

GCSE MATHEMATICS (9-1) — TUTORIALS & WORKED EXAMPLES

Grade 9 Mathematics — Tutorials & Examples

(Higher Tier)

Higher Tier Tutorials, Common Mistake Warnings & Step-by-Step Worked Examples

- Higher Tier Core Concept Notes & Precision Geometric/Graph Diagrams
- Exam-Focused "Common Mistake" Warnings on every single step
- Model Worked Examples tagged **Calculator** or **Non-Calculator**
- Streamlined Higher Revision Guide (Tutorials & Worked Examples Only)

Student Name: _____

Class / Teacher: _____

Target Grade: _____



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BreakingDownMaths.com

Interactive Step-by-Step GCSE Mathematics Mastery

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Algebraic Proof

1 Setting up Proofs

□ CORE CONCEPT & METHOD

To prove something algebraically, you must use letters instead of numbers.

- Any integer: n
- An even number: $2n$
- An odd number: $2n + 1$
- Consecutive integers: $n, n + 1, n + 2$
- Consecutive even numbers: $2n, 2n + 2, 2n + 4$

COMMON MISTAKE: Never use $2n$ and $2m$ for consecutive even integers — link both to the SAME integer n (use $2n$ and $2n + 2$ for evens, or $2n + 1$ and $2n + 3$ for consecutive odds).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

✎ Non-Calculator

Prove algebraically that the sum of any five consecutive integers is always a multiple of 5.

STEP-BY-STEP SOLUTION:

Let the five consecutive integers be $n, n + 1, n + 2, n + 3$, and $n + 4$.

$$\text{Sum} = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10$$

Factorise: $5(n + 2)$

Since it is a multiple of 5, the statement is proven.

2 Expanding to Prove

□ CORE CONCEPT & METHOD

Most algebraic proofs just require you to **expand all brackets and simplify**.

If you are asked to prove an expression is a multiple of 12, your final simplified expression will usually look like $12n$.

COMMON MISTAKE: Do not stop at $8n^2 + 8n + 4$ — to prove a multiple of k , you MUST factor k out explicitly as $4(2n^2 + 2n + 1)$ and write a brief concluding statement.

MODEL WORKED EXAMPLES

EXAMPLE 2.1 **Non-Calculator**

Prove that $(n + 3)^2 - (n - 3)^2$ is always a multiple of 12.

STEP-BY-STEP SOLUTION:

Expand both brackets:

$$(n^2 + 6n + 9) - (n^2 - 6n + 9)$$

Simplify:

$$n^2 + 6n + 9 - n^2 + 6n - 9 = 12n$$

Since $12n$ is a multiple of 12 for any integer n , the statement is proven.

Equation of a Circle

1 Centre and Radius

CORE CONCEPT & METHOD

The equation of a circle centered at the origin $(0, 0)$ is $x^2 + y^2 = r^2$, where r is the radius.

Remember to **square root** the number to find r !

COMMON MISTAKE: In $x^2 + y^2 = r^2$, the constant on the right is r^2 , NOT r — always take the positive square root to find the radius (e.g. $x^2 + y^2 = 225 \implies r = 15$).

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 **Non-Calculator**

A circle has the equation $x^2 + y^2 = 81$. Find the radius.

STEP-BY-STEP SOLUTION:

$$r^2 = 81, \text{ so } r = \sqrt{81} = 9.$$

9

2 Points on a Circle

CORE CONCEPT & METHOD

To check if a point (x, y) lies on the circle $x^2 + y^2 = r^2$, **substitute** the coordinates into the equation and check if it equals r^2 .

COMMON MISTAKE: Always wrap negative coordinates in brackets before squaring: $(-8)^2 + 15^2 = +64 + 225 = 289$, NEVER $-64 + 225$.

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 **Non-Calculator**

Show that $(6, 8)$ lies on $x^2 + y^2 = 100$.

