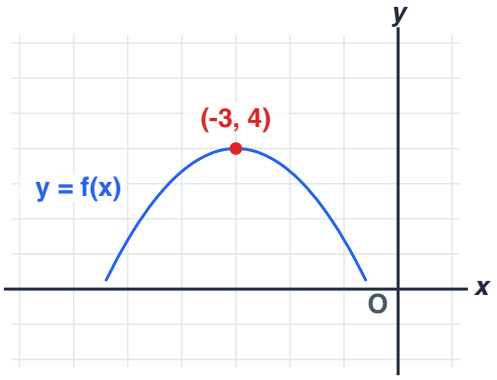
1. The negative sign is **inside** the bracket, which reflects the graph in the y**-axis**.
2. Under a reflection in the y-axis, $(x, y) \rightarrow (-x, y)$, so $(4, -2)$ moves to $(-4, -2)$.

$$(-4, -2)$$

EXAMPLE 2.3

Non-Calculator

The curve $y = f(x)$ on the grid has a turning point at $(-3, 4)$.



Find the coordinates of the turning point on:

(a) $y = -f(x)$

(b) $y = f(-x)$

STEP-BY-STEP SOLUTION:

1. For $y = -f(x)$, reflect in the x-axis (negate y): $(-3, 4) \to (-3, -4)$.
2. For $y = f(-x)$, reflect in the y-axis (negate x): $(-3, 4) \to (3, 4)$.

$$\text{(a) } (-3, -4), \quad \text{(b) } (3, 4)$$

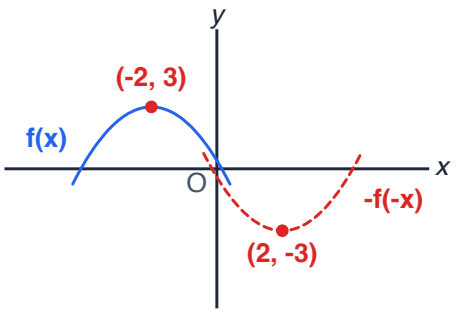
3 Combined Transformations & Identifying Equations

CORE CONCEPT & METHOD

Combined Transformations & Identifying Equations

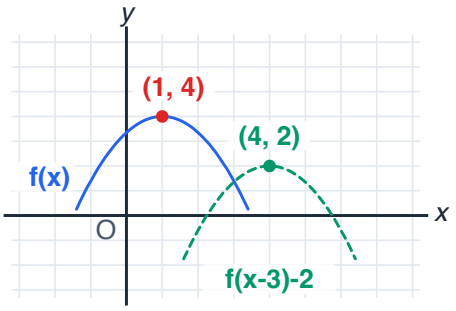
When a function has both an inside transformation (affecting x) and an outside transformation (affecting y), apply the x-change to the x-coordinate and the y-change to the y-coordinate independently!

1. Combined Coordinate Mapping



- $y = -f(x + a)$: $(x, y) \to (x - a, -y)$
- $y = f(-x) + b$: $(x, y) \to (-x, y + b)$
- $y = -f(x) + c$: $(x, y) \to (x, -y + c)$

2. Identifying a Transformation from a Grid



Compare the turning point (x_1, y_1) of $y = f(x)$ with (x_2, y_2) on the transformed curve:

- Same way up & shifted by $\begin{pmatrix} h \\ k \end{pmatrix} \to$ $y = f(x - h) + k$.

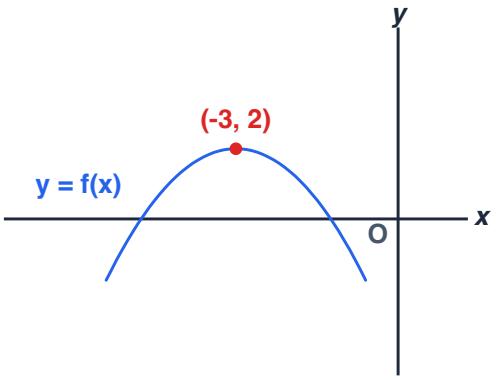
COMMON MISTAKE: When translating an algebraic curve like $y = x^2 - 6x + 11$ by $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$, replace EVERY x with $(x + 2)$ inside brackets before expanding and adding 3.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

Non-Calculator

Shown below is the curve $y = f(x)$ with a maximum turning point at $(-3, 2)$.