STEP-BY-STEP SOLUTION:

$$6^2 + 8^2 = 36 + 64 = 100. \text{ This equals the radius squared, so it lies on the circle.}$$

100

3 Area and Circumference

□ CORE CONCEPT & METHOD

Once you know r^2 , you can easily find the area (πr^2)! You don't even need to square root it first.

COMMON MISTAKE: Check whether the question asks for exact AREA (πr^2 , using r^2 directly) or CIRCUMFERENCE ($2\pi r$, square-rooting r^2 first), and leave π in your answer.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

 Non-Calculator

Find the exact area of the circle $x^2 + y^2 = 49$.

STEP-BY-STEP SOLUTION:

Here, $r^2 = 49$.

Area = $\pi r^2 = 49\pi$.

$$49\pi$$

4 Translations

□ CORE CONCEPT & METHOD

If a circle centered at $(0, 0)$ is translated by a vector $\begin{pmatrix} a \\ b \end{pmatrix}$, the new centre becomes (a, b) .

COMMON MISTAKE: Translating a circle moves its CENTRE $(0, 0) \rightarrow (a, b)$ without changing its radius r , and inside $(x - a)^2 + (y - b)^2 = r^2$ the signs are OPPOSITE to (a, b) .

MODEL WORKED EXAMPLES

EXAMPLE 4.1

 Non-Calculator

A circle centered at $(0, 0)$ is translated by $\begin{pmatrix} -4 \\ 7 \end{pmatrix}$. What is its new centre?

STEP-BY-STEP SOLUTION:

The centre moves 4 units left and 7 units up.

New centre: $(-4, 7)$.

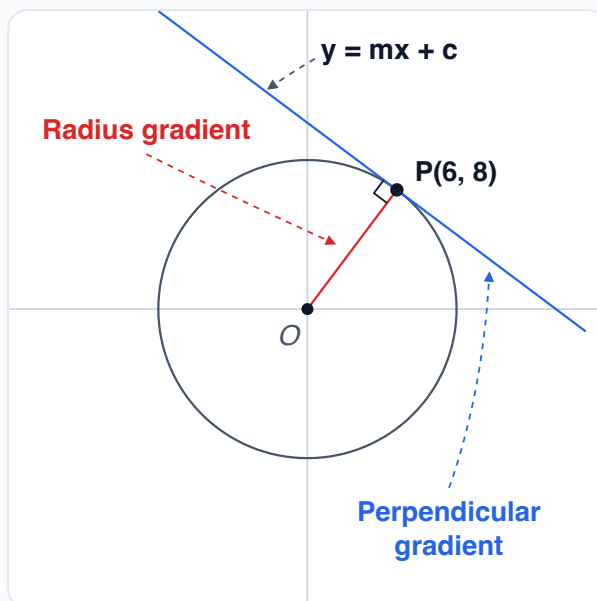
$$(-4, 7)$$

5 Tangents to Circles

□ CORE CONCEPT & METHOD

To find the **equation of a tangent**, follow 3 steps:

1. Find the gradient of the radius (from the origin to the point).
2. Find the perpendicular gradient (negative reciprocal).
3. Substitute the coordinates into $y = mx + c$ to find c !



COMMON MISTAKE: A tangent is PERPENDICULAR to the radius — after finding $m_{\text{radius}} = \frac{y_1}{x_1}$, take the negative reciprocal $m_{\text{tangent}} = -\frac{x_1}{y_1}$ before substituting (x_1, y_1) .

MODEL WORKED EXAMPLES

EXAMPLE 5.1

Non-Calculator

Find the equation of the tangent to the circle $x^2 + y^2 = 100$ at the point $(6, 8)$.

STEP-BY-STEP SOLUTION:

1. **Radius gradient:** The centre is $(0, 0)$. The gradient to $(6, 8)$ is $\frac{8}{6} = \frac{4}{3}$.2. **Tangent gradient:** The negative reciprocal is $m = -\frac{3}{4}$.3. **Find c:** Substitute $x = 6, y = 8$ into $y = -\frac{3}{4}x + c$:

$$8 = -\frac{3}{4}(6) + c$$

$$8 = -\frac{9}{2} + c$$

$$c = 8 + \frac{9}{2} = \frac{25}{2}$$

$$\text{Equation: } y = -\frac{3}{4}x + \frac{25}{2}.$$

$$y = -\frac{3}{4}x + \frac{25}{2}$$

EXAMPLE 5.2

Non-Calculator

Find the equation of the tangent to the circle $x^2 + y^2 = 289$ at the point $(-8, 15)$.

STEP-BY-STEP SOLUTION:

1. **Radius gradient:** The gradient to $(-8, 15)$ is $\frac{15}{-8} = -\frac{15}{8}$.2. **Tangent gradient:** The negative reciprocal is $m = \frac{8}{15}$.3. **Find c:** Substitute $x = -8, y = 15$ into $y = \frac{8}{15}x + c$:

$$15 = \frac{8}{15}(-8) + c$$

$$15 = -\frac{64}{15} + c$$

$$c = 15 + \frac{64}{15} = \frac{289}{15}$$

$$\text{Equation: } y = \frac{8}{15}x + \frac{289}{15}.$$

$$y = \frac{8}{15}x + \frac{289}{15}$$

Gradients & Rates of Change

1 Acceleration on Curves

□ CORE CONCEPT & METHOD

If a velocity-time or distance-time graph is curved, the rate of change is constantly changing.

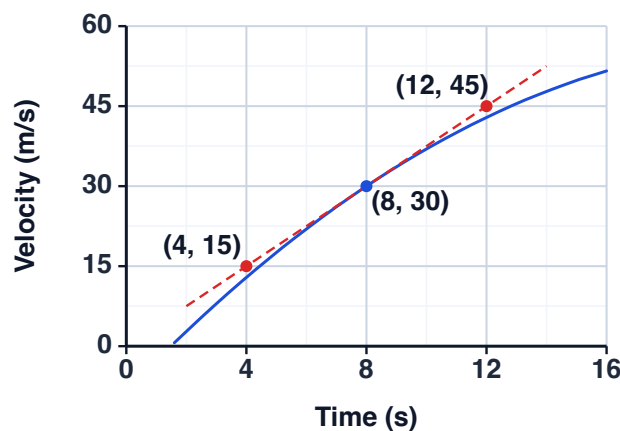
To find the **instantaneous rate of change** at a specific time t , use the **tangent line** touching the curve at that point and calculate its gradient $(\frac{y_2 - y_1}{x_2 - x_1})$.

COMMON MISTAKE: Find the gradient of the TANGENT using two widely spaced points (t_1, v_1) and (t_2, v_2) ON the tangent line — never divide $v \div t$ from a single point on the curve.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

✎ Non-Calculator



A curved velocity-time graph shows a car accelerating. A tangent is drawn at $t = 8$ s, passing through the points $(4, 15)$ and $(12, 45)$.

Find the instantaneous acceleration at $t = 8$ s.

STEP-BY-STEP SOLUTION:

The instantaneous acceleration is the **gradient of the tangent** at $t = 8$ s.

$$a = \frac{\text{change in } v}{\text{change in } t} = \frac{45 - 15}{12 - 4} = \frac{30}{8} = 3.75 \text{ m/s}^2$$

Note: Because the graph is curved, the tangent gives the exact *instantaneous* rate of change at $t = 8$ s.

2 Distance on Curves (Trapezium Rule)

▣ CORE CONCEPT & METHOD

To estimate the distance travelled under a curved velocity-time graph, we split the area into vertical strips of equal width h and treat them as **trapeziums**.