Write down the coordinates of the turning point of the curve with equation $y = -f(x + 4)$.

STEP-BY-STEP SOLUTION:

1. Inside the bracket, $x + 4$ shifts the x-coordinate 4 units to the left:

$$x = -3 - 4 = -7$$

2. Outside the bracket, the negative sign reflects the curve in the x-axis (negating the y-coordinate):

$$y = -(2) = -2$$

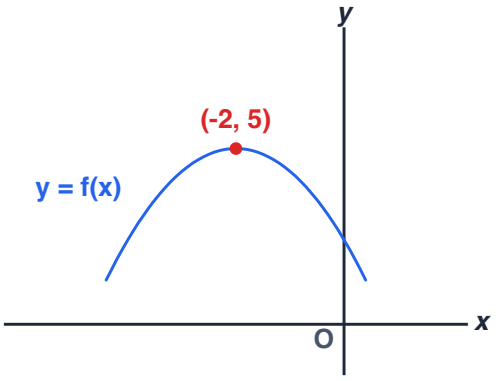
3. Therefore, the new turning point is $(-7, -2)$.

$$(-7, -2)$$

EXAMPLE 3.2

Non-Calculator

The curve $y = f(x)$ has a turning point at $(-2, 5)$.



Find the coordinates of the turning point of $y = f(-x) - 3$.

STEP-BY-STEP SOLUTION:

1. Inside the bracket, $-x$ reflects the curve in the y-axis (negating the x-coordinate):

$$x = -(-2) = 2$$

2. Outside the bracket, -3 translates the curve 3 units downwards:

$$y = 5 - 3 = 2$$

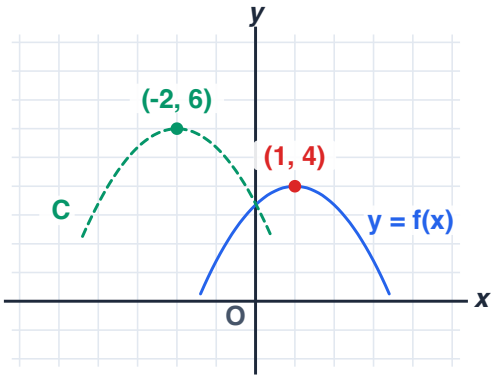
3. Therefore, the new turning point is $(2, 2)$.

$$(2, 2)$$

EXAMPLE 3.3

Non-Calculator

The grid shows the curve $y = f(x)$ with maximum point $(1, 4)$ and a transformed curve C with maximum point $(-2, 6)$.



Write down the equation of curve C in the form $y = f(x + a) + b$.

STEP-BY-STEP SOLUTION:

1. Compare the maximum points: $(1, 4) \rightarrow (-2, 6)$ is a shift of 3 units left (-3 in x) and 2 units up ($+2$ in y).
2. A horizontal shift of 3 units left is written inside the bracket as $f(x + 3)$.
3. A vertical shift of 2 units up is written outside the bracket as $+2$, giving $y = f(x + 3) + 2$.

$$y = f(x + 3) + 2$$

RATIO, PROPORTION AND RATES OF CHANGE — RATES OF CHANGE

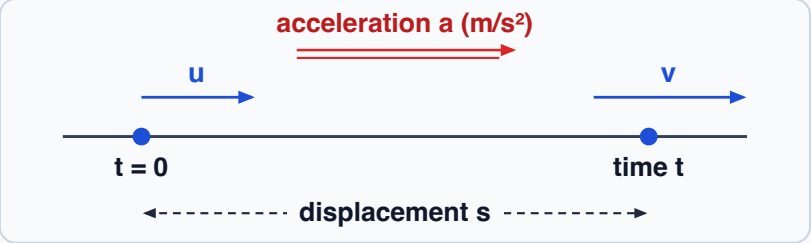
Kinematics

1 Constant Acceleration Formulae (SUVAT)

CORE CONCEPT & METHOD

When an object moves in a straight line with **constant acceleration**, five variables link its motion:

- s = **displacement** (distance in a given direction, in m)
- u = **initial velocity** ($\mathrm{m/s}$) — note $u = 0$ if starting *from rest*
- v = **final velocity** ($\mathrm{m/s}$) — note $v = 0$ if coming *to rest*
- a = **constant acceleration** ($\mathrm{m/s^2}$) — **negative** if decelerating!
- t = **time taken** (s)



Three Core SUVAT Equations (choose the one missing the unneeded variable):

1. $v = u + at$
(no s)

2. $s = ut + \frac{1}{2}at^2$
(no v)

3. $v^2 = u^2 + 2as$
(no t)

COMMON MISTAKE: If an object slows down (decelerates), substitute a as a NEGATIVE value into $v = u + at$ or $v^2 = u^2 + 2as$, and set $u = 0$ ("from rest") or $v = 0$ ("comes to rest").

MODEL WORKED EXAMPLES

EXAMPLE 1.1 Non-Calculator

A car accelerates uniformly from $u = 6\ \text{m/s}$ at $a = 2.5\ \text{m/s}^2$ for $t = 8\ \text{s}$.
Find:
(a) its final velocity v in m/s,
(b) the distance travelled s in m.

STEP-BY-STEP SOLUTION:

(a) List knowns: $u = 6$, $a = 2.5$, $t = 8$. Use $v = u + at$:

$$v = 6 + (2.5)(8) = 6 + 20 = 26\ \text{m/s}$$

(b) Use $s = ut + \frac{1}{2}at^2$:

$$s = (6)(8) + \frac{1}{2}(2.5)(8^2) = 48 + 80 = 128\ \text{m}$$

$$\text{(a) } 26\ \text{m/s}, \quad \text{(b) } 128\ \text{m}$$

EXAMPLE 1.2 Calculator

A train travelling at $28\ \text{m/s}$ brakes with a constant deceleration of $1.5\ \text{m/s}^2$ until its speed is $16\ \text{m/s}$. Find the distance travelled while braking, in metres.

STEP-BY-STEP SOLUTION:

1. Identify the known SUVAT values: $u = 28$, $v = 16$, and $a = -1.5$ (negative because the train is decelerating).

2. Since time t is not involved, substitute into $v^2 = u^2 + 2as$:

$$16^2 = 28^2 + 2(-1.5)s$$
$$256 = 784 - 3s$$
$$3s = 528 \implies s = 176\ \text{m}$$

$$176\ \text{m}$$

2 Algebraic Multi-Stage Journey Problems

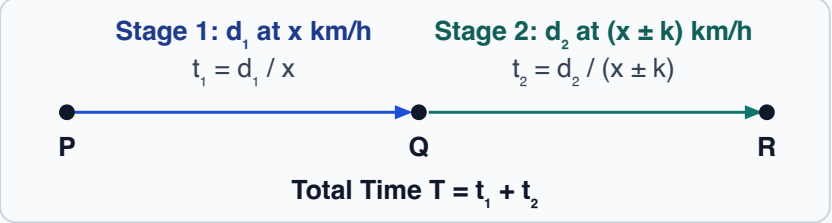
CORE CONCEPT & METHOD

In Higher GCSE multi-stage journey problems, divide the trip into clear stages and link them through **total distance** and **total time**:

- $\text{Average Speed} = \dfrac{\text{Total Distance}}{\text{Total Time}}$ *(never simply average the two stage speeds!)*.
- When stage speeds are algebraic (e.g. x km/h and $(x \pm k)$ km/h), write each stage time using $t = \dfrac{d}{s}$ and set their sum equal to the total time T:

$$\frac{d_1}{x} + \frac{d_2}{x \pm k} = T$$

- Multiply every term by $x(x \pm k)$ to clear the fractions, rearrange into $ax^2 + bx + c = 0$, and solve (rejecting any negative or invalid speed).



COMMON MISTAKE: NEVER average the two stage speeds — always use $\text{Average Speed} = \text{Total Distance} \div \text{Total Time}$, finding each stage time separately via $t = d \div s$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 Calculator

A cyclist rides $24\ \text{km}$ at a constant speed of $x\ \text{km/h}$ and then rides a further $14\ \text{km}$ at a constant speed of $(x+2)\ \text{km/h}$.
The total time for the whole $38\ \text{km}$ journey is 3 hours.
Form and solve a quadratic equation to find the value of x.