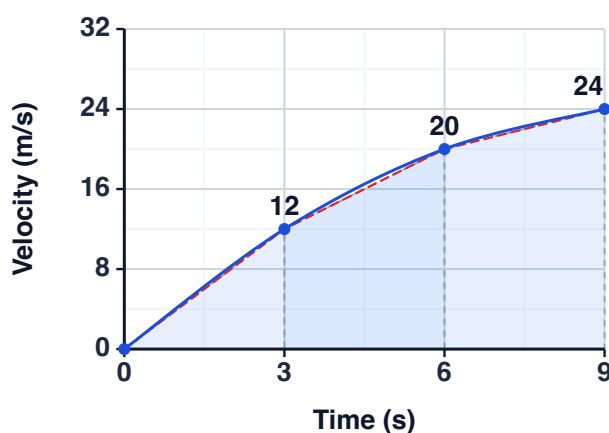
Area of each strip = $\frac{1}{2}(a + b)h$, where a and b are the vertical velocities at the left and right edges of the strip.

Overestimate or Underestimate? Compare the straight top chord of each trapezium with the curve: if the curve bends downwards (concave down, decreasing acceleration), the chords lie **below** the curve so the estimate is an **underestimate**; if the curve bends upwards (concave up, increasing acceleration), the chords lie **above** the curve so it is an **overestimate**.

COMMON MISTAKE: Count the STRIPS (gaps), not the vertical lines, to get strip width h , and justify under/overestimates geometrically ("chords lie below/above the curve").

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Calculator


The curved velocity-time graph has been split into 3 strips of equal width $h = 3$ s. The velocities at each boundary are $v_0 = 0$, $v_1 = 12$, $v_2 = 20$, and $v_3 = 24$ m/s. Use the trapezium rule to estimate the total distance travelled.

STEP-BY-STEP SOLUTION:

Area of each trapezium strip = $\frac{1}{2}(a + b) \times h$, where $h = 3$ s:

Strip 1: $\frac{1}{2}(0 + 12) \times 3 = 18$ m

Strip 2: $\frac{1}{2}(12 + 20) \times 3 = 48$ m

Strip 3: $\frac{1}{2}(20 + 24) \times 3 = 66$ m

$$\text{Total distance} \approx 18 + 48 + 66 = 132 \text{ m}$$

Vector Proofs

1 Vectors in Triangles

📖 CORE CONCEPT & METHOD

To find a position vector like $\vec{OZ}$, first find the vector for the whole line (e.g., $\vec{XY}$). Then use the given ratio to find $\vec{XZ}$, and finally use $\vec{OZ} = \vec{OX} + \vec{XZ}$.

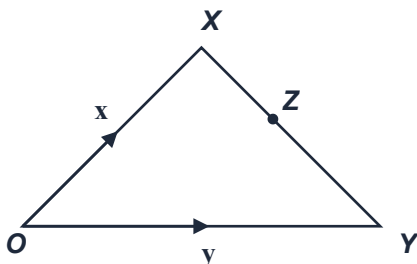
COMMON MISTAKE: Watch arrow directions: $\vec{AB} = \vec{OB} - \vec{OA} = -\vec{OA} + \vec{OB}$, and remember $AP : PB = 3 : 2$ means $AP = \frac{3}{5}AB$, NOT $\frac{3}{2}AB$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

 Non-Calculator

In triangle OXY , $\vec{OX} = \mathbf{x}$ and $\vec{OY} = \mathbf{y}$.
 Z is the point on XY such that $XZ : ZY = 3 : 4$.
 $\vec{OZ} = k(4\mathbf{x} + 3\mathbf{y})$. Find the value of k .



STEP-BY-STEP SOLUTION:

1. Find $\vec{XY}$:

$$\vec{XY} = -\mathbf{x} + \mathbf{y}$$

2. Find $\vec{XZ}$:

The ratio is $3 : 4$, so XZ is $\frac{3}{7}$ of the line.

$$\vec{XZ} = \frac{3}{7}(-\mathbf{x} + \mathbf{y}) = -\frac{3}{7}\mathbf{x} + \frac{3}{7}\mathbf{y}$$

3. Find $\vec{OZ}$:

$$\vec{OZ} = \vec{OX} + \vec{XZ}$$

$$\vec{OZ} = \mathbf{x} - \frac{3}{7}\mathbf{x} + \frac{3}{7}\mathbf{y} = \frac{4}{7}\mathbf{x} + \frac{3}{7}\mathbf{y}$$

4. Find k :

Factorise out $\frac{1}{7}$:

$$\vec{OZ} = \frac{1}{7}(4\mathbf{x} + 3\mathbf{y})$$

$$\text{So, } k = \frac{1}{7}.$$

$$\frac{1}{7}$$

2 Proving Collinearity

□ CORE CONCEPT & METHOD

To prove three points X , Y and Z are on the same straight line (collinear), you must prove that the vectors $\vec{XY}$ and $\vec{XZ}$ (or $\vec{YZ}$) are parallel. You do this by showing one is a **scalar multiple** of the other, e.g. $\vec{XZ} = 2\vec{XY}$.

COMMON MISTAKE: Showing $\vec{CD} = 4\vec{CM}$ only proves the vectors are PARALLEL — you MUST also write "and they share the common point C , so C, M, D are collinear".

MODEL WORKED EXAMPLES

EXAMPLE 2.1

 Non-Calculator

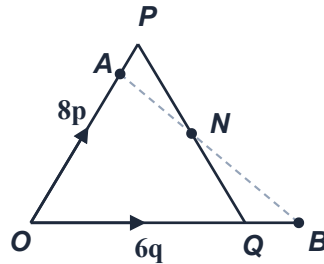
In the diagram, $\vec{OP} = 8\mathbf{p}$ and $\vec{OQ} = 6\mathbf{q}$.

A is the point on OP such that $OA : AP = 3 : 1$.

N is the midpoint of PQ .

B is the point on OQ extended such that $OQ : QB = 2 : 1$.

Show that A , N and B are on the same straight line.



STEP-BY-STEP SOLUTION:

1. Find $\vec{AN}$:

$$\vec{PQ} = -8\mathbf{p} + 6\mathbf{q}$$

$$\vec{PN} = \frac{1}{2}(-8\mathbf{p} + 6\mathbf{q}) = -4\mathbf{p} + 3\mathbf{q}$$

$$\vec{AP} = 2\mathbf{p} \text{ (since ratio is } 3 : 1, OP = 8\mathbf{p}\text{)}$$

$$\vec{AN} = \vec{AP} + \vec{PN} = 2\mathbf{p} - 4\mathbf{p} + 3\mathbf{q} = -2\mathbf{p} + 3\mathbf{q}$$

2. Find $\vec{AB}$:

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$\vec{AO} = -6\mathbf{p}$$

$$OB \text{ is } \frac{3}{2} \text{ of } OQ \text{ (ratio } 2 : 1\text{), so } \vec{OB} = 9\mathbf{q}$$

$$\vec{AB} = -6\mathbf{p} + 9\mathbf{q}$$

3. Compare vectors:

$$\vec{AN} = -2\mathbf{p} + 3\mathbf{q}$$

$$\vec{AB} = 3(-2\mathbf{p} + 3\mathbf{q})$$

Since $\vec{AB}$ is a scalar multiple of $\vec{AN}$ (specifically, $\vec{AB} = 3\vec{AN}$) and they share the common point A , they must lie on the same straight line.

Proven

3 Parallelogram Proofs

□ CORE CONCEPT & METHOD

In a parallelogram, opposite sides are equal and parallel. So if $\vec{OP} = \mathbf{p}$ and $\vec{OQ} = \mathbf{q}$, then $\vec{QR} = \mathbf{p}$ and $\vec{PR} = \mathbf{q}$. Use these parallel pathways to find your vectors!

COMMON MISTAKE: Check vertex order around the parallelogram carefully: in $OACB$, $\vec{OC} = \vec{OA} + \vec{OB}$ is the diagonal, whereas in $OABC$, $\vec{OB}$ is the diagonal.

MODEL WORKED EXAMPLES

EXAMPLE 3.1

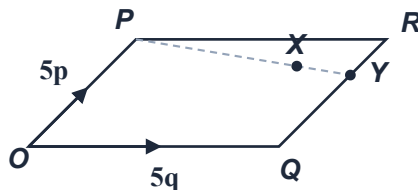
 Non-Calculator

The diagram shows a parallelogram $OPRQ$. $\vec{OP} = 5\mathbf{p}$ and $\vec{OQ} = 5\mathbf{q}$.