STEP-BY-STEP SOLUTION:

1. Write the time for each stage using $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$:

$$\frac{24}{x} + \frac{14}{x+2} = 3$$

2. Multiply every term by $x(x+2)$ to clear the denominators:

$$24(x+2) + 14x = 3x(x+2)$$
$$38x + 48 = 3x^2 + 6x \implies 3x^2 - 32x - 48 = 0$$

3. Factorise and solve:

$$(3x+4)(x-12) = 0$$

Since speed $x > 0$, reject $x = -\dfrac{4}{3}$, so $x = 12\ \text{km/h}$.

$$\boxed{x = 12}$$

RATIO, PROPORTION AND RATES OF CHANGE — RATES OF CHANGE

Velocity-Time Graphs

CORE CONCEPT & METHOD

On a velocity-time graph:

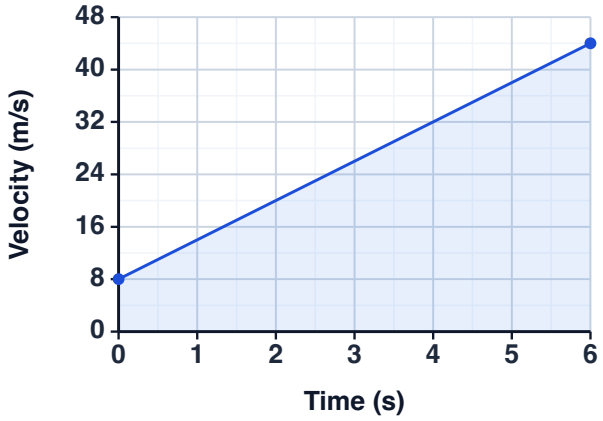
- **Acceleration** is the **GRADIENT** ($\frac{\text{change in velocity}}{\text{change in time}}$).
- **Distance travelled** is the **AREA under the graph** (split into triangles, rectangles, or trapeziums).

COMMON MISTAKE: On a velocity-time graph, Distance is the AREA under the graph (not peak velocity $\times$ total time), and read the numbered axis scales rather than counting squares.

MODEL WORKED EXAMPLES

EXAMPLE 1

Non-Calculator



A velocity-time graph shows a straight line from $(0, 8)$ to $(6, 44)$.
(a) Calculate the acceleration.
(b) Calculate the total distance travelled.

STEP-BY-STEP SOLUTION:

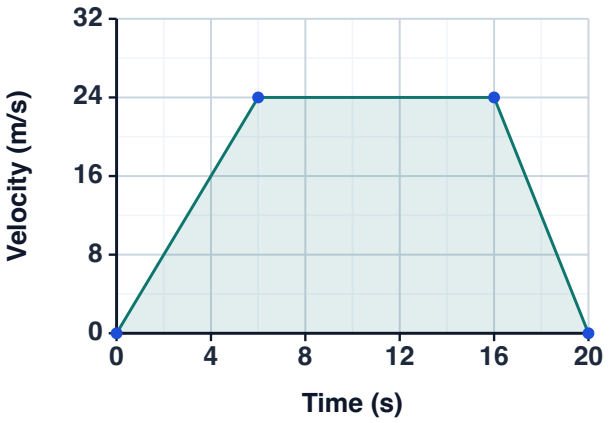
(a) Acceleration = gradient $= \frac{44 - 8}{6 - 0} = \frac{36}{6} = 6\ \text{m/s}^2$.

(b) Distance = area under graph (a trapezium with parallel vertical sides 8 and 44, width 6):

$$d = \frac{1}{2}(8 + 44) \times 6 = \frac{1}{2} \times 52 \times 6 = 156\ \text{m}$$

EXAMPLE 2

Non-Calculator



The velocity-time graph shows a 20-second journey.
(a) Find the acceleration in the first 6 seconds.
(b) Find the total distance travelled over the 20 seconds.

STEP-BY-STEP SOLUTION:

(a) Acceleration $= \dfrac{24 - 0}{6 - 0} = 4\ \text{m/s}^2$.

(b) Total distance = area of trapezium with base 20 s, top width $16 - 6 = 10$ s, and height 24 m/s:

$$d = \frac{1}{2}(20 + 10) \times 24 = 360\ \text{m}$$