X is the point on OR such that $OX : XR = 4 : 1$.

Y is the point on QR such that $QY : YR = 3 : 1$.

Show that P , X and Y are on the same straight line.



STEP-BY-STEP SOLUTION:

1. Find $\vec{PX}$:

$$\vec{OR} = \vec{OP} + \vec{PR} = 5\mathbf{p} + 5\mathbf{q}$$

$$\vec{OX} = \frac{4}{5}\vec{OR} = 4\mathbf{p} + 4\mathbf{q}$$

$$\vec{PX} = \vec{PO} + \vec{OX} = -5\mathbf{p} + 4\mathbf{p} + 4\mathbf{q} = -\mathbf{p} + 4\mathbf{q}$$

2. Find $\vec{PY}$:

Y is on QR ($QY : YR = 3 : 1$). $\vec{QR} = \vec{OP} = 5\mathbf{p}$.

$$\vec{QY} = \frac{3}{4}\vec{QR} = \frac{15}{4}\mathbf{p}$$

$$\vec{PY} = \vec{PQ} + \vec{QY} = (-5\mathbf{p} + 5\mathbf{q}) + \frac{15}{4}\mathbf{p}$$

$$\vec{PY} = -\frac{5}{4}\mathbf{p} + 5\mathbf{q}$$

3. Compare:

$$\vec{PX} = 1(-\mathbf{p} + 4\mathbf{q})$$

$$\vec{PY} = \frac{5}{4}(-\mathbf{p} + 4\mathbf{q})$$

Since $\vec{PY} = \frac{5}{4}\vec{PX}$ and they share point P , they are collinear.

Proven

4 Finding Unknown Extensions

CORE CONCEPT & METHOD

If you are told points are collinear, you can set an unknown line extension (like $\vec{OC}$) to be a multiple of its direction vector, e.g. $\vec{OC} = k\mathbf{y}$. Then equate the scalar multiples of $\mathbf{x}$ and $\mathbf{y}$ to solve for k !

COMMON MISTAKE: Never try to divide vectors — write the collinear vector in TWO independent ways using scalars (e.g. k and μ), then equate coefficients of $\mathbf{a}$ and $\mathbf{b}$ separately.

MODEL WORKED EXAMPLES

EXAMPLE 4.1

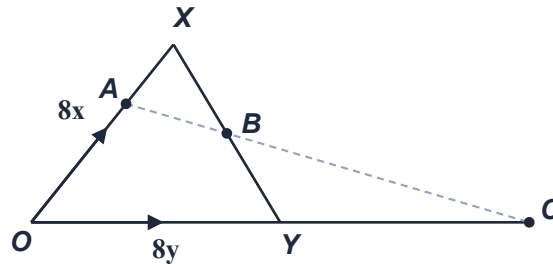
Non-Calculator

In the diagram, $\vec{OX} = 8\mathbf{x}$ and $\vec{OY} = 8\mathbf{y}$.

A is on OX such that $OA : AX = 3 : 1$.

B is the midpoint of XY .

The line OY is extended to point C . Given that A, B and C are on the same straight line, find $\vec{YC}$.



STEP-BY-STEP SOLUTION:

1. Find $\vec{AB}$:

$$\vec{XY} = -8\mathbf{x} + 8\mathbf{y}$$

$$\vec{XB} = \frac{1}{2}\vec{XY} = -4\mathbf{x} + 4\mathbf{y}$$

$$\vec{AX} = 2\mathbf{x}$$

$$\vec{AB} = \vec{AX} + \vec{XB} = 2\mathbf{x} - 4\mathbf{x} + 4\mathbf{y} = -2\mathbf{x} + 4\mathbf{y}$$

2. Express $\vec{AC}$:

$$\text{Let } \vec{OC} = k\mathbf{y}.$$

$$\vec{AC} = \vec{AO} + \vec{OC} = -6\mathbf{x} + k\mathbf{y}$$

3. Solve for k :

Since A, B, C are collinear, $\vec{AC}$ is a multiple of $\vec{AB}$.

$$\vec{AB} = -2\mathbf{x} + 4\mathbf{y}$$

$$\vec{AC} = -6\mathbf{x} + k\mathbf{y}$$

The x-component is multiplied by 3 (from -2 to -6).

So the y-component must also be multiplied by 3.

$$k = 4 \times 3 = 12.$$

$$\text{So } \vec{OC} = 12\mathbf{y}.$$

$$\vec{YC} = \vec{YO} + \vec{OC} = -8\mathbf{y} + 12\mathbf{y} = 4\mathbf{y}.$$

$$4\mathbf{y}$$

Probability Equations

1 Equations from Ratios

CORE CONCEPT & METHOD

If you are given a ratio like 2:1, use algebra! Let the number of marbles be $2x$ and x . The total is $3x$.

Then, set up your standard 'without replacement' probability fractions and solve the equation for x .

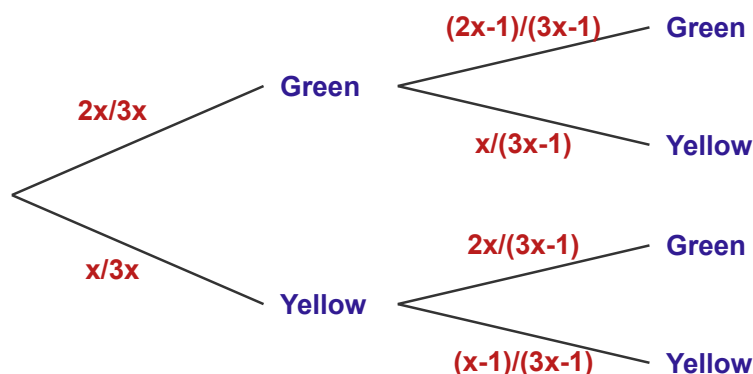
COMMON MISTAKE: When picking WITHOUT replacement, reduce BOTH the numerator (if same colour) AND the total denominator by 1 on the second pick — e.g. $\frac{2x}{5x} \times \frac{2x-1}{5x-1}$.

MODEL WORKED EXAMPLES

EXAMPLE 1.1

Non-Calculator

A box contains green and yellow marbles in the ratio 2:1. Two marbles are taken **without replacement**. The probability of picking two green marbles is $\frac{2}{5}$. How many marbles are in the box in total?



STEP-BY-STEP SOLUTION:

Step 1: Set up variables. Let the number of green marbles be $2x$ and yellow be x . Total = $3x$.

Step 2: Write the probability equation.

$$\frac{2x}{3x} \times \frac{2x-1}{3x-1} = \frac{2}{5}$$

Simplify $\frac{2x}{3x} = \frac{2}{3}$:

$$\frac{2}{3} \times \frac{2x-1}{3x-1} = \frac{2}{5}$$

Step 3: Cross-multiply and solve.

$$\begin{aligned} 2(2x-1) \times 5 &= 2 \times 3(3x-1) \\ 10(2x-1) &= 6(3x-1) \\ 20x-10 &= 18x-6 \\ 2x &= 4 \implies x = 2 \end{aligned}$$

Total marbles = $3x = 6$.

2 Equations With Replacement

CORE CONCEPT & METHOD

If the item is **replaced**, the fractions are identical. For example, if there are 4 purple and y white beads, the probability of picking two purples is:

$$\frac{4}{4+y} \times \frac{4}{4+y}$$

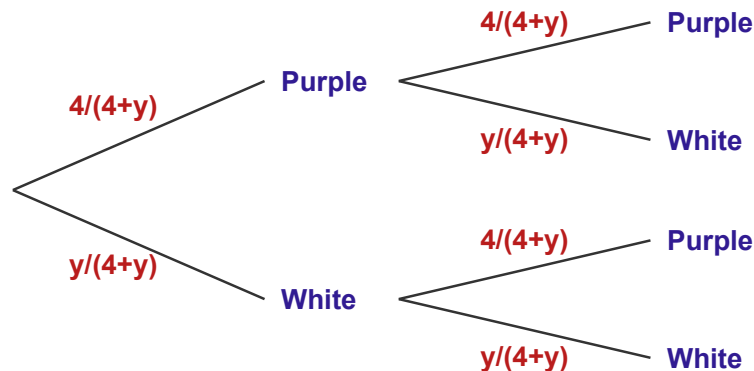
COMMON MISTAKE: For "one of each colour" (e.g. one red and one blue), remember to add BOTH orders (RB + BR) unless the question specifies "first red, then blue".

MODEL WORKED EXAMPLES

EXAMPLE 2.1

Non-Calculator

A jar contains 4 purple and y white beads. A bead is picked, its colour noted, and then **replaced**. This is done twice. The probability of picking purple both times is $\frac{16}{81}$. Find y .



STEP-BY-STEP SOLUTION:

Because the bead is replaced, the total stays the same for both picks.

The probability of picking purple twice is:

$$\frac{4}{4+y} \times \frac{4}{4+y} = \frac{16}{81}$$

So:

$$\left(\frac{4}{4+y} \right)^2 = \frac{16}{81}$$

Take the square root of both sides:

$$\frac{4}{4+y} = \frac{4}{9}$$

Therefore $4 + y = 9$, giving $y = 5$.

3 Quadratic Probability Equations

□ CORE CONCEPT & METHOD

For 'without replacement' problems with an unknown total n , you will often need to expand brackets and form a quadratic equation (e.g., $n^2 - 13n - 30 = 0$), then factorise it to find n .

COMMON MISTAKE: Cross-multiply carefully, rearrange to $an^2 + bn + c = 0$ before factorising, and always reject negative or non-integer roots since a count n must be a positive integer.

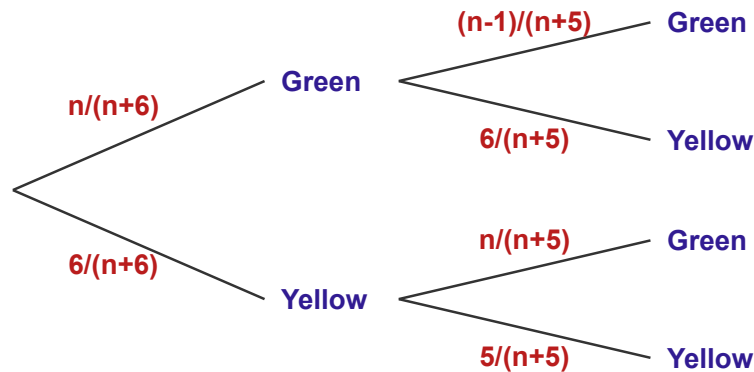
MODEL WORKED EXAMPLES

EXAMPLE 3.1



A box contains n green counters and 6 yellow counters. Two counters are drawn at random **without replacement**.

The probability that both counters are green is $\frac{1}{2}$. Show that $n^2 - 13n - 30 = 0$, and hence find the value of n .



STEP-BY-STEP SOLUTION:

First draw: Total counters = $n + 6$.

$$P(\text{Green}_1) = \frac{n}{n+6}$$

Second draw: One green counter is removed, so $n - 1$ greens remain out of a total of $n + 5$.

$$P(\text{Green}_2) = \frac{n-1}{n+5}$$

Equation:

$$\begin{aligned} \frac{n}{n+6} \times \frac{n-1}{n+5} &= \frac{1}{2} \\ \frac{n(n-1)}{(n+6)(n+5)} &= \frac{1}{2} \\ 2(n^2 - n) &= (n+6)(n+5) \end{aligned}$$

$$2n^2 - 2n = n^2 + 11n + 30$$

$$n^2 - 13n - 30 = 0$$

Factorise and solve:

$$(n-15)(n+2) = 0$$

Since $n > 0$, $n = 15$